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معيار استقرار جديد معتمد على التأخير لنظام خطي ذو تأخير متغير مع الوقت باستخدام خاصية ويرتينجر لعدم المساواة

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الخلاصة

تم اقتراح معيار الاستقرار جديد يعتمد على تأخير متغير مع الوقت وعلى دم المساواة حسب خاصية Wirtinger للنظم الخطية ذات وقت تأخير متغير. وباستخدام خاصية التفاوت ويرتينجر واختيار مناسب لخاصية ليابوناف وكارسوفسكي، استمدت النتيجة المقترحة من حيث عدم مساواة المصفوفات الخطية (LMIs). تطبيق معيار الاستقرار المقترح يوفر أقل تقدير متحفظ للاستقرار من حيث تحسين حدود تأخير العليا المسموح بها. وقد تم التحقق من الفعالية والخفض المحافظ على المعيار المقترح وأثبتت النتائج أنها متفوقة على النتائج الحالية.

New delay dependent stability criterion for linear system with time varying delay using Wirtinger's inequality

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ABSTRACT

A new delay dependent stability criterion has been proposed based on Wirtinger's inequality, for linear systems incorporating time varying delays. By use of Wirtinger's inequality and suitably chosen Lyapunov-Krasovskii functional, the proposed result has been derived in terms of Linear Matrix Inequalities (LMIs). The application of the proposed stability criterion provides less conservative estimate of system stability in terms of improved allowable upper delay bounds. The efficacy and reduced conservatism of the proposed criterion has been investigated and has been shown superior to existing results.

Keywords: Linear matrix inequalities (LMIs); linear systems; stability; time-varying delay; Wirtinger's inequality.

INTRODUCTION

Delays, irrespective of how significant they are, affect almost all types of physical systems e.g. transportation systems, networked systems, chemical processes, economic systems, financial systems etc. They have since long known for their negative impact on system performance as well as stability, due to introduction of undesirable phase lag and infinite dimensions. Although insignificantly small delays are usually neglected, when present in significant amount, their non-inclusion may lead to inaccurate judgment of system stability and may drastically affect subsequent control design. Due to the impact of appreciable delays on system behavior, the problem of stability assessment in the face of wide class of time delays has received considerable attention in the past decade.

Stability analysis of time delay systems for various types of delays have been extensively studied in several monographs like Wu *et al.* (2010), Fridman (2014), Bhatia & Szeg (1970) and La Salle (1976). Several researchers like Gu *et al*. (2003), Richard (2003) and Mohmoud (2000) have addressed the effects of delays over performance and stability. Stability of linear time delay systems incorporating constant delays can be analyzed in both frequency domain as well as time domain. The frequency domain methods involve distribution of roots of characteristic equation, and are in general applicable to linear systems with constant delays (Mori & Kokame, 1989). In contrast, time domain methods involving construction of an appropriate Lyapunov-Krasovskii functional provide assessment of system stability for a broader class of time delay systems.

The stability assessment results can be broadly classified into two: delay independent and delay dependent ones. The former ones do not explicitly demand candidature of delay terms, whereas the latter ones utilize the information about the delays involved and are therefore less conservative compared to the former ones. Majority of time domain delay dependent stability results utilize the derivative of Lyapunov- Krasovskii functional with certain inequalities and free weighting matrix expressing relationships among different terms in Newton-Leibnitz formula. Fridman & Shaked (2002) proposed a delay dependent stability criterion using Park's and Moon's inequalities. An improved delay dependent stability criterion based on Jensen's inequality, for systems with time varying delays was proposed by Wu *et al*. (2004). Stability of systems with time varying delay was also considered via ignoring certain terms in simplified derivative of Lyapunov-Krasovskii functional, resulting in considerably reduced conservativeness by Wu *et al*. (2004), He *et al*. (2006), Han (2004), Jiang & Han (2006), Han & Yue (2007), Shao (2008). The delay dependent stability problem has been studied through many variations and transformation for different types of delay systems e.g. Han (2004) investigated stability condition for uncertain neutral system with discrete delay using model transformation and Park's inequality. Suplin *et al*. (2006) proposed delay dependent stability conditions based on the augmented Lyapunov-Krasovskii's functional, using Finsler's lemma. By using the idea of introducing free- weighting matrices, which brings flexibility in solving LMI's a parameter dependent technique is proposed by Lin *et al*. (2006). Similar results were obtained by He *et al*. (2007) without ignoring some cross product terms, while estimating the upper bound of the derivative of the Krasovskii functional considered. Later, some improvement in results were reported by Park & Ko (2007), Ko & Park(2011) by using not only time varying delayed state, but also in the delay in upper bound state, to exploit all the possible information for the relationship among the current state and an exactly delayed state and designing integral inequality technique. State augmentation technique was used to enhance the stability radius of linear time delay system by Arabia & Gouaisbaut (2009). Further, Arabia, *et al.* (2010) using Cauchy-Schwartz inequality derived less conservative results for time varying delays. Delay decomposition approach was exploited by Zhu & Yang (2010) to derive new results for such systems. A delay range dependent stability criterion using free weighting matrices and appropriate Lyapunov -Krasovskii functional was proposed by Yan *et al*. (2010). By choosing different Lyapunov matrices in the decomposed integral intervals and estimating the upper bounds of cross terms, less conservative results were reported. A note on the stability of linear system with time varying delay has been given by Kim (2011). This note presents the stability condition derivation without using delay decomposition method. By considering a simple Lyapunov-Krasovskii functional and by multiplying the cross terms and quadratic terms with a higher degree scalar function results, in terms of LMIs has been proposed. Zang *et.al* (2013) reported improvement in maximum allowable upper delay, bound by using delay partition approach and compared with the existing results.The stability results for neutral systems have been studied using decomposition approach by Chao *et al.* (2014).

Improvement in conservatism of stability assessment for delay systems have been achieved using another well-known inequality in Fourier analysis; Wirtinger's inequality. Seuret & Gouaisbaut (2013) proposed the applicability of the Wirtinger's inequality, instead of using Jensen's inequality, for studying system stability problems. However, Jensen's inequality introduces an undesirable conservativeness in the stability conditions. By considering a new Lyapunov function and Wirtinger's inequality, stability conditions for sampled-data system were proved by Liu & Fridman (2012). The main advantage of Wirtinger's inequality lies in its dependence on Fourier theory, that helps reduce the gap of undesirable conservativeness in the stability conditions. The inequality derives advantage from sawtooth evolution of the delays introduced, due to sample and hold. Wirtinger's integral inequality is used in computing the derivative of the considered Lyapunov-Krasovskii functional. In this paper, a less conservative stability criterion has been derived in terms of linear matrix inequalities (LMIs), using an appropriate Lyapunov-Krasovskii functional and Wirtinger's inequality for adjudging stability of a given linear system with time varying delay.

The remaining paper is organized as follows: Section 2 describes preliminaries and the problem formulation, in which the representation of the system and conditions of delay are defined. In section 3, the proposed asymptotic stability criteria of time delay system is presented using an appropriate Lyapunov functional and Wirtinger's inequality. The simulation results of proposed and existing criteria are discussed in Section 4. Section 5 concludes the paper.

Notations

$\Re^n$	: n-dimensional Euclidean space with vector norm $\lVert . \rVert$
$\Re^{n\times m}$	: set of $n\times m$ real matrices
$P>0$, $P\in\Re^{n\times n}$	: P is a symmetric positive definite matrix of size $n\times n$.

$\begin{bmatrix} A & B \\ * & C \end{bmatrix}$: implies symmetric matrix as $\begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$.

$\|z\|$: For any differentiable $z:[a,b]\to\mathbb{R}^n$ with norm $\|z\|_2^2 = \frac{1}{b-a}\int_a^b z^T(u)z(u)du$

PRELIMINARIES AND PROBLEM FORMULATION

Consider a class of linear state delayed systems with time varying delay as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-d(t)) \qquad t \geq 0 \\ x(t) &= \varphi(t), \qquad t \in [-\tau, 0] \end{aligned} \tag{1}$$

where $x(t)\in R^n$ represents system state vector, the $A\in\Re^{n\times n}$ and $A_d\in\Re^{n\times n}$ are known constant system matrices representing delay-free and state-delayed dynamics, respectively. $d(t)=h$ is a bounded and real valued time varying continuous function lying given by

$$0 \leq d(t) \leq \tau \tag{2}$$

which is assumed to be rate bounded as

$$\begin{aligned} \dot{d}(t) &\leq \mu \leq 1 \\ \ddot{d}(t) &\leq \gamma \leq 1 \end{aligned} \tag{3}$$

Here τ, μ and γ are real finite positive constants. $\varphi(t)$ is a continuous vector valued initial function on $t\in[-\tau,0]$ and is known a-priori.

The following well established lemmas serve key role to derive the proposed stability criterion, and are stated below:

Lemma 1. Boyd *et.al* (1994) (*Schur complement*)

For a given symmetric matrix $S = S^T = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix}$, where $S_{11}\in\Re^{r\times r}$, the following conditions are true

- $S < 0;$
- $S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0;$
- $S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0;$

Lemma 2. Wu *et al.* (2004) (*Jensen inequality*): For a given matrix $R>0$ and for any differentiable signal ω in $[a,b]\to\Re^n$, the following inequality holds:

$$\int_a^b \omega^T(u) R\,\dot{\omega}(u)du \geq \frac{(\omega(b)-\omega(a))^T R(\omega(b)-\omega(a))}{b-a}.$$

Lemma 3. Seuret & Gouaisbaut (2013) (*Wirtinger's inequality*):

Let z a differentiable function such that $z(a) = z(b)$ and

$z(a\int_a^b z(u)du = 0) = z(b)$, then for any $n \times n$ matrix $R > 0$, the following holds

$$\int_a^b \dot{z}^T(u)R\dot{z}(u)du \geq \frac{4\pi^2}{(b-a)^2}\int_a^b z^T(u)Rz(u)du.$$

MAIN RESULTS

In this section, we derive delay-range and delay-rate based stability conditions for the considered class of system in Equation (1) for a-priori known rate of variation of time delay interval, as well as arbitrary rate of time delay interval. The proposed result theoretically computes the upper bound for the admissible time varying delay that can be tolerated without compromising overall system stability.The derived criterion uses standard lemmas and Wirtinger's inequality. The criterion for the unknown rate of variation of delay interval appears as natural corollary to the problem. The result below proposes stability condition for system in Equation (1) for a-priori known rate of variation of delay interval.

Theorem 1: Given scalars $\tau > 0, \mu, \gamma$, the linear system Equation (1) with time varying delay satisfying Equation (2) is asymptotically stable, if there exist matrices $P = P^T > 0, R = R^T > 0, Z = Z^T > 0$ such that the following LMI holds:

$$\begin{bmatrix} \Pi & * & * \\ A_d{}^T P + \tau A_d{}^T ZA & -R(1-\mu) + \tau A_d{}^T ZA_d & * \\ 0 & 0 & -\frac{4\pi^2}{\tau^2}Z(1-\gamma) \end{bmatrix} < 0$$

Proof:

Consider a Lyapunov- Krasovskii functional as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{4}$$

where

$$V_1(t) = x^T(t)Px(t)$$

$$V_2(t) = \int_{t-h}^{t} x^T(\alpha)Rx(\alpha)d\alpha$$

$$V_3(t) = \int_{-\tau}^{0}\int_{t+\beta}^{t} \dot{x}^T(\alpha)Z\,\dot{x}(\alpha)d\alpha d\beta$$

The derivative of the Lyapunov-Krasovskii functional Equation (4) is given as

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) \tag{5}$$

$$\begin{aligned} \dot{V}_1(t) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \\ &= x^T(t)A^T Px(t) + x^T(t-h)A_d{}^T Px(t) + x^T(t)PAx(t) + x^T(t)PA_d x(t-h) \end{aligned} \tag{6}$$

Following Wu *et al.* (2010),

$$\dot{V}_2(t) = x^T(t)Rx(t) - (1-\mu)x^T(t-h)Rx(t-h) \tag{7}$$

$$\dot{V}_3(t) = \int_{-\tau}^{0} \dot{x}^T(t)Z\,\dot{x}(t)d\beta - \int_{-\tau}^{0} \dot{x}^T(t+\beta)Z\,\dot{x}(t+\beta)d\beta \tag{8}$$

Using, $s = t + \beta$ we get

$$\dot{V}_3 = \int_{-\tau}^{0} \dot{x}^T(t)Z\,\dot{x}(t)d\beta - \int_{t-\tau}^{t} \dot{x}^T(s)Z\,\dot{x}(s)ds \tag{9}$$

Using Newton-Leibnitz formula, the first term in Equation (9) becomes

$$\dot{V}_3 = \tau\dot{x}^T(t)Z\dot{x}(t) - \int_{t-\tau}^{t} \dot{x}^T(s)Z\dot{x}(s)ds \tag{10}$$

From the following form of Wirtinger's inequality

$$\int_a^b \dot{z}^T(u)R\dot{z}(u)du \geq \frac{4\pi^2}{(b-a)^2}\int_a^b z^T(u)Rz(u)du \tag{11}$$

The Equation (10) becomes,

$$\begin{aligned} \dot{V}_3 &= \tau\dot{x}^T(t)\,\mathrm{Z}\dot{x}(t) - \frac{4\pi^2}{\tau^2}\int_{t-\tau}^{t} x^T(s)\,\mathrm{Z}x(s)ds \\ &= \tau\dot{x}^T(t)\,\mathrm{Z}\dot{x}(t) - \frac{4\pi^2}{\tau^2}[x^T(t)\,\mathrm{Z}x(t) - x^T(t-\tau)\,\mathrm{Z}x(t-\tau)(1-\gamma)] \end{aligned} \tag{12}$$

Using Equation (1) in Equation (12) results,

$$\begin{aligned} \dot{V}_3 &= \tau[Ax(t) + A_d x(t-h)]^T\,\mathrm{Z}[Ax(t) + A_d x(t-h)] \\ &\quad - \frac{4\pi^2}{\tau^2}[x^T(t)\mathrm{Z}x(t) - x^T(t-\tau)\,\mathrm{Z}x(t-\tau)(1-\gamma)] \\ &= \tau[x^T(t)A^T + x^T(t-h)A_d{}^T]\mathrm{Z}[Ax(t) + A_d x(t-h)] \\ &\quad - \frac{4\pi^2}{\tau^2}x^T(t)\mathrm{Z}x(t) - \frac{4\pi^2}{\tau^2}x^T(t-\tau)\,\mathrm{Z}x(t-\tau)(1-\gamma) \end{aligned} \tag{13}$$

$$\begin{aligned}\dot{V}_3 &= \tau x^T A^T ZAx + x^T \tau A^T ZA_d x(t-h) \\ &+ x^T(t-h)\tau A_d{}^T ZAx + x^T(t-h)\tau A_d{}^T ZA_d x(t-h) \\ &- \frac{4\pi^2}{\tau^2} x^T Zx - \frac{4\pi^2}{\tau^2} x^T(t-\tau) Z x(t-\tau)(1-\gamma)\end{aligned} \tag{14}$$

Substituting Equations (6), (7) and (14) in Equation (5), we get

$$\begin{aligned}\dot{V} &= x^T(t)A^T Px(t) + x^T(t-h)A_d{}^T Px(t) + x^T(t)PAx(t) \\ &+ x^T(t)PA_d x(t-h) + x^T(t)Rx(t) - x^T(t-h)Rx(t-h)(1-\mu) \\ &+ \tau x^T(t)A^T ZAx(t) + x^T \tau A^T ZA_d x(t-h) + x^T(t-h)\tau A_d{}^T ZAx(t) \\ &+ x^T(t-h)\tau A_d{}^T ZA_d x(t-h) - \frac{4\pi^2}{\tau^2} x^T(t)Zx(t) \\ &- \frac{4\pi^2}{\tau^2} x^T(t-\tau) Z x(t-\tau)(1-\gamma)\end{aligned} \tag{15}$$

Rearranging the terms,

$$\begin{aligned}\dot{V} &= x^T[A^T P + PA + R + \tau A^T ZA \\ &- \frac{4\pi^2}{\tau^2} Z]x + x^T(t-h)[A_d{}^T P + \tau A_d{}^T ZA]x \\ &+ x^T[PA_d + \tau A^T ZA_d]x(t-h) \\ &+ x^T(t-h)[-R(1-\mu) + \tau A_d{}^T ZA_d]x(t-h) \\ &+ x^T(t-\tau)[-\frac{4\pi^2}{\tau^2} Z(1-\gamma)]x(t-\tau)\end{aligned} \tag{16}$$

Equation (16) can be rearranged in form $\dot{V}(t) = \zeta^T(t)\Phi\zeta(t)$

$$\dot{V}(t) = [x^T \quad x^T(t-h) \quad x^T(t-\tau)] \begin{bmatrix} \Pi & * & * \\ A_d{}^T P + \tau A_d{}^T ZA & -R(1-\mu) + \tau A_d{}^T ZA_d & 0 \\ 0 & 0 & -\frac{4\pi^2}{\tau^2} Z(1-\gamma) \end{bmatrix} \begin{bmatrix} x \\ x(t-h) \\ x(t-\tau) \end{bmatrix} \tag{17}$$

where $\zeta^T(t) = [x^T(t) \;\; x^T(t-h) \;\; x^T(t-\tau)]$, and

$$\Phi = \begin{bmatrix} \Pi & * & * \\ A_d{}^T P + \tau A_d{}^T ZA & -R(1-\mu) + \tau A_d{}^T ZA_d & 0 \\ 0 & 0 & -\frac{4\pi^2}{\tau^2} Z(1-\gamma) \end{bmatrix}$$

with

$$\Pi = A^T P + PA + R + \tau A^T ZA - \frac{4\pi^2}{\tau^2} Z$$

The system in Equation (1) is therefore asymptotically stable, if and only if $\dot{V}(t) < 0$, which implies $\Phi < 0$.

Corollary1. For given scalars $\tau > 0$, μ and γ, the linear time delay system (1) subject to (2) with no information about the time derivative of the delay, is asymptotically stable, if there exist symmetric positive definite matrices P, R and Z such that the following LMI holds:

$$\begin{bmatrix} \Pi & * & * \\ A_d^{\ T} P + \tau A_d^{\ T} ZA & -R + \tau A_d^{\ T} ZA_d & 0 \\ 0 & 0 & -\frac{4\pi^2}{\tau^2} Z \end{bmatrix} < 0$$

Proof:

For the given system in Equation (1), consider a Lyapunov-Krasovskii functional as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{18}$$

where

$$V_1(t) = x^T(t) Px(t)$$

$$V_2(t) = \int_{t-h}^{t} x^T(\alpha) Rx(\alpha) d\alpha$$

$$V_3(t) = \int_{-\tau}^{0} \int_{t+\beta}^{t} \dot{x}^T(\alpha) Z \dot{x}(\alpha) d\alpha d\beta$$

Taking derivative of the Lyapunov-Krasovskii functional and rearranging the terms results Equation (17)

$$\dot{V}(t) = [x^T \quad x^T(t-h) \quad x^T(t-\tau)] \begin{bmatrix} \Pi & * & * \\ A_d^{\ T} P + \tau A_d^{\ T} ZA & -R(1-\mu) + \tau A_d^{\ T} ZA_d & 0 \\ 0 & 0 & -\frac{4\pi^2}{\tau^2} Z(1-\gamma) \end{bmatrix} \begin{bmatrix} x \\ x(t-h) \\ x(t-\tau) \end{bmatrix} \tag{19}$$

Following Kim (2011), the terms may be treated as $\mu = \gamma = 0$ due to non-availability of delay derivative. It should be noted that $\mu = 0$ does not mean a constant delay (i.e., $d(t) = 0$, $\forall t \geq 0$), but it means a time-varying delay with $d(t) \leq 0$, $\forall t \geq 0$. Therefore,

$$\dot{V}(t) = \zeta^T(t)\Phi\zeta(t)$$

where

$$\zeta^T(t) = [x^T(t) \;\; x^T(t-h) \;\; x^T(t-\tau)]$$

and

$$\Phi = \begin{bmatrix} \Pi & * & * \\ A_d^T P + \tau A_d^T Z A & -R + \tau A_d^T Z A_d & 0 \\ 0 & 0 & -\frac{4\pi^2}{\tau^2} Z \end{bmatrix}$$

with

$$\Pi = A^T P + PA + R + \tau A^T Z A - \frac{4\pi^2}{\tau^2} Z$$

Therefore, the system in Equation (1) subject to Equation (2) with unknown rate of variation of delay interval, is asymptotically stable, if the following holds

$$\Phi < 0 \tag{20}$$

NUMERICAL EXAMPLES

The results proposed in this paper are applicable to any linear system with time varying state delays. To illustrate reduced conservativeness of the proposed results and compare the allowable delay bounds, we consider a set of examples as in He *et al.* (2007) and Fridman (2014).

Example 1: Consider system (1) with nominal and state delayed-dynamic matrices as

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \qquad A_d = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}$$

The simulations for given system, with various a-priori values of rate of variation in time delay intervals, were carried out using the YALMIP LMI solver package **(**Lofberg, 2004) on the Matlab platform. The state trajectories of the considered system are as shown in Figure 1.

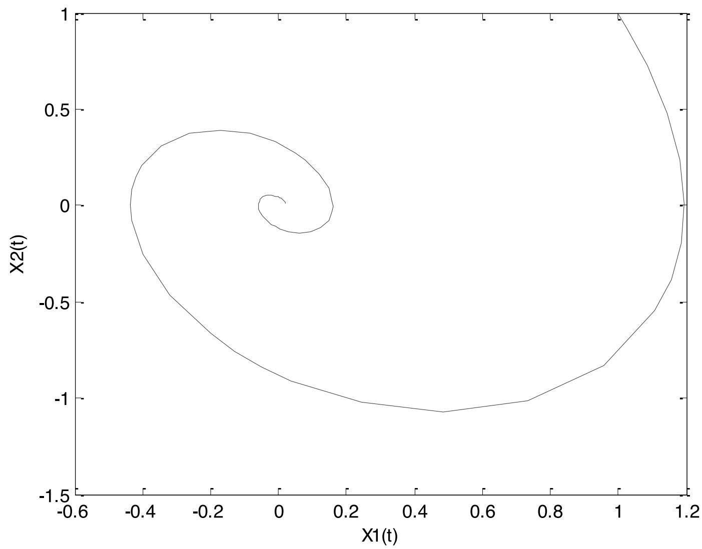

Fig. 1. State trajectories of Example 1.

Table 1. Maximum allowable upper delay bound for various μ

μ	**0**	**0.1**	**0.5**	**0.9**
Lin *et al.* (2006)	4.47	3.60	2.0	1.18
He *et al.* (2007)	4.47	3.60	2.0	1.18
Park & Ko (2007)	4.47	3.66	2.33	1.87
Ko & Park (2011)	5.55	4.41	2.4	2.12
Zang *et al.* (2013)	5.87	4.43	2.46	2.22
Proposed Theorem	6.03	4.57	2.89	2.54

Table 1 shows the comparison results of maximum allowable delay bound of time varying delay for various values of μ with existing methods and the proposed criterion. From Table 1 it can be seen that the proposed method allows maximum allowable delay bounds larger than others, which shows the effectiveness of Wirtinger's inequality based approach.

Example 2. Consider system (1) with the following parameters to illustrate the conservativeness of the proposed criteria compared to the results reported by some approaches in the book by Fridman (2014). The state trajectories of the considered system are as shown in Figure 2.

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \qquad A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$$

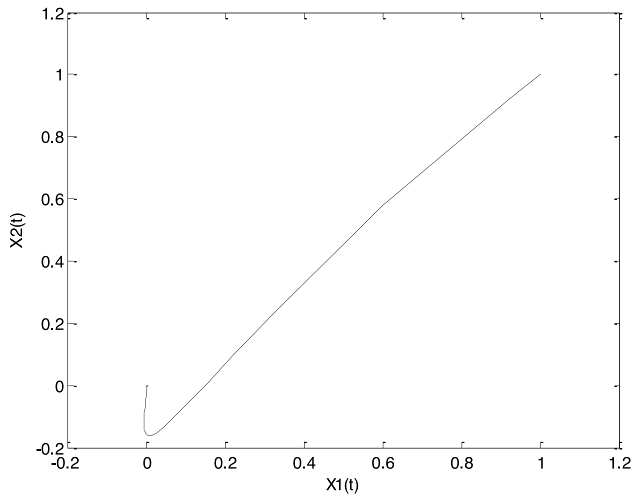

Fig. 2. State trajectories of Example 2.

Table 2. Maximum allowable upper delay bound for various μ

μ	**0**	**0.2**	**0.4**	**0.6**	**0.8**
LMI 3.111 Fridman (2014)	4.77	3.03	2.30	1.82	1.49
Theorem 3.5 Fridman (2014)	4.77	3.16	2.54	2.16	1.93
Proposed Theorem	5.04	3.34	3.22	2.94	2.12

Table 2 shows that the proposed criteria is less conservative than existing ones as evident from computed admissible upper bound of time delay. The proposed bounds are larger than others, indicating that larger delays can be tolerated before system becomes unstable. LMI 3.111 in Fridman (2014) is based on Lyapunov-Krasovskii functional and Jensen's inequality, whereas Theorem 3.5 is its extension in descriptor form. The Jensen's inequality generally induces some conservatism difficulty and the same can be overcome by a wide class of new parameterized inequalities like Wirtinger's inequality as proposed.

CONCLUSION

A new delay dependent stability criterion for the analysis of linear systems with time varying delay has been proposed. The proposed stability criterion results in less conservative bounds for allowable delays by considering an appropriate Lyapunov-

Krasovskii functional with Wirtinger's inequality. The results show improved maximum allowable upper time delay bounds over the existing ones. The advantage of the proposed criterion stems from the fact that it can reduce the conservatism introduced by Jensen's inequality.

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