Adaptive Backstepping Control for a Unicycle-Type Mobile Robot

Saleh Alshamali

Department of Electrical Engineering, Kuwait University, P.O. Box 5965, Safat 13060, Kuwait Corresponding Author: s.alshamali@ku.edu.kw

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ABSTRACT

This paper considers the trajectory tracking of a unicycle-type mobile robot. The nonlinear dynamics of the mobile robot is treated as two subsystems for the robot's orientation and position. A Lyapunov-based adaptive control law is proposed for asymptotic tracking of the robot's orientation, while an adaptive backstepping control scheme is proposed for asymptotic tracking of the robot's position. The synthesized controllers and derived tuning laws were shown to drive the error dynamics to zero; hence, trajectory tracking was achieved. Simulations were conducted to illustrate the effectiveness of the proposed approach.

Keywords: Adaptive control; Backstepping control; Lyapunov-based control; Unicycle-type mobile robot.

INTRODUCTION

The control of nonholonomic mobile robots has received a great deal of attention in recent years. Mobile robots have been applied in many fields, such as military, security, and agriculture. Despite their widespread application, mobile robots pose a challenge to control engineers owing to the underactuated nature of their dynamics (Kim et al., 2002). The two standard control problems discussed in the literature are point stabilization and trajectory tracking (Chwa, 2004). The tracking problem of nonholonomic wheeled mobile robots (WMRs) has been addressed by many researchers. Some of the literature related to this work is reviewed below.

The work in (Lee et al., 2015) presented an approach to the trajectory tracking problem for a class of mobile robots via sliding mode control, where a new sliding surface design is proposed to ensure the convergence of the error dynamics. Lyapunov-based controllers for the kinematic and dynamic models of a mobile robot are proposed to adjust the velocities of the two wheels and ensure trajectory tracking (Fareh et al., 2016). An output feedback tracking controller for mobile robots with delayed measurements, where the delay is estimated via a sliding mode-based observer is presented in (Guechi et al., 2012). The trajectory tracking of a mobile robot via model predictive control (MPC) is introduced by (Chen et al., 2019), where a neural network based adaptive controller is utilized to estimate the unknown dynamics. A tracking control law for a leader-following formation utilizing onboard cameras and adaptive observers for position and velocity estimation is presented by (Liang et al., 2018). The authors in (Petrov et al., 2010) considered the design of a path tracking controller for the dynamic model of a skid-steered mobile robot is proposed in (Hwang et al., 2013). Recently, a model predictive based control law for the reference tracking of a WMR under control input constraints is presented (Yang et al., 2017). The stabilization and tracking problems of a WMR are solved via a Lyapunov-based controller (Wang et al., 2015).

Because of modeling errors and parameter uncertainties, the control community has shown great interest in adaptive control techniques. Several adaptive control strategies have been proposed for the stabilization and tracking of WMRs. The work in (Park et al., 2009) proposed an adaptive controller for the trajectory tracking of mobile robots with parametric uncertainties, where the dynamic surface control (DSC) method is utilized in the control design. In (Hu et al., 2011), an adaptive feedback control law for the trajectory tracking of the dynamic model of WMRs is introduced. An adaptive fuzzy logic control law for the tracking of a WMR under parameter uncertainties and external disturbances is addressed in (Peng et at., 2018). The authors in (Rossomando et al., 2011) proposed a model reference adaptive controller (MRAC) that utilizes a radial basis function neural network (RBF-NN) for the trajectory tracking problem of mobile robots.

This paper addresses the trajectory tracking problem of a nonholonomic mobile robot with unknown parameters. The design involves two steps. First, the error in the robot's orientation is driven to zero via a Lyapunov-based adaptive control law. Next, a simple trick is used to put the system in a form where the backstepping control technique can be applied. This is achieved by interchanging the error dynamics of the robot's (x, y) position. Finally, an adaptive backstepping control law is designed to drive the error in the robot's position to zero.

The rest of the paper is organized as follows. First, the kinematic model of the mobile robot and the necessary assumptions are presented. Then, the control design approach is introduced. Afterwards, an adaptive control law based on the Lyapunov theory for the stabilization of the mobile robot's orientation angle, followed by the design of an adaptive backstepping control law for the stabilization of the mobile robot's position are proposed. Next, simulation results are presented and discussed. Finally, some concluding remarks are given.

PROBLEM FORMULATION

The kinematic model of a unicycle-type mobile robot is described by the following dynamics:

$$\begin{aligned} \dot{x} &= v \cos(\theta) \\ \dot{y} &= v \sin(\theta) \\ \dot{\theta} &= \omega \end{aligned} \tag{1}$$

where x and y represent the (x, y) Cartesian coordinates of the center of mass of the mobile robot, θ is the orientation angle between the x-axis and heading direction of the robot, and v and ω are the control inputs that represent the robot's linear and angular velocities, respectively.

A unicycle-type mobile robot consists of two independently controlled driving wheels. Figure 1 shows a schematic diagram of the differential drive robot considered.

The objective is to design a controller based on the backstepping control scheme to track a reference trajectory (x_r, y_r, θ_r) that satisfies

$$\begin{aligned} \dot{x}_r &= v_r \cos(\theta_r) \\ \dot{y}_r &= v_r \sin(\theta_r) \\ \dot{\theta}_r &= \omega_r \end{aligned} \tag{2}$$



Figure 1. Model of the wheeled mobile robot.

The following error coordinates were introduced by (Kanayama et al., 1990):

$[x_e]$		[cosθ	sinθ	[0	$\begin{bmatrix} x_r - x \end{bmatrix}$
y_e	=	–sinθ	cosθ	0	$ y_r - y $
$\left[\theta_{e} \right]$		lo	0	1	$\left[\theta_r - \theta\right]$

These can be used to express the error model given below:

$$\dot{x}_e = \omega \ y_e - \nu + \nu_r \ \cos(\theta_e) \tag{3a}$$

$$\dot{y}_e = -\omega \, x_e + v_r \, \sin(\theta_e) \tag{3b}$$

$$\dot{\theta}_e = \omega_r - \omega \tag{3c}$$

For convenience, the control variables v and ω are denoted by u_1 and u_2 , respectively. The following assumption was adopted in this work:

Assumption 1 The parameters v_r and ω_r are constant, bounded, but uncertain with $v_r \neq 0$ and $\omega_r \neq 0$.

PROPOSED APPROACH

In this section, the control approach is presented. If the error dynamics (Equation (3)) are examined, it seems natural to treat the system as two subsystems. Subsystem (I) is given by Equation (3c) and represents the error dynamics of the robot's orientation. Subsystem (II) is given by Equations (3a)–(3b) and represents the error dynamics of the robot's position.

For subsystem (I), a Lyapunov-based controller is proposed to drive the robot's angle error to zero. Furthermore, since ω_r is uncertain, an adaptation law is derived to estimate the unknown parameter. For subsystem (II), a backstepping control law is synthesised to drive the robot's position error to zero. However, to put subsystem (II) in a form where the backstepping control technique is applicable, the dynamic equations (Equation (3a)–(3b)) are interchanged. The error dynamics can now be written as

$$\dot{y}_e = v_r \sin(\theta_e) - u_2 x_e \tag{4a}$$

$$\dot{x}_e = v_r \cos(\theta_e) + u_2 y_e - u_1 \tag{4b}$$

$$\dot{\theta}_e = \omega_r - u_2 \tag{4c}$$

As a common practice in the backstepping control methodology, the state x_e in Equation (6) is viewed as a virtual control input that is designed to regulate the *y*-axis error to zero. Then, the control law u_1 is designed to ensure the stability of the overall system and guarantee the asymptotic convergence of the position error dynamics to zero. This

guarantees tracking of the reference trajectory. However, note that the developed control law must deal with the fact that the reference linear velocity v_r is unknown. This can be resolved by incorporating an adaptation law with the backstepping control design.

ADAPTIVE CONTROL DESIGN

This section presents the developed adaptive nonlinear control law for trajectory tracking of a unicycle-type mobile robot. The design of an adaptive Lyapunov-based control law for subsystem (I) is first introduced. Then, the design of an adaptive backstepping tracking controller for subsystem (II) is proposed.

Lyapunov-based Adaptive Control Design

Consider the angle error dynamics given in Equation (4c), where ω_r is an unknown parameter. Let $\hat{\omega}_r$ be the estimate of ω_r . Then, the estimation error can be defined as

$$\widetilde{\omega}_r = \widehat{\omega}_r - \omega_r \tag{5}$$

Proposition 1. The robot's orientation subsystem (Equation (4c)) is asymptotically stable under the control law

$$u_2 = k_1 \theta_e + \widehat{\omega}_r \tag{6}$$

and the adaptation law

$$\hat{\omega}_r = \theta_e \tag{7}$$

where k_1 is a positive scalar.

Proof: Consider the Lyapunov function candidate

$$V_0(t) = \frac{1}{2}\theta_e^2 + \frac{1}{2}\widetilde{\omega}_r^2$$

By using the dynamic equation Equation (4c) and estimation error Equation (5), the time derivative of $V_0(t)$ is given by

$$\begin{split} \dot{V}_0 &= \theta_e(\omega_r - u_2) + \widetilde{\omega}_r \ \dot{\widetilde{\omega}}_r \\ &= -\theta_e \ u_2 + \theta_e(\widehat{\omega}_r - \widetilde{\omega}_r) + \widetilde{\omega}_r \ \dot{\widetilde{\omega}}_r \end{split}$$

By substituting for the adaptive control law in Equations (6)-(7), it can be shown that

$$\dot{V}_0 = -k_1 \, \theta_e^2 \le 0$$

Because \dot{V}_0 is negative semi-definite, the angle error subsystem (I) is stable. However, to show that θ_e in fact goes to zero, we can invoke LaSalle's theorem. Thus, the compact set can be defined such that

$$\Omega_c = \{ [\theta_e \ \widetilde{\omega}_r]^T, \dot{V}_0 = 0 \}$$

Solving for $\dot{V}_0 = 0$ gives

$$-k_1 \theta_e^2 = 0 \implies \theta_e = 0$$

Because $\theta_e = 0$, this implies that $\dot{\theta}_e = 0$. Now, by setting $\dot{\theta}_e = 0$ in Equation (4c), it is clear that $\omega_r - (k_1\theta_e + \hat{\omega}_r) = 0$. Because $\theta_e = 0$, then $\omega_r - \hat{\omega} = -\tilde{\omega}_r = 0$. This means that the only solution that exists in the set Ω_c is the trivial solution { $(\theta_e, \tilde{\omega}) = (0,0)$ }. Therefore, $\theta_e \to 0$ as $t \to \infty$.

Adaptive Backstepping Control Design

This subsection presents the proposed adaptive backstepping control law for subsystem (II). By substituting for the control law u_2 , the dynamic equations for subsystem (II) become

$$\dot{y}_e = v_r \sin(\theta_e) - (k_1 \theta_e + \hat{\omega}_r) x_e \tag{8a}$$

$$\dot{x}_e = v_r \cos(\theta_e) + (k_1 \theta_e + \widehat{\omega}_r) y_e - u_1$$
(8b)

Let \hat{v}_r be the estimate of v_r . Then, the estimation error can be defined as

$$\tilde{\nu}_r = \hat{\nu}_r - \nu_r \tag{9}$$

Proposition 2. The dynamic model of subsystem (II) is asymptotically stable under the control law

$$u_{1} = \hat{v}_{r} \cos(\theta_{e}) - \hat{v}_{r} \,\overline{w} \sin(\theta_{e}) + \overline{w}^{2} \,x_{e} - \overline{w}^{-1} \sin(\theta_{e}) \,\dot{v}_{r}$$
$$-(k_{1} \,y_{e} - k_{1} \,\overline{w}^{-2} \,\hat{v}_{r} \sin(\theta_{e}) + \overline{w}^{-1} \,\hat{v}_{r} \cos(\theta_{e}))(\hat{\omega}_{r} - \overline{w})$$
$$-(y_{e} - \overline{w}^{-2} \,\hat{v}_{r} \sin(\theta_{e}))\dot{\omega}_{r} + k_{2} \,z$$
(10)

and the adaptation law

$$\dot{\hat{v}}_r = \sin(\theta_e) y_e - z \left(\overline{w} \sin(\theta_e) - \cos(\theta_e) \right)$$
(11)

where $k_2 > 0$ is a scalar, $\overline{w} := u_2 = k_1 \theta_e + \widehat{\omega}_r$, $z = x_e - \phi(\cdot)$, and where

$$\phi(\cdot) = \phi(y_e, \theta_e, \hat{v}_r, \hat{w}_r) = \overline{w} y_e + \overline{w}^{-1} \hat{v}_r \sin(\theta_e)$$
(12)

Proof: Viewing x_e in Equation (8a) as a virtual control input, the control law $x_e = \phi(y_e, \theta_e, \hat{v}_r, \hat{\omega}_r)$ is designed to stabilize the dynamic Equation (8a) in subsystem (II). For convenience, we use the short notation $\phi(\cdot)$ for $\phi(y_e, \theta_e, \hat{v}_r, \hat{\omega}_r)$. Consider the following Lyapunov function candidate:

$$V_1(y_e, \hat{v}_r) = \frac{1}{2}(y_e^2 + \tilde{v}_r^2)$$

Taking the time derivative of V_1 along the trajectories of Equation (8a) gives

$$\dot{V}_1 = v_r \sin(\theta_e) y_e - \overline{w} x_e y_e + \tilde{v}_r \dot{v}_r$$

Using the parameter error of Equation (9), and substituting for the virtual control input of Equation (12) give

$$\dot{V}_1 = -\overline{w}^2 \ y_e^2 - \tilde{v}_r \sin(\theta_e) \ y_e + \tilde{v}_r \ \dot{\hat{v}}_r \tag{13}$$

Finally, by selecting $\dot{\hat{v}}_r = y_e \sin(\theta_e)$ for the adaptation law, Equation (13) becomes

$$\dot{V}_1 \leq -\overline{w}^2 y_e^2$$

Since $\dot{V}_1 \leq 0$, then the dynamic Equation (8a) is bounded. Now, by following the steps of the backstepping control design, the error signal $z = x_e - \phi(\cdot)$ is defined. The design of the control law u_1 starts by considering the following Lyapunov function candidate

$$V_2(y_e, \hat{v}_r, x_e) = \frac{1}{2}(y_e^2 + \tilde{v}_r^2 + z^2)$$

Taking the time derivative of V_2 along the trajectories of Equations (8a)–(8b) gives

$$\dot{V}_2 = y_e \left(v_r \sin(\theta_e) - \overline{w} \, x_e \right) + \tilde{v}_r \, \dot{\hat{v}}_r + z \left(v_r \cos(\theta_e) + \overline{w} \, y_e - \, u_1 - \, \phi(\cdot) \right) \tag{14}$$

where it is clear from Equation (12), and the expression of \overline{w} that

$$\dot{\phi}(\cdot) = \frac{\partial \phi}{\partial y_e} \dot{y}_e + \frac{\partial \phi}{\partial \hat{v}_r} \dot{\hat{v}}_r + \frac{\partial \phi}{\partial \hat{\omega}_r} \dot{\hat{\omega}}_r + \frac{\partial \phi}{\partial \theta_e} \dot{\theta}_e$$
(15)

Adding and subtracting the term $\overline{w} \phi(\cdot) y_e$ to Equation (14), using the fact that $z = x_e - \phi(\cdot)$, and invoking the expression (15) for $\phi(\cdot)$, we have the following after some algebraic manipulations

$$\begin{split} \dot{V}_2 &= v_r \sin(\theta_e) \ y_e - \overline{w} \ z \ y_e - \overline{w} \ \phi(\cdot) y_e + \tilde{v}_r \ \dot{v}_r \\ &+ z \ (v_r \cos(\theta_e) + \overline{w} \ y_e - u_1 - v_r \ \overline{w} \ \sin(\theta_e) \\ &+ \overline{w}^2 \ x_e - \overline{w}^{-1} \sin(\theta_e) \ \dot{v}_r - (y_e - \overline{w}^{-2} \ \hat{v}_r \sin(\theta_e)) \dot{\omega}_r \\ &- (k_1 \ y_e - k_1 \ \overline{w}^{-2} \ \hat{v}_r \sin(\theta_e) + \overline{w}^{-1} \ \hat{v}_r \cos(\theta_e)) \ (\omega_r - \overline{w})) \end{split}$$

Now, substituting for ω_r by its estimate, $\hat{\omega}_r$, generated from the dynamic Equation (7), and using Equation (9) to replace v_r with $\hat{v}_r - \tilde{v}_r$, the above expression can be written as

$$\begin{split} \dot{V}_2 &= \hat{v}_r \sin(\theta_e) \ y_e - \overline{w} \ \phi(\cdot) y_e + \tilde{v}_r \ (\dot{v}_r - \sin(\theta_e) \ y_e \\ &+ z \ (\overline{w} \sin(\theta_e) - \cos(\theta_e))) + z(\hat{v}_r \cos(\theta_e) - \hat{v}_r \ \overline{w} \sin(\theta_e) \\ &+ \overline{w}^2 \ x_e - \overline{w}^{-1} \sin(\theta_e) \ \dot{v}_r - (y_e - \overline{w}^{-2} \ \hat{v}_r \sin(\theta_e)) \dot{\omega}_r \\ &- (k_1 \ y_e - k_1 \ \overline{w}^{-2} \ \hat{v}_r \sin(\theta_e) + \overline{w}^{-1} \ \hat{v}_r \cos(\theta_e)) \ (\hat{\omega}_r - \overline{w}) - u_1) \end{split}$$

Finally, substituting for $\phi(\cdot)$ from Equation (12), and for the control law (10)-(11), the above expression can be algebraically simplified and reduced to

$$\dot{V}_2 = -\overline{w}^2 y_e^2 - k_2 z^2$$

The stability of the closed-loop system can be established using Barbalat's lemma (see Appendix) as follows. Given the fact that $\theta_e \to 0$ as $t \to \infty$ and the fact that v_r and ω_r are bounded signals (Assumption 1), it can be shown that \dot{V}_2 is also bounded. Hence, \dot{V}_2 is uniformly continuous. Invoking Barbalat's lemma, it can be concluded that $\dot{V}_2 \to 0$ as $t \to \infty$, i.e., both y_e and z tend to zero as $t \to \infty$, which shows that trajectory tracking is successfully achieved.

SIMULATION STUDIES

The nonlinear dynamic model of the unicycle-type mobile robot was simulated with the proposed adaptive controllers given in Equations (6)–(7) and (10)–(12). The system parameters for the simulation were chosen such that the reference linear velocity was $v_r = 0.5$ and the reference angular velocity was $\omega_r = 0.6$. The initial conditions of the error system were set to $[x_e(0), y_e(0), \theta_e(0)]^T = [0.5, 0.4, 0.5]^T$. The controller gains were $k_1 = 1.25$ and $k_2 = 3.5$.

Figures 2–6 show the results of the simulation study. Figure 2 shows the asymptotic convergence of the errors in the orientation angle and the (x, y) position to zero as $t \to \infty$, which indicates successful tracking. Figure 3 depicts the

trajectories of the original system Equation (1) and the reference model Equation (2). Figures 3(a)-3(b) show a smooth convergence of the mobile robot's (x, y) coordinates towards the reference (x_r, y_r) coordinates. Figure 3(c) shows the convergence of the mobile robot's orientation angle towards the reference angle θ_r . Figure 4 depicts the convergence of the robot's trajectory towards the reference trajectory.

The adaptive tracking controllers are shown in Figure 5. Figure 5(a) shows the evolution of the control law u_1 , which represents the mobile robot's linear velocity v. Figure 5(b) depicts the evolution of the controller u_2 , which represents the mobile robot's angular velocity ω . Finally, Figure 6 shows the responses of the estimated angular velocity $\hat{\omega}_r$ and linear velocity \hat{v}_r , respectively. The estimates approached their steady-state values as $t \to \infty$.

CONCLUSION

This paper addresses the problem of trajectory tracking of a unicycle-type mobile robot, where it is assumed that the virtual robot's (reference) linear and angular velocities are not known exactly. The system dynamics is treated as two subsystems. Subsystem (I) describes the angle error dynamics and subsystem (I) describes the error dynamics of the robot's x-axis and y-axis coordinates. For subsystem (I), an adaptive control law that uses Lyapunov techniques is proposed, where the adaptation law is designed such that the error tends to zero despite the uncertainty in the reference angular velocity ω_r . For subsystem (II), a simple trick is used where the error dynamics of the x-axis and y-axis Cartesian coordinates are interchanged to allow for the application of a backstepping control design. A backstepping-based adaptive control scheme and a tuning law are then synthesised to force the position error to zero. The simulations showed that the proposed controllers ensured the asymptotic convergence of the robot's (x, y, θ) states towards the reference trajectory (x_r, y_r, θ_r) .



Figure 2. Tracking errors x_e , y_e , and θ_e .



Figure 3. Tracking trajectories of the (a) *x*-position, (b) *y*-position, and (c) θ -direction.



Figure 4. Reference trajectory (dashed line) vs actual trajectory (solid line).



Figure 5. Control inputs: (a) linear velocity $u_1 := v$ and (b) angular velocity $u_2 := \omega$.



Figure 6. Estimated parameters $\hat{\omega}_r$ and \hat{v}_r .

APPENDIX

Lemma [Barbalat's Lemma] If $\lim_{t\to\infty} \int_0^t f(\tau) d\tau = \alpha < \infty$ (i.e., exists and is finite), and f(t) is uniformly continuous, then $\lim_{t\to\infty} f(t) = 0$.

Proof: The proof of the lemma can be found in (Krstic et al., 1995).

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