Control of a nonlinear system utilizing analytical target cascading

Sulaiman F. Alyaqout*, Anwar F. Alyaqout**

*Mechanical Engineering Department, Kuwait University.
**Civil Engineering Department, Kuwait University.
***Corresponding Author: s.alyaqout@ku.edu.kw

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ABSTRACT

The present article introduces an approach that uses the analytical target cascading (ATC) method for managing nonlinear optimal control problems. Using this ATC approach, a decomposed nonlinear optimal control problem is obtained. This decomposed nonlinear control problem is then used to solve a synchronous machine optimal control problem. The numerical behavior of this decomposed synchronous machine optimal control problem is then examined and the solution time properties are also investigated. We demonstrated ATC as a valuable tool to solving nonlinear optimal control problems. Results show that as we increase the control parameter q_1 the total solution time will decrease.

Keywords: ATC; decomposition; hierarchical optimization; optimal control; optimization; control.

INTRODUCTION

Many researchers have used ATC (Alexander et al., 2011, Chan, 2011, Guarneri et al., 2011 & Wang et al., 2013) to solve vehicle (Kim et al., 2003, Kang et al., 2014 & Bayrak et al., 2016), thermal (Choudhary et al., 2005), and airplane (Allison et al., 2006) problems. Michalek & Papalambros (2005) introduced an approach that modifies some components in ATC (Han & Papalambros, 2010 & Dormohammadi & Rais-rohani, 2013) to obtain a satisfactory answer. Lassiter et al. (2005) utilized the subgradient strategy to solve ATC problems. Kim et al. (2006) provided a technique for determining the optimal ATC components. Moussouni et al. (2009) introduced a multi-criteria approach for solving ATC problems. Li et al. (2008) used a precise approach to solve subproblems simultaneously at the same time. While, Tosserams et al. (2006) enhanced the performance of ATC by utilizing an approach that solves part of the problem on just an individual occasion.

The optimization of hierarchical control systems has been studied by many authors (Jamshidi, 1997, Masmoudi et al., 2009, Picasso et al., 2010 & Sadati & Berenji, 2016). Li et al. (2013) used a decomposition approach to convert the optimal control problem into smaller problems. This decomposition approach never falls short of achieving a solution. Tang et al. (1991) provided an algorithm that divides problems utilizing time. Tang et al. (2006) presented an approach comprised of solving an upper problem and a lower problem. This approach was then applied to a manufacturing case-study. Hassan & Singh (1976) proposed an approach where some of the expression is supposed to be constant, afterwards this constant expression is calculated from another level. After applying his procedure, Hassan et al. (1978) was able to solve a very easy problem at the bottom level. While, Fawzy (1981) improved Hassan's work and added an extra level to his algorithm. Sadati & Ramezani (2010) optimized their control problem utilizing the gradient information. A procedure utilizing the gradient has also been used to solve other problems (Sadati & Babazadeh, 2006 & Sadati & Ramezani, 2008).

Much of the research on ATC has been concerned with improvement and extension of its formulation to improve the computational behavior. Dormohammadi & Rais-rohani (2013) used an exponential penalty function approach to solve hierarchical optimization problems. He then compared his method numerical efficiency to some other ATC algorithms. While, Wang et al. (2013) introduced two cutting plane methods to compute the dual variable. He reported comparable performance to the method of multipliers approach. On the other hand, Alexander et al. (2011) utilized artificial neural networks for ATC. Jung et al. (2018) proposed a formulation with parallelization that uses the subgradient method. In addition, a procedure for selecting the suitable step size was presented. Zhang et al. (2013) provided a procedure for managing the pareto set using a genetic algorithm. For this procedure, there is no need to select weights. Leverenz et al. (2016) introduced a multiparametric subgradient algorithm and compared the computational performance to another approach. He then improved a slow algorithm computational effort by utilizing fast function evaluations. While, the nonhierarchical approach of ATC delivered additional choices for coordination (Tosserams et al., 2010). Therefore, improving the numerical performance has been one of the main topics that has been selected by researchers.

Even though many researchers have chosen to use ATC to solve a lot of different engineering problems, the design of controllers using ATC topic did not receive a lot of attention from authors. The first goal of this article is to decompose our nonlinear optimal control problem using the ATC formulation introduced by Tosserams (Tosserams et al., 2006), then solve it utilizing an optimization algorithm. The second goal of this article is to study the computational cost of ATC solving this nonlinear optimal control problem. This ATC computational cost could help future researchers on optimal control because these researchers could use this ATC cost for comparison with other ATC formulations. Another goal of this article is to investigate the relationship between the total solution time and some of the control parameters. Such a relationship should help engineering designers in choosing control parameters that have good total solution time.

Our aim in this article remains optimizing nonlinear control systems using the ATC approach. This ATC method is used to decompose the synchronous machine optimal control problem. We then utilize an SQP procedure which obtains an answer for this decomposed problem. We will start this article with a review of some of the preceding work on the ATC topic that we will be utilizing. Then we talk about the synchronous machine optimal control problem. After that we will utilize our knowledge of ATC to decompose this synchronous optimal control problem into a two-level structure. Then we will discuss the computational results and present our conclusions.

PREVIOUS WORK ON ATC

The i means level and the j means element. A very important formula is the augmented Lagrangian function. The augmented Lagrangian function can be used with the universal problem to obtain the universal problem with augmented Lagrangian function. The universal ATC problem with augmented Lagrangian function for an element P_{ij} is stated as follows (Tosserams et al., 2006).

$$\min_{\bar{\mathbf{x}}_{ij}} f_{ij}\,(\bar{\mathbf{x}}_{ij}) - \mathbf{v}_{ij}^T\mathbf{r}_{ij} + \sum_{k\in D_{ij}} \mathbf{v}_{(i+1)k}^T\,\mathbf{t}_{(i+1)k}$$

$$+\left\|\mathbf{w}_{ij} o(\mathbf{t}_{ij} - \mathbf{r}_{ij})\right\|_2^2 + \sum_{k\in D_{ij}} \left\|\mathbf{w}_{(i+1)k} o(\mathbf{t}_{(i+1)k} - \mathbf{r}_{(i+1)k})\right\|_2^2$$

$$\text{subject to} \qquad \mathbf{g}_{ij}(\bar{\mathbf{x}}_{ij}) \le \mathbf{0}$$

$$\mathbf{h}_{ij}(\bar{\mathbf{x}}_{ij}) = \mathbf{0} \qquad (1)$$

$$\text{where} \qquad \bar{\mathbf{x}}_{ij} = [\mathbf{x}_{ij}, \mathbf{r}_{ij}, \mathbf{t}_{(i+1)k_1}, \dots, \mathbf{t}_{(i+1)k_{m_{ij}}}]$$

and $\mathbf{x}_{ij}$ is the local variables vector, $\mathbf{t}_{ij}$ is the targets with the responses denoted by $\mathbf{r}_{ij}$, $D_{ij} = \{k_1, \dots, k_{m_{ij}}\}$ is the children's set, N_a symbolizes total ATC levels, M_a symbolizes ATC elements, $\mathbf{c}_{ij} = \mathbf{t}_{ij} - \mathbf{r}_{ij}$ are the discrepancies, and $\mathbf{c} = [\mathbf{c}_{22}, \dots, \mathbf{c}_{N_a M_a}]$ is the vector of discrepancies.

Then, the following expression has to be calculated

$$\mathbf{v}^{(\kappa+1)} = \mathbf{v}^{(\kappa)} + 2\mathbf{w}^{(\kappa)} o \mathbf{w}^{(\kappa)} o \mathbf{c}^{(\kappa)} \tag{2}$$

We should also evaluate the following expression

$$\mathbf{w}^{(\kappa+1)} = \beta \mathbf{w}^{(\kappa)} \tag{3}$$

The ATC method with augmented Lagrangian function introduced by Tosserams et al. (2006) is used in this paper.

OPTIMAL CONTROL OF A SYNCHRONOUS MACHINE

In this section, ATC ability in solving difficult problems is investigated by solving the synchronous machine optimal control problem. Several researchers have utilized this problem to investigate the performance of hierarchical control algorithms (Mukhopadhyay & Malik, 1972 & Singh & Hassan, 1977). The continuous-time model of the machine is a nonlinear model. Sadati & Ramezani (2010) discretized this nonlinear continuous-time synchronous machine model into a discrete-time nonlinear machine model with sampling period τ. The model that we will use in this paper is this discrete-time system. Therefore, the system we utilize can be written as

$$S_1(x_1(k+1), x_1(k), x_2(k)) =$$

$$-x_1(k+1) + x_1(k) + \tau x_2(k) = 0 \tag{4}$$

$$S_2(x_2(k+1), x_1(k), x_2(k), x_3(k)) = -x_2(k+1) + (1 - \frac{\tau}{3.69959304})x_2(k)$$
$$- \frac{\tau}{0.08325008} x_3(k) \sin x_1(k) - \frac{\tau}{-0.04162504} \sin 2x_1(k) + \frac{\tau}{0.02551723} = 0 \tag{5}$$

$$S_3(x_3(k+1), x_1(k), x_3(k), u(k)) = -x_3(k+1)$$
$$+(1 - \frac{\tau}{3.10366232})x_3(k) + \frac{\tau}{0.52631578} \cos x_1(k) + \tau u(k) = 0 \tag{6}$$

In addition, we can see that the parameter value is $\tau = 0.01$. The aim of the synchronous machine optimal control problem is to minimize the following performance index

$$J = \frac{\tau}{2} \sum_{k=0}^{N} (q_1(x_1(k) - 0.7461)^2 + q_2 x_2^2(k) + q_3(x_3(k) - 7.7438)^2$$

$$+ r(u(k) - 1.1)^2) \tag{7}$$

Now this machine optimal control problem can be created by using the performance index in Equation (7) with the constraints in Equations (4), (5) and (6) of the discrete-time model. Therefore, this optimal control problem can be written as

$$\min_{x_1(1),\dots,x_1(N+1),x_2(1),\dots,x_2(N+1),x_3(1),\dots,x_3(N+1),u(0),\dots,u(N)} \frac{\tau}{2}\sum_{k=0}^{N}(q_1(x_1(k)-0.7461)^2 + q_2x_2^2(k) + q_3(x_3(k)-7.7438)^2 + r(u(k)-1.1)^2)$$

subject to

$$S_1(x_1(k+1), x_1(k), x_2(k)) = 0$$

$$S_2(x_2(k+1), x_1(k), x_2(k), x_3(k)) = 0 \tag{8}$$

$$S_3(x_3(k+1), x_1(k), x_3(k), u(k)) = 0 \qquad k = 0,1,\dots,N$$

where $\quad x_1(0) = 0.5,\; x_2(0) = -0.4,\; x_3(0) = 8$

OPTIMAL CONTROL OF ATC

Let us write this synchronous machine problem in Equation (8) in terms of the ATC formulation in Equation (1). We then decompose this optimal control problem in Equation (8) into a two-level structure using ATC.

Now let us write this synchronous machine optimal control problem in terms of this two-level ATC structure as shown in Figure 1. The upper level problem has only one ATC subproblem P_{11}. This upper level problem can be written as follows.

$$\min_{\bar{\mathbf{x}}_{11}} \frac{\tau}{2}\sum_{k=0}^{N}(q_1(x_1(k)-0.7461)^2 + q_2x_2^2(k) + q_3(x_3(k)-7.7438)^2 + r(u(k)-1.1)^2)$$

$$+\mathbf{v}_{22}^T\mathbf{t}_{22} + \mathbf{v}_{23}^T\mathbf{t}_{23} + \|\mathbf{w}_{22}o(\mathbf{t}_{22}-\mathbf{r}_{22})\|_2^2 + \|\mathbf{w}_{23}o(\mathbf{t}_{23}-\mathbf{r}_{23})\|_2^2$$

$$\text{where} \quad \bar{\mathbf{x}}_{11} = [\mathbf{t}_{22}, \mathbf{t}_{23}] \tag{9}$$

$$\mathbf{t}_{22} = [x_1(1),\dots,x_1(N), x_3(1),\dots,x_3(N), x_2(1),\dots,x_2(N)]$$

$$\mathbf{t}_{23} = [x_1(1),\dots,x_1(N), x_3(1),\dots,x_3(N), u(0),\dots,u(N)]$$

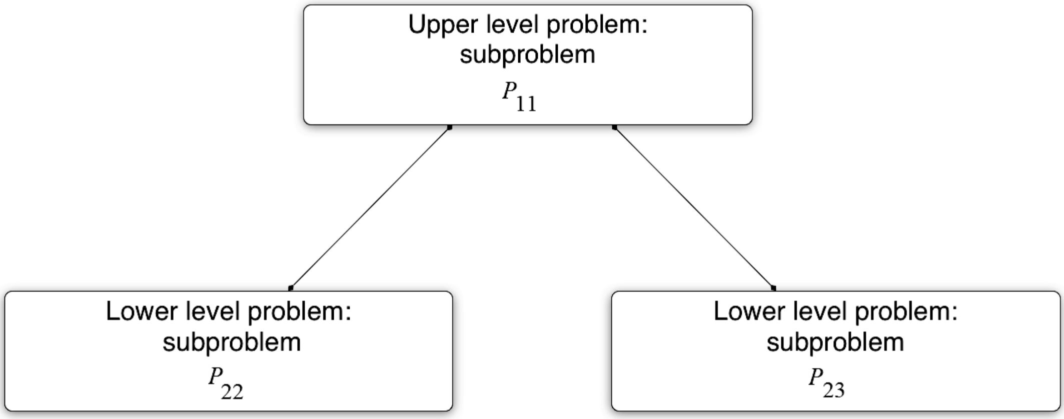


Fig. 1. The two-level structure for the synchronous problem.

The lower level problem has two ATC subproblems P_{22} and P_{23}. The constraints of subproblem P_{22} are Equation (4) and Equation (5) of the discrete-time model, while the constraint of subproblem P_{23} is Equation (6) of the discrete-time model. Let us now see how the $\mathbf{t}_{22}$ and $\mathbf{t}_{23}$ have been created. The $x_1(1), \dots, x_1(N)$ are located in both P_{22} and P_{23} with P_{11} managing, therefore they are linking variables. The $x_3(1), \dots, x_3(N)$ are also located in both P_{22} and P_{23} with P_{11} managing, therefore they are linking variables. On the other hand, the $x_2(1), \dots, x_2(N)$ are only located in P_{22} with P_{11} managing, therefore they are responses. Moreover, the $u(0), \dots, u(N)$ are only located in P_{23} with P_{11} managing, therefore they are also responses. This lower level ATC subproblem P_{22} is stated as follows.

$$\min_{\bar{\mathbf{x}}_{22}} -\mathbf{v}_{22}^T\mathbf{r}_{22} + \|\mathbf{w}_{22} o(\mathbf{t}_{22} - \mathbf{r}_{22})\|_2^2$$

subject to

$$S_1(x_1(k+1), x_1(k), x_2(k)) = 0 \qquad k = 0,1, \dots, N$$

$$S_2(x_2(k+1), x_1(k), x_2(k), x_3(k)) = 0 \qquad (10)$$

where $\bar{\mathbf{x}}_{22} = [\mathbf{x}_{22}, \mathbf{r}_{22}]$

$$\mathbf{x}_{22} = [x_1(N+1), x_2(N+1)]$$

$$\mathbf{r}_{22} = [x_1(1), \dots, x_1(N), x_3(1), \dots, x_3(N), x_2(1), \dots, x_2(N)]$$

$$x_1(0) = 0.5,\ x_2(0) = -0.4$$

,

Note that the $x_1(N+1), x_2(N+1)$ in $\mathbf{x}_{22}$ are only located in P_{22}, hence they are local variables. While, the lower level ATC subproblem P_{23} is stated as follows.

$$\min_{\bar{\mathbf{x}}_{23}} -\mathbf{v}_{23}^T\mathbf{r}_{23} + \|\mathbf{w}_{23} o(\mathbf{t}_{23} - \mathbf{r}_{23})\|_2^2$$

subject to

$$S_3(x_3(k+1), x_1(k), x_3(k), u(k)) = 0 \qquad k = 0,1,\dots,N \qquad (11)$$

where $\bar{\mathbf{x}}_{23} = [\mathbf{x}_{23}, \mathbf{r}_{23}]$

$$\mathbf{x}_{23} = [x_3(N+1)]$$

$$\mathbf{r}_{23} = [x_1(1),\dots,x_1(N), x_3(1),\dots,x_3(N), u(0),\dots,u(N)]$$

$$x_3(0) = 8$$

Note that the $x_3(N+1)$ in $\mathbf{x}_{23}$ is only located in P_{23}, consequently they are local variables. The ATC subproblems P_{11}, P_{22} and P_{23} are all solved using the fmincon optimization tool from MATLAB. This optimization tool employs an SQP algorithm (Papalambros & Wilde, 2000).

RESULTS AND DISCUSSIONS

Now we consider investigating efficiency for ATC synchronous machine problem in Equations (9), (10), and (11). Also, we investigate the relationship between the total solution time and some of the control parameters. The MATLAB program was used to obtain the numerical solution of this ATC synchronous machine optimal control problem.

To check the validity of the solution obtained using the MATLAB program, we compare our solution to the results obtained by Sadati & Ramezani (2010). The parameters for this Sadati case are $q_1 = 10$, $q_2 = 10$, $q_3 = 1$, $r = 10$, $x_1(0) = 0.5$, $x_2(0) = -0.4$ and $x_3(0) = 8$. Using these parameters, the ATC synchronous machine optimal control problem was solved using MATLAB. Figure 2 shows the optimal rotor angle x_1^* versus time.

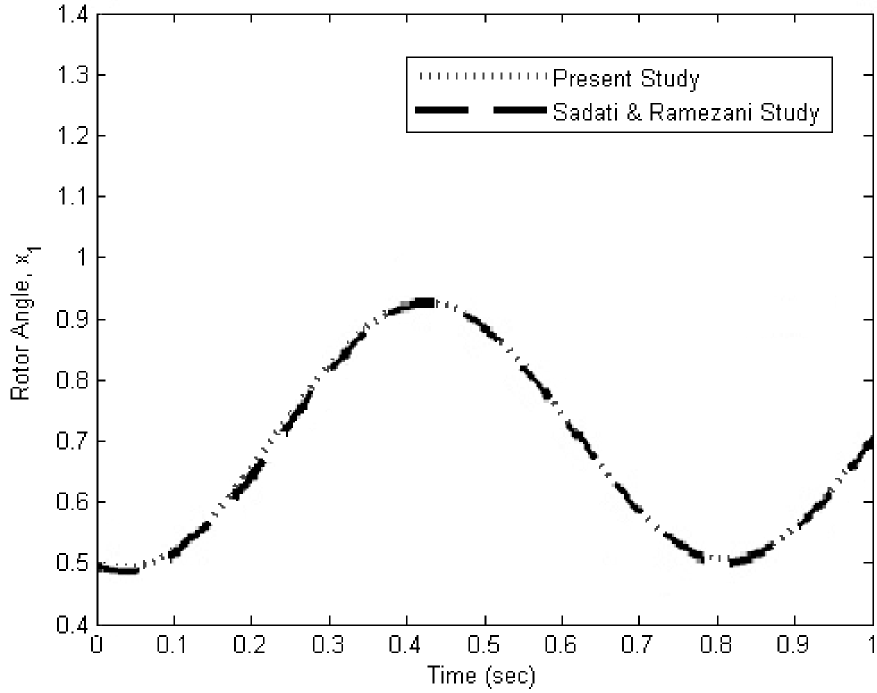


Fig. 2. The optimal rotor angle versus time for $q_1 = 10$, $q_2 = 10$, $q_3 = 1$, and $r = 10$.

Figure 3 shows the optimal speed deviation x_2^* versus time. The ATC responses in Figure 2 and Figure 3 are nearly identical to the optimal responses from Sadati & Ramezani (2010). The optimal field flux linkage x_3^* versus time is shown in Figure 4. The ATC response in this Figure 4 is very close to the Sadati response. The optimal control variable u^* versus time is shown in Figure 5. The ATC response in this Figure 5 is also nearly identical to the optimal response from Sadati.

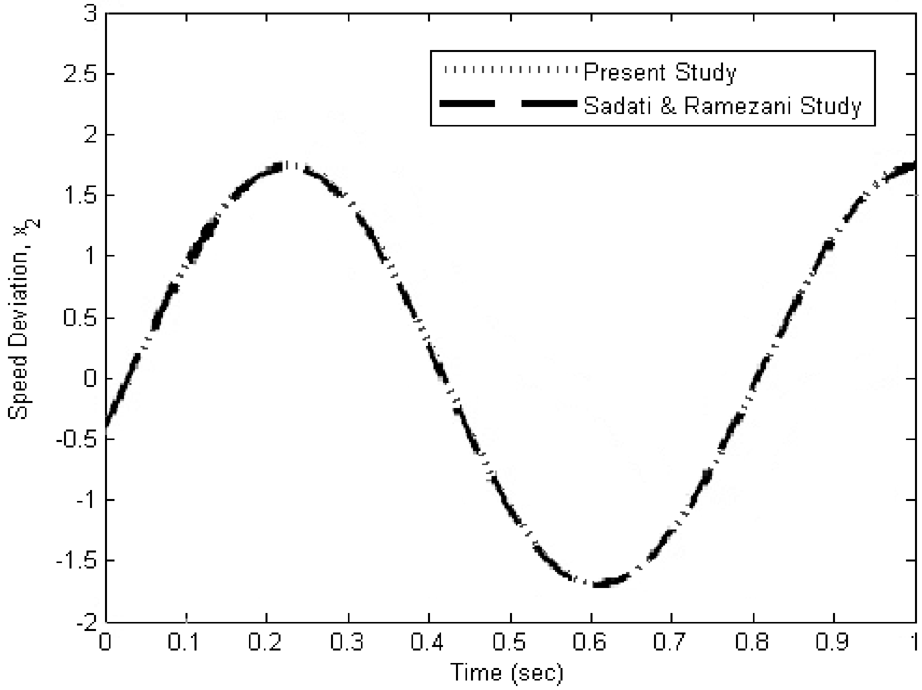


Fig. 3. The optimal speed deviation versus time for $q_1 = 10, q_2 = 10, q_3 = 1$, and $r = 10$

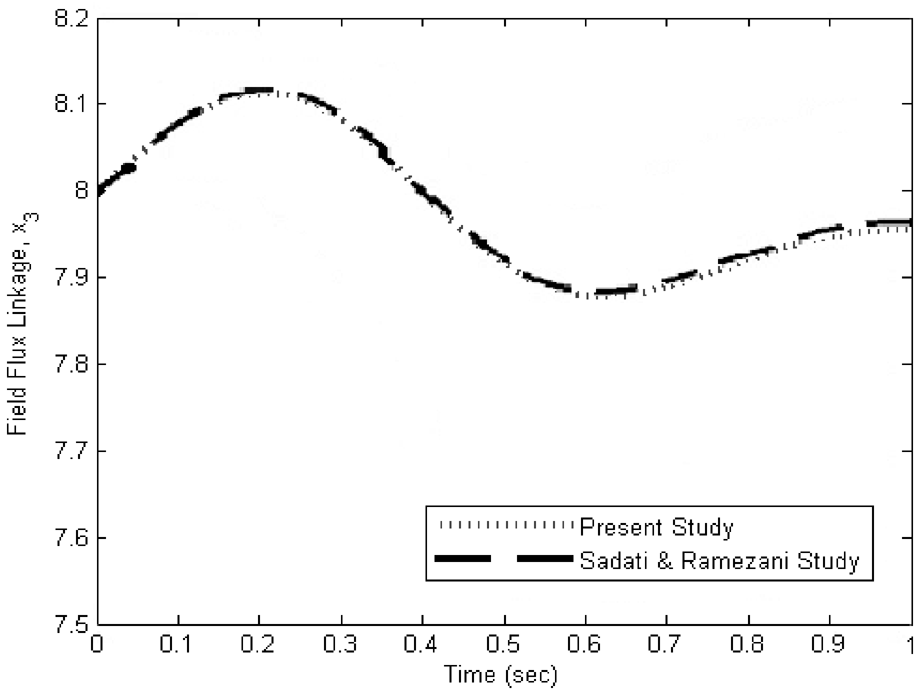


Fig. 4. The optimal field flux linkage versus time for

$q_1 = 10, q_2 = 10, q_3 = 1$, and $r = 10$.

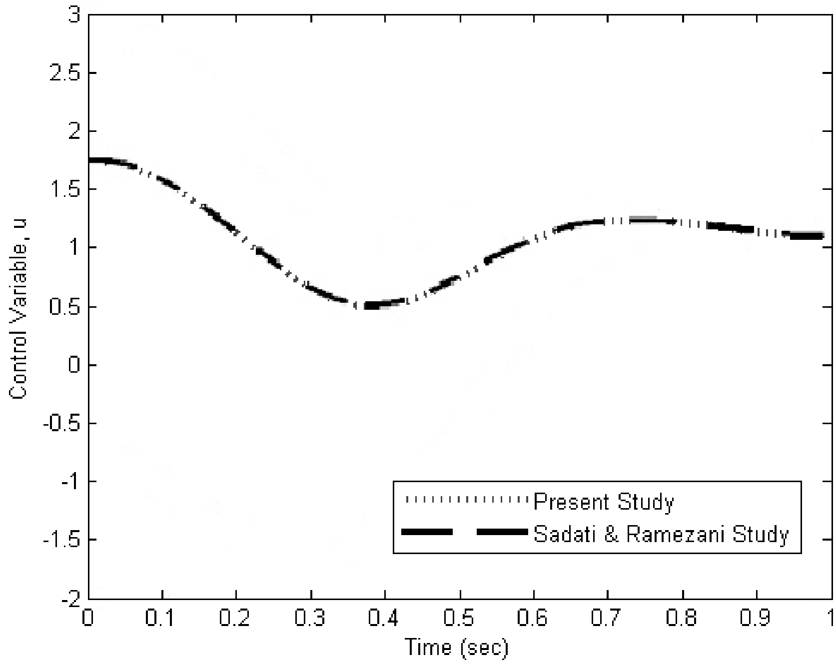
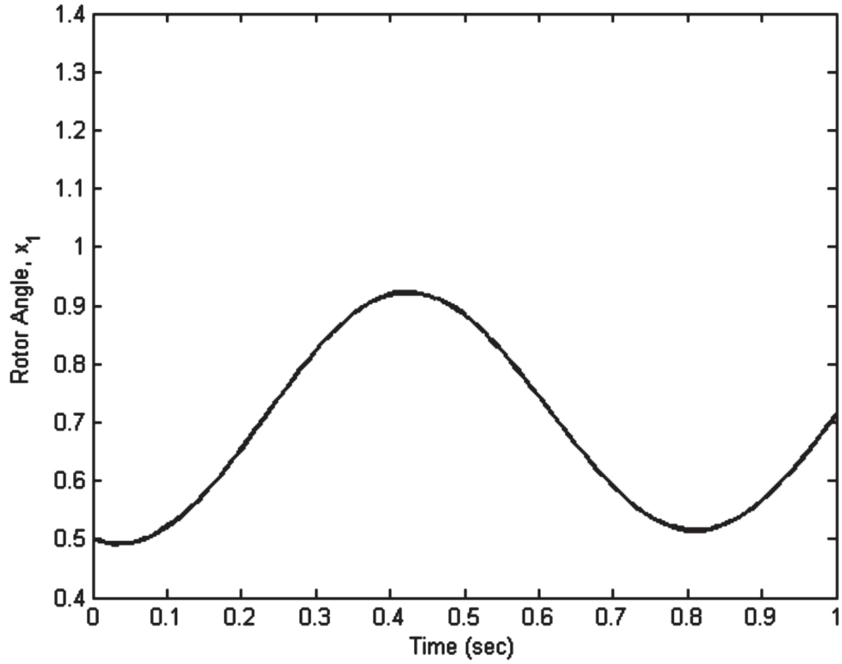


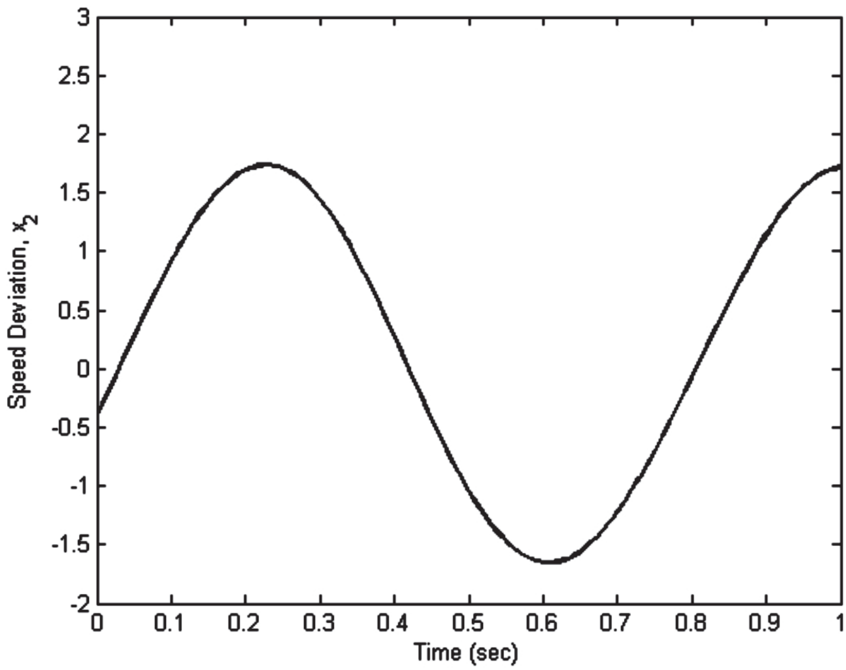
Fig. 5. The optimal control variable versus time for $q_1 = 10$, $q_2 = 10$, $q_3 = 1$, and $r = 10$.

To investigate the performance of the ATC optimal control problem, let us contrast the ATC problem with the performance from the control problem in Equation (8). The optimal ATC control objective is $J^*_{ATC} = 7.8717$, while the optimal control problem is $J^* = 7.8709$. Since J^*_{ATC} and J^* are very close, therefore the answer from ATC match very accurately to answer from optimal control problem. To achieve this, ATC performed 256 iterations between the upper and lower levels. The total solution time of ATC to reach the optimal solution is 943.33 seconds (or 15.722 min).

Let us now investigate the effect of changing the q_1 control parameter on the accuracy of ATC. The parameters for this new case are $q_1 = 150$, $q_2 = 10$, $q_3 = 1$, $r = 10$, $x_1(0) = 0.5$, $x_2(0) = -0.4$ and $x_3(0) = 8$. Using ATC, the optimal ATC control objective is $J^*_{ATC} = 9.7335$ while the optimal control objective is $J^* = 9.7323$. Although q_1 is different, ATC answer remains very accurate to optimal control answer. Figure 6 shows the optimal rotor angle and speed deviation responses. While, the field flux linkage and control variable responses are shown in Figure 7.

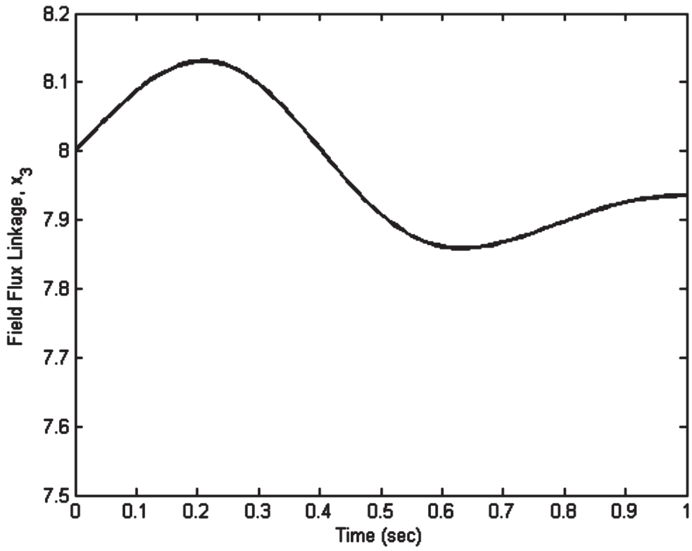


(a)

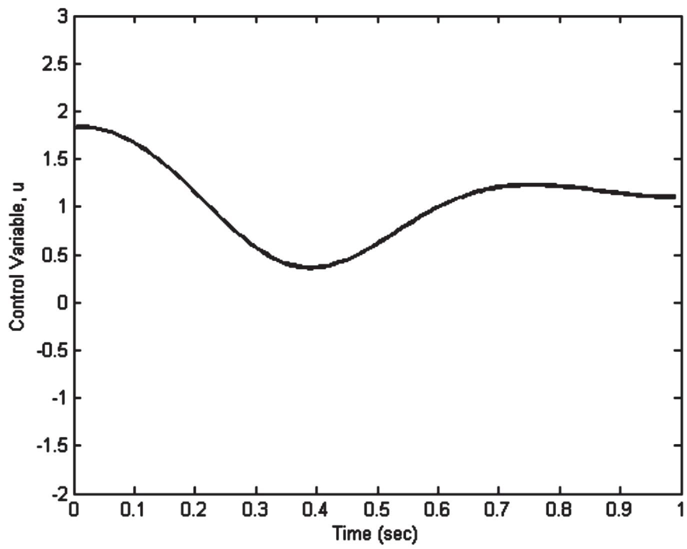


(b)

Fig. 6. The optimal (a) rotor angle (b) and speed deviation versus time for $q_1 = 150$, $q_2 = 10$, $q_3 = 1$, and $r = 10$.



(a)



(b)

Fig. 7. The optimal (a) field flux linkage (b) and control variable versus time for

$$q_1 = 150,\, q_2 = 10,\, q_3 = 1,\, r = 10.$$

Note that the control parameter q_1 has been changed to investigate the accuracy of ATC. The result of this change is optimum ATC control objective with optimum control objective are always almost equal. Hence, one can conclude ATC answer match very accurately to optimal control answer.

Relationship between the total solution time and the control parameter q_1 is shown in Figure 8. Note that in this case q_1 is varied, while the other parameters are $q_2 = 10, q_3 = 1, r = 10$. The data points were fitted with a polynomial that had a degree equal to 5. One can notice that increasing the total solution time decreases the control parameter q_1. Hence, as we increase the control parameter q_1 the total solution time will decrease.

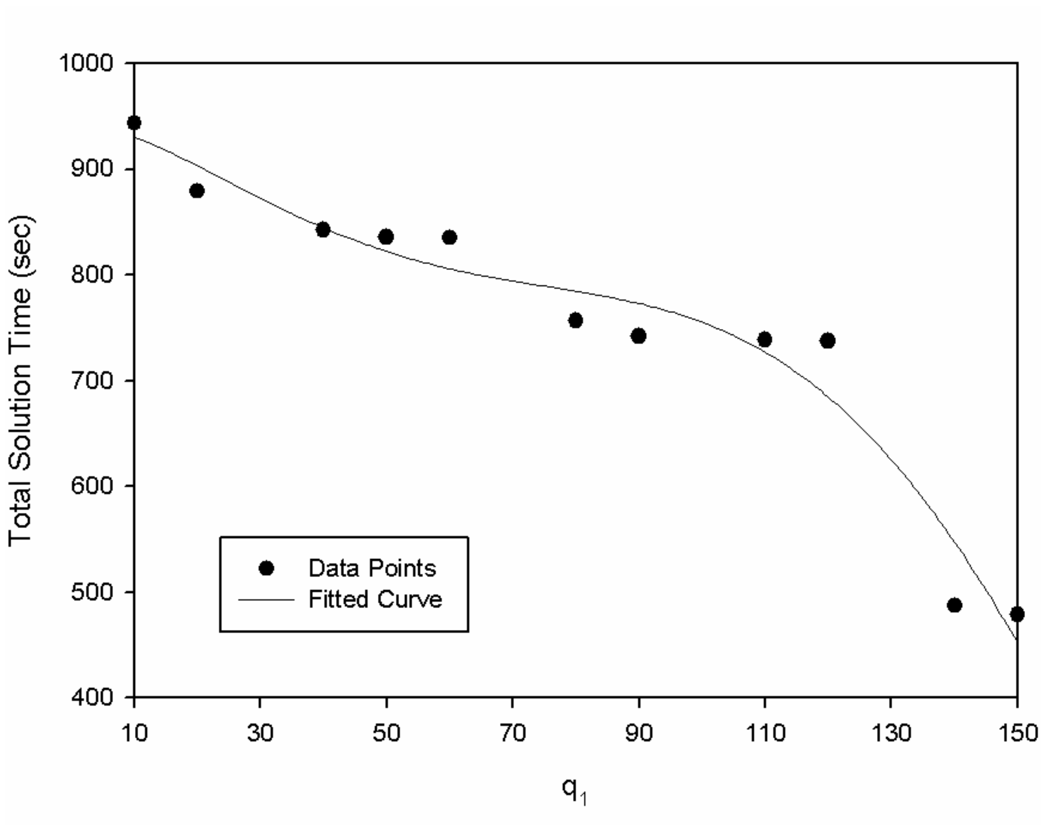


Fig. 8. Relationship between the total solution time and control parameter q_1

The effect of the control parameter q_2 on the total solution time is shown in Figure 9. Observe that in this case q_2 is varied, while the other parameters are $q_1 = 10$, $q_3 = 1$, $r = 10$. A polynomial of a degree 5 is used to fit the data points. Looking at the fitted curve, we can conclude that the total solution time tends to increase as the control parameter q_2 is increased.

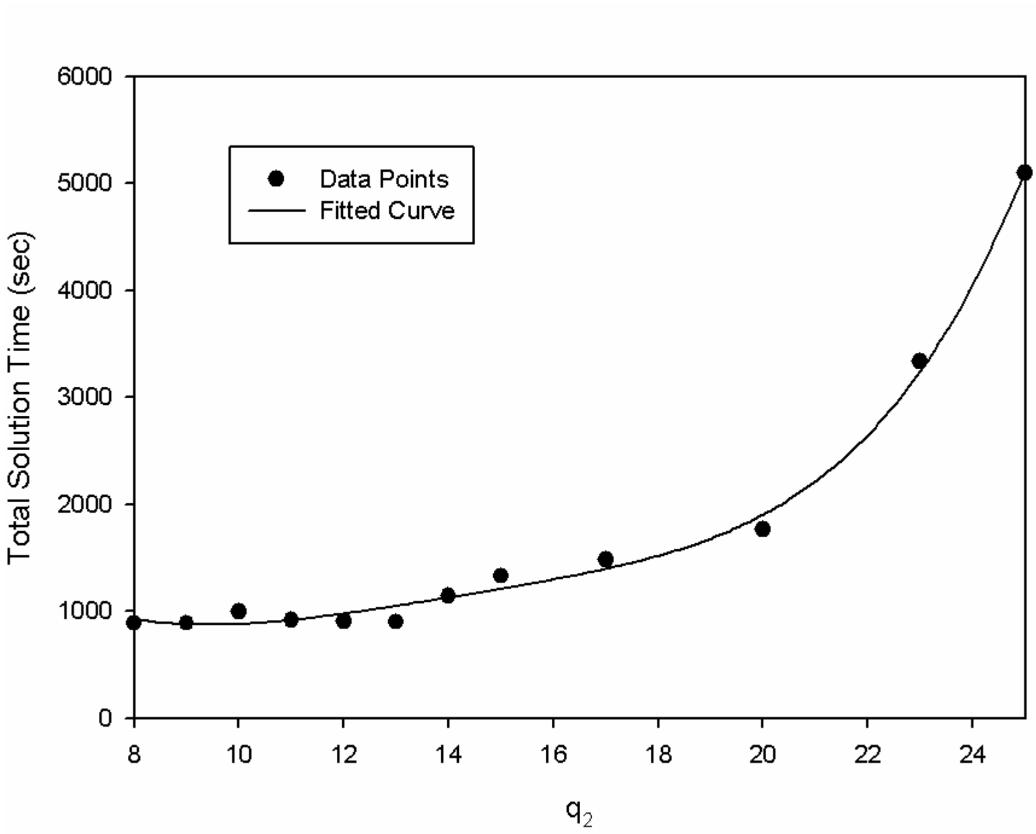


Fig. 9. Relationship between the total solution time and control parameter q_2

Figure 10 shows the total solution time versus the control parameter q_3 Now in this final case q_3 is varied, while $q_1 = 10$, $q_2 = 10$, $r = 10$, This is also fitted with a degree equal to 5.

Looking at this curve, we can conclude that the total solution time tends to be lower on large q_3.

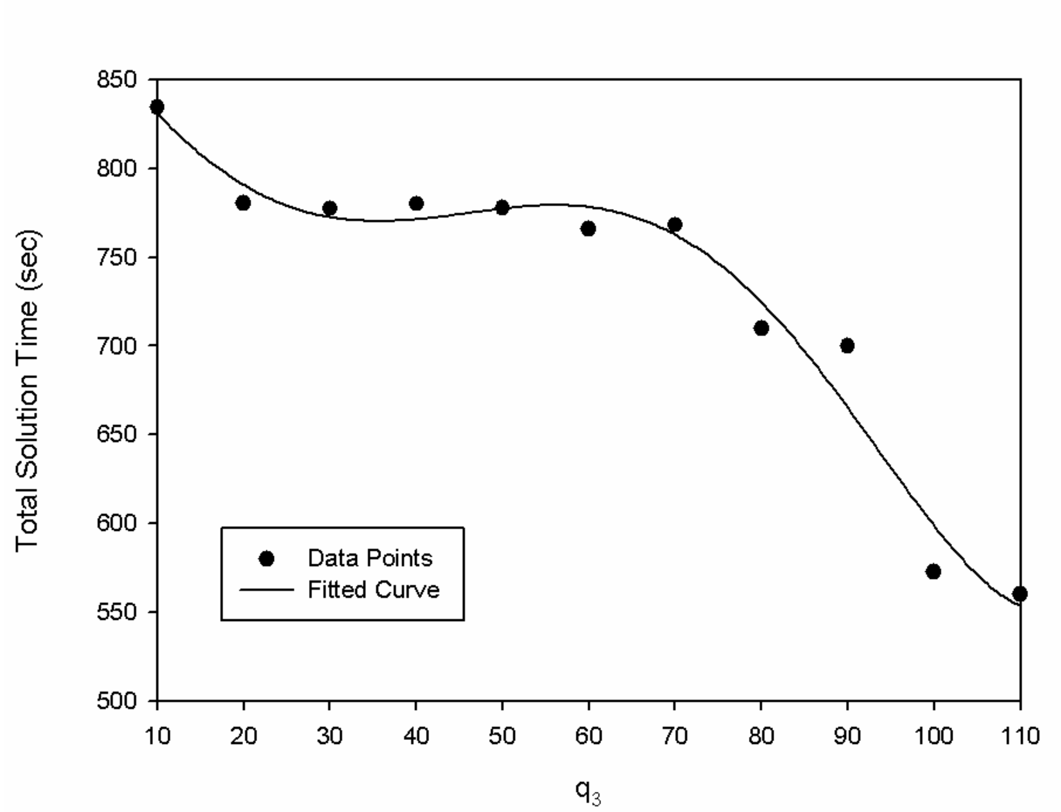


Fig. 10. Relationship between the total solution time and control parameter q_3

The effect of the control parameter q_1 on the optimal ATC control objective J^*_{ATC} is shown in Figure 11. Note that in this case q_1 is varied, while the other parameters are $q_2 = 10$, $q_3 = 1$, $r = 10$. The data points were fitted with a polynomial that had a degree equal to 1. We can notice that the optimal ATC control objective J^*_{ATC} tends to increase as the control parameter q_1 is increased.

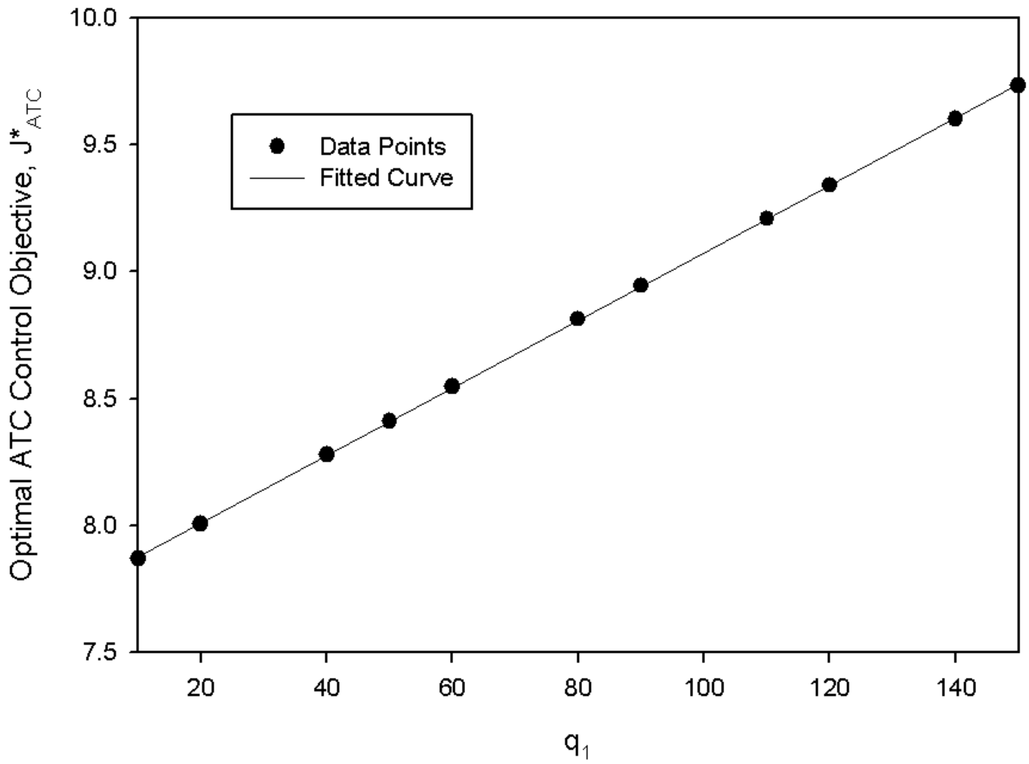


Fig. 11. Relationship between the optimal ATC control objective and control parameter q_1

Figure 12 shows the relationship between the optimal field flux linkage and time for different values of the control parameter q_1. Note that in this case q_1 is varied, while the other parameters are $q_2 = 10$, $q_3 = 1$, $r = 10$.

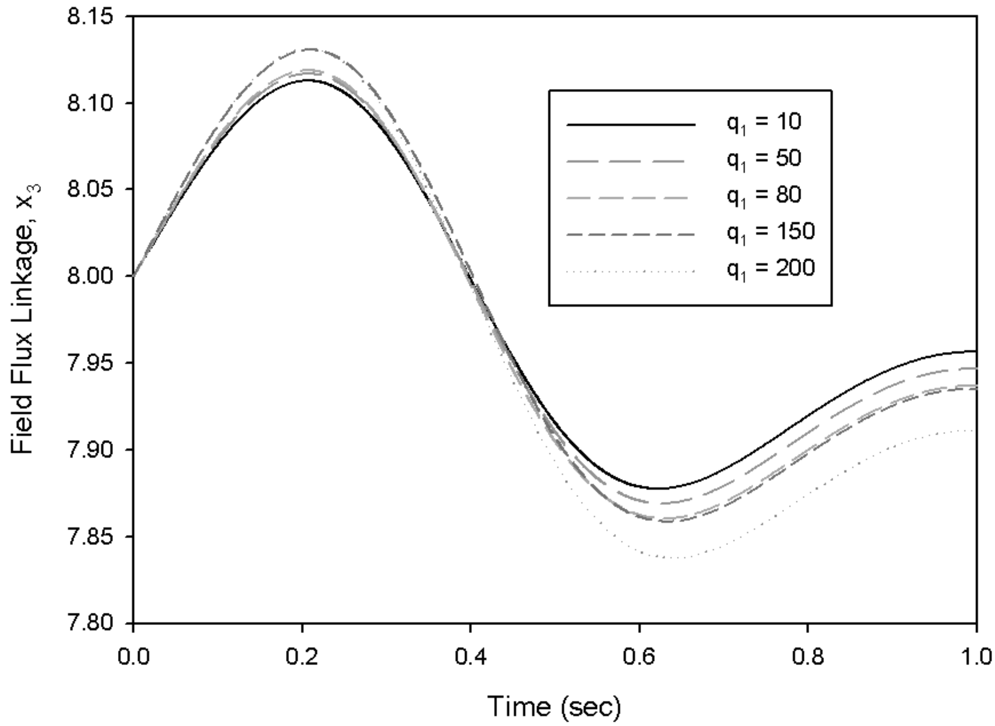


Fig. 12. The optimal field flux linkage versus time for different control parameter q_1

CONCLUSION

This manuscript proposed a procedure that solved nonlinear optimal control problems utilizing ATC. A synchronous machine optimal control problem was divided into smaller problems and then coordinated utilizing this ATC scheme. A solution is then obtained for this coordinated machine optimal control problem using SQP.

The performance associated with ATC is then studied. For our work, we demonstrated that the ATC optimal controller was very close to the optimal controller. In addition, nonlinear optimal control problems were successfully solved using the ATC formulation. Also, we investigated the computational cost of solving nonlinear optimal control problems utilizing ATC. We then studied the effect of some control parameters on total solution time. We noticed that the total solution time tended to decrease as the control parameter q_1 tended to increase. We also noticed that the total solution time tended to increase as the control parameter q_2 tended to increase. Future work should consider investigating the performance of two additional ATC formulations. These two are the exponential penalty function formulation and the cutting plane methods formulation.

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