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اختيار الاستراتيجية باستخدام تقنية اتخاذ القرار متعدد المعايير الضبابية ذات الفاصل الزمني من النوع الثاني وتطبيقاتها

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الخلاصة

اليوم أصبحت ظروف السوق التنافسية أكثر تحدياً في كل القطاعات. يجب على الشركات تحديد خرائط الطرق الخاصة بها من حيث الاستراتيجيات القصيرة والمتوسطة والطويلة المدى. يعتبر اختيار الإستراتيجية الأنسب من قبل مجموعة من صناع القرار عملية معقدة للغاية. وحيث أن التعبير اللغوي نفسه يمكن أن يمثل معاني مختلفة لأشخاص مختلفين، فإن عملية صنع القرار الغامضة هي أداة جيدة للاستخدام في قرارات المجموعة. يتم استخدام طرق متعددة ومصطلحات لغوية مختلفة لتعريف المجموعات الضبابية. في السنوات الأخيرة، تم إدخال طرق التحليل الهرمي الضبابية (AHP) وتقنية ترتيب الأفضلية بالتشابه مع الحل المثالي (TOPSIS) ذات الفاصل الزمني من النوع الثاني في المنشورات العلمية. الهدف من هذه الدراسة هو تقديم نموذج لاختيار استراتيجية جديدة عن طريق صنع القرار الجماعي. في الخطوة الأولى من النموذج، تم تحديد المعايير باستخدام بطاقة النتيجة المتوازنة. بعد ذلك، تم حساب أوزان المعايير باستخدام طريقة التحليل الهرمي الضبابية ذات الفاصل الزمني من النوع الثاني. وأخيراً، تم ذكر الاستراتيجيات البديلة المنظمة مع تقنية ترتيب الأفضلية بالتشابه مع الحل المثالي ذات الفاصل الزمني من النوع الثاني والاستراتيجيات الأكثر ملاءمة للشركة. تم تطبيق النموذج المُقترح في عملية اختيار الاستراتيجية لشركة نقل عام واسعة النطاق تعمل في اسطنبول / تركيا.

Strategy selection by using interval type-2 fuzzy mcdm and an application

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ABSTRACT

Competitive market conditions today are becoming more challenging in every sector. Companies must determine their road maps in terms of short-, mid-, and long-term strategies. The selection of the most suitable strategy by a group of decision makers is a very complicated process. As the same linguistic expression can represent different meanings to different people the fuzzy decision-making process is a good tool to use in group decisions. Various methods and different linguistic terms are used to define fuzzy sets. In recent years, the interval type-2 fuzzy AHP and interval type-2 TOPSIS methods have been introduced in the literature. The aim of this study is to present a new strategy selection model by using group decision-making. In the first step of model, criteria were determined by using Balanced Score Card. Then, criteria weights were calculated with interval type-2 fuzzy AHP. Finally, alternative strategies ordered with interval type-2 TOPSIS and the most suitable strategies for company were stated. The proposed model was applied in the strategy selection process of a large-scale public transportation company that operates in Istanbul/Turkey.

Keywords: balanced score card; group decision-making; interval type-2 fuzzy AHP; interval type-2 fuzzy TOPSIS; strategy selection.

INTRODUCTION

For every business, determining short-, mid-, and long-term strategies is an important key point of competition. Performance management tools can be used for both the performance monitoring and business strategy selection processes. In the literature, Balanced Score Card (BSC) is one of the most used performance measurement tools. According to Kaplan and Norton, measuring performance is as complicated as managing an organization (Kaplan & Norton, 1992). Managers need to measure performance in many different areas at the same time. Initially the focus and practice of balanced score card are directed towards profit-making institutions, but they also provide an excellent opportunity for the development of management in state-owned and nonprofit institutions. In any case, the success of state institutions and nonprofit organizations should be measured by how effective and efficient they are in meeting public needs (Kaplan, 2009). To perform this measurement, the following four dimensions were included in the method: financial dimension, customer dimension, inner process dimension, and learning and innovation dimension. These dimensions can also be used in the strategy selection process with AHP (Lee, Chen, & Chang, 2008).

AHP, which was proposed by Saaty, is an extremely popular method used in a variety of decision-making problems (Saaty, 1980). On the other hand, TOPSIS is a very famous decision-making method due to its simple calculations (Hwang & Yoon, 1981). In the AHP and TOPSIS methods, people's evaluations are shown as integers. However, in some cases, it may not be possible to explain the linguistic evaluations of decision makers by using integers. The main reasons behind this problem are that words can represent different meanings to different people, decision-making groups can have conflicting opinions, and decision makers cannot express their preferences with all numerical values.

In the literature, there are numerous fuzzy AHP methodologies proposed by different authors (Kahraman, Onar, & Oztaysi, 2015). These methods have introduced a systematic approach for the alternative selection, using the analysis of hierarchical structure and fuzzy set theory. Generally, decision-makers are not clear about their preferences because of the complexity in the pairwise comparison process. For that reason, in many cases, it is more dependable to use intermediate values rather than fixed-value judgments. In addition, fuzzy sets are useful for group decisions and linguistic evaluations where it is difficult to determine a complete membership function.

Fuzzy sets were first introduced by Zadeh (Lotfi A. Zadeh, 1965). In the crisp numbers, the ratio is either 0 or 1. If an element in the set belongs to the same set, the membership value is assumed to be 1. If it does not belong to that set, the membership value is assumed to be 0. Contrarily, there are uncertain limits in fuzzy sets (Akdag, Kalaycı, Karagöz, Zülfikar, & Giz, 2014). This means that the membership functions may take infinite possibilities between 0 and 1.

The concept of type-2 fuzzy set was introduced as an extension of the concept of type-1 fuzzy set (L. A. Zadeh, 1975). If the membership values are ambiguous and cannot be determined clearly, it would be more appropriate to use a type-2 fuzzy set instead of a type-1 fuzzy set (Mendel & Wu, 2006). Triangular or trapezoidal fuzzy numbers can be used in membership functions (Vahdani, Zandieh, & Tavakkoli-Moghaddam, 2011). In type-1 triangular membership functions, the center of the triangle, the base width, and the degree of overlapping are important decisions. On the other hand, many of those decisions are not needed in type-2 membership functions.

Uncertainty in the primary membership of the type-2 fuzzy set consists of a limited region, expressed as footprint of uncertainty (Özek, 2010). While type-1 membership functions are two-dimensional, type-2 membership functions are three-dimensional. The evaluation of type-2 fuzzy sets is more difficult than that of type-1 fuzzy sets because their computation is more complicated due to the extra dimension.

The interval type-2 fuzzy set is a special case of the generalized type-2 fuzzy set. Type-2 fuzzy sets require complex and cumbersome computation procedures. Interval type-2 fuzzy sets are more common because of their simple and reduced computations.

Chen et al. (S.-M. Chen & Lee, 2010a) proposed a method to solve fuzzy multicriteria decision problems with interval type-2 fuzzy sets by developing TOPSIS method. They also proposed a new ranking based method to handle fuzzy multiple attributes group decision-making problems and the arithmetic operations of interval type-2 fuzzy sets (S. M. Chen & Lee, 2010b).

Chiao used trapezoidal interval type-2 fuzzy sets for AHP method to select the best professor in terms of mathematical creativity, application creativity, administrative ability, and human maturity criteria (Chiao, 2012). A study by Erdoğan and Kaya, in which interval type-2 fuzzy set and TOPSIS method were used to rank the private universities in Istanbul, can be given as an example of fuzzy multicriteria decision-making (f-MCDM) usage in academics (Erdoğan & Kaya, 2014).

Personnel selection is a very popular application area of multicriteria decision-making (MCDM) techniques. A study conducted by Kahraman and Öztayşi using interval type-2 fuzzy AHP method for personnel selection can be given as an example (Kahraman & Öztayşi, 2013). The fuzzy TOPSIS method with veto threshold was also used in personnel selection problems (Kelemenis & Askounis, 2010).

There are many different applications for interval type-2 fuzzy MCDM. For example, using interval type-2 fuzzy set Zamri and Abdullah selected the best institutional system from five different alternatives with 34 criteria (Zamri & Abdullah, 2013). In MCDM area, Mardani made a generalized literature research in 1994-2014 with 403 papers (Mardani, Jusoh, & Zavadskas, 2015).

In this study, a new interval type-2 decision-making method was proposed for group decision-making. Defining the suitable criteria for strategy selection can be a complicated process. Because of that, criteria were determined by using Balanced Score Card method. And, in all variants of AHP methodologies, the number of criteria and alternatives should be 7±2. However, in some real-life situations, the number of alternatives may exceed the given limits. In this case, AHP cannot be a sufficient methodology. On the other hand, TOPSIS methodology can be used efficiently for alternative selection process but in this case, criteria weights must be determined with another methodology. For these reasons, we combined all these methodologies together. The proposed method consists of a combination of the interval type-2 fuzzy AHP method (Kahraman, Öztayşi, Uçal Sarı, & Turanoğlu, 2014) and the interval type-2 fuzzy TOPSIS method (S.-M. Chen & Lee, 2010a).

INTERVAL TYPE-2 FUZZY SETS

The interval type-2 fuzzy sets theory was proposed by Zadeh as an extended version of the type-1 fuzzy set (L. A. Zadeh, 1975). There are many usage areas of the interval type-2 fuzzy set. It can be combined with AHP in a supplier selection problem (Kahraman et al., 2014) or in the assessment of early warning ratings in occupational safety (Abdullah & Najib, 2014). In type-1 fuzzy sets, membership functions are completely defined, whereas, in type-2 fuzzy sets, membership functions are fuzzy.

The description of the type-2 fuzzy set in the literature is as follows:

Definition 2.1 Let X be a universal set ($X \neq \emptyset$). The membership function that defines $\tilde{\tilde{A}}$ type-2 fuzzy set is represented as $\mu_{\tilde{\tilde{A}}}$ (Mendel & Wu, 2006)

$$\tilde{\tilde{A}} = \left\{\left((x.u), \mu_{\tilde{\tilde{A}}}(x,u)\right) \middle| \forall x \in X, \forall u \in J_x \subseteq [0,1], 0 \leq \mu_{\tilde{\tilde{A}}}(x,u) \leq 1\right\} \qquad (1)$$

where x, the primary variable, has domain X; u, the secondary variable, has domain at each J_x at each $x \in X$; J_x is called the primary membership of x; and the secondary grades of $\tilde{\tilde{A}}$ all equal 1.

Definition 2.2 Let X be a universal set ($X \neq \emptyset$). The membership function that defines $\tilde{\tilde{A}}$ type-2 fuzzy set is represented as $\mu_{\tilde{\tilde{A}}}$. When all $\mu_{\tilde{\tilde{A}}}(x,u) = 1$, set $\tilde{\tilde{A}}$ is called the interval type-2 fuzzy set (Mendel & Wu, 2006). Set $\tilde{\tilde{A}}$, which is an interval type-2 fuzzy set, is considered to be a special case of a type-2 fuzzy set and can be represented as follows:

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} 1/(x,u), \text{ where } J_x \subseteq [0,1] \qquad (2)$$

Definition 2.3 The upper and lower membership functions of the interval type-2 fuzzy set are type-1 membership functions, respectively (Mendel & Wu, 2006). The reference point of interval type-2 fuzzy sets and the highest-lowest membership functions are used to characterize type-2 fuzzy sets. Figure 1 demonstrates a fuzzy set of type-2 fuzzy sets, where $\widetilde{A}$ represents type-1 fuzzy sets.

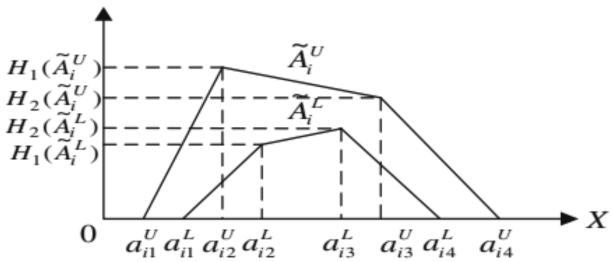

Figure 1. A trapezoidal interval type-2 fuzzy set graph (S.-M. Chen & Lee, 2010b)

According to the figure equation 3 can be written as

$$\tilde{\tilde{A}}_i = \left(\tilde{A}_i^U, \tilde{A}_i^L\right) = \left(\left(a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1\left(\tilde{A}_i^U\right), H_2\left(\tilde{A}_i^U\right)\right), \left(a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1\left(\tilde{A}_i^L\right), H_2\left(\tilde{A}_i^L\right)\right)\right) \tag{3}$$

where $\tilde{A}_i^U$ and $\tilde{A}_i^L$ type-1 fuzzy sets (S.-M. Chen & Lee, 2010b).

$a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U, a_{i1}^L, a_{i2}^L, a_{i3}^L$ and a_{i4}^L reference points of the interval type-2 fuzzy set of $\tilde{\tilde{A}}_i$.

$H_j\left(\tilde{A}_i^U\right)$ indicates the membership value of the $a_{i(j+1)}^U$ element in the $\tilde{A}_i^U$ upper trapezoidal membership function where $1 \leq j \leq 2$ and $1 \leq \mathrm{i} \leq \mathrm{n}, \forall i, j \in \mathrm{Z}$; $H_1\left(\tilde{A}_i^U\right) \in [0,1]$, $H_2\left(\tilde{A}_i^U\right) \in [0,1]$

$H_j\left(\tilde{A}_i^L\right)$ indicates the membership value of the $a_{i(j+1)}^U$ element in the $\tilde{A}_i^L$ lower trapezoidal membership function where $1 \leq j \leq 2$ and $1 \leq \mathrm{i} \leq \mathrm{n}, \forall i, j \in \mathrm{Z}$; $H_1\left(\tilde{A}_i^L\right) \in [0,1]$, $H_2\left(\tilde{A}_i^L\right) \in [0,1]$.

Definition 2.4 Addition operation of the trapezoidal interval type-2 fuzzy sets for (S.-M. Chen & Lee, 2010b). Assume that two fuzzy numbers are defined in equation 4 and 5.

$$\tilde{\tilde{A}}_1 = \left(\tilde{A}_1^U, \tilde{A}_1^L\right) = \left(\left(a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_1^U\right)\right), \left(a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_1^L\right)\right)\right) \tag{4}$$

$$\tilde{\tilde{A}}_2 = \left(\tilde{A}_2^U, \tilde{A}_2^L\right) = \left(\left(a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1\left(\tilde{A}_2^U\right), H_2\left(\tilde{A}_2^U\right)\right), \left(a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1\left(\tilde{A}_2^L\right), H_2\left(\tilde{A}_2^L\right)\right)\right) \tag{5}$$

The addition operation is defined as follows in equation 6:

$$\tilde{\tilde{A}}_1 \oplus \tilde{\tilde{A}}_2 = \left(\tilde{A}_1^U, \tilde{A}_1^L\right) \oplus \left(\tilde{A}_2^U, \tilde{A}_2^L\right)$$

$$= \left((a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; min\left(H_1\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_2^U\right)\right), min\left(H_2\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_2^U\right)\right)\right),$$

$$\left((a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L; min\left(H_1\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_2^L\right)\right), min\left(H_2\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_2^L\right)\right)\right) \tag{6}$$

Definition 2.5 Subtraction operation of the trapezoidal interval type-2 fuzzy sets for the operation is defined as follows in equation 7 (S.-M. Chen & Lee, 2010b):

$$\tilde{\tilde{A}}_1 \ominus \tilde{\tilde{A}}_2 = \left(\tilde{A}_1^U, \tilde{A}_1^L\right) \ominus \left(\tilde{A}_2^U, \tilde{A}_2^L\right) =$$

$$\left(\left(a_{11}^U - a_{24}^U, a_{12}^U - a_{23}^U, a_{13}^U - a_{22}^U, a_{14}^U - a_{21}^U; min\left(H_1\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_2^U\right)\right), min\left(H_2\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_2^U\right)\right)\right),\right.$$

$$\left.\left((a_{11}^L - a_{24}^L, a_{12}^L - a_{23}^L, a_{13}^L - a_{22}^L, a_{14}^L - a_{21}^L;\ min\left(H_1\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_2^L\right)\right), min\left(H_2\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_2^L\right)\right)\right)\right) \tag{7}$$

Definition 2.6 Multiplication operation of the trapezoidal interval type-2 fuzzy sets for the operation is defined as follows (S.-M. Chen & Lee, 2010b):

$$\tilde{\tilde{A}}_1 \otimes \tilde{\tilde{A}}_2 = \left(\tilde{A}_1^U, \tilde{A}_1^L\right) \otimes \left(\tilde{A}_2^U, \tilde{A}_2^L\right)$$

$$= \left((a_{11}^U \times a_{21}^U, a_{12}^U \times a_{22}^U, a_{13}^U \times a_{23}^U, a_{14}^U \times a_{24}^U;\ min\left(H_1\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_2^U\right)\right), min\left(H_2\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_2^U\right)\right)\right),$$

$$\left(\left(a_{11}^L \times a_{21}^L, a_{12}^L \times a_{22}^L, a_{13}^L \times a_{23}^L, a_{14}^L \times a_{24}^L;\ min\left(H_1\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_2^L\right)\right), min\left(H_2\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_2^L\right)\right)\right)\right) \tag{8}$$

Definition 2.7 Arithmetic operations between the trapezoidal interval type-2 fuzzy sets and k, where k is a real number, the operation is defined as follows in equation 9 and. 10 (S.-M. Chen & Lee, 2010b):

$$\tilde{\tilde{A}}_1 = (\tilde{A}_1^U, \tilde{A}_1^L)\ k\tilde{\tilde{A}}_1 = \left(\left(k \times a_{11}^U, k \times a_{12}^U, k \times a_{13}^U, k \times a_{14}^U;\ H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)\right)\left(\left(k \times a_{11}^L, k \times a_{12}^L, k \times a_{13}^L, k \times a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)\right)\right)\right) \quad (9)$$

$$\frac{\tilde{A}_1}{k} = \left(\left(\frac{1}{k} \times a_{11}^U, \frac{1}{k} \times a_{12}^U, \frac{1}{k} \times a_{13}^U, \frac{1}{k} \times a_{14}^U;\ H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)\right), \left(\left(\frac{1}{k} \times a_{11}^L, \frac{1}{k} \times a_{12}^L, \frac{1}{k} \times a_{13}^L, \frac{1}{k} \times a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)\right)\right)\right) \quad \text{where } k > 0; \quad (10)$$

Ranking in interval type-2 fuzzy sets

Chen and Lee revealed the fuzzy ranking method within a trapezoidal type-2 fuzzy set given in equation 4. According to this method (S.-M. Chen & Lee, 2010b) the Rank $\left(\tilde{\tilde{A}}_i\right)$ of the fuzzy interval type-2 fuzzy set ranking value can be expressed as

$$Rank\left(\tilde{\tilde{A}}_i\right) = M_1(\tilde{A}_i^U) + M_1(\tilde{A}_i^L) + M_2(\tilde{A}_i^U) + M_2(\tilde{A}_i^L) + M_3(\tilde{A}_i^U) + M_3(\tilde{A}_i^L) - \frac{1}{4}\left(S_1(\tilde{A}_i^U) + S_1(\tilde{A}_i^L) + S_2(\tilde{A}_i^U) + S_2(\tilde{A}_i^L) + S_3(\tilde{A}_i^U) + S_3(\tilde{A}_i^L) + S_4(\tilde{A}_i^U) + S_4(\tilde{A}_i^L)\right) + H_1(\tilde{A}_i^U) + H_1(\tilde{A}_i^L) + H_2(\tilde{A}_i^U) + H_2(\tilde{A}_i^L) \quad (11)$$

while $M_P(\tilde{A}_i^j)$, a_{ip}^j and $a_{i(p+1)}^j$ demonstrate the mean values of the elements;

$$M_P(\tilde{A}_i^j) = \left(a_{iP}^j + a_{i(p+1)}^j\right)/2, 1 \le p \le 3 \quad (12)$$

$S_q(\tilde{A}_i^j)$, a_{iq}^j and $a_{i(q+1)}^j$ demonstrate the standard deviation of the elements.

$$S_q(\tilde{A}_i^j) = \sqrt{\frac{1}{2}\sum_{k=q}^{q+1}\left(a_{ik}^j - \frac{1}{2}\sum_{k=q}^{q+1} a_{ik}^j\right)^2}, \quad 1 \le q \le 3, (12) \quad (13)$$

If $S_4(\tilde{A}_i^j)$, $a_{i1}^j, a_{i2}^j, a_{i3}^j, a_{i4}^j$ demonstrate the standard deviation of the elements,

$$S_4(\tilde{A}_i^j) = \sqrt{\frac{1}{4}\sum_{k=1}^{4}\left(a_{ik}^j - \frac{1}{4}\sum_{k=1}^{4} a_{ik}^j\right)^2}, \quad (14)$$

with $H_P(\tilde{A}_i^j), \tilde{A}_i^j$ trapezoidal membership function, $a_{i(p+1)}^j$ demonstrates the membership value of the elements. $(1 \le p \le 2, j \in \{U, L\}, ve\ 1 \le i \le n)$.

In Equation 11, total value of $M_1(\tilde{A}_i^U)$, $M_1(\tilde{A}_i^L)$, $M_2(\tilde{A}_i^U)$, $M_2(\tilde{A}_i^L)$, $M_3(\tilde{A}_i^U)$, $M_3(\tilde{A}_i^L)$ and $H_1(\tilde{A}_i^U)$, $H_1(\tilde{A}_i^L)$, $H_2(\tilde{A}_i^U)$, $H_2(\tilde{A}_i^L)$ denotes the simple ranking value while the mean of the standard deviations of $S_1(\tilde{A}_i^U), S_1(\tilde{A}_i^L)$ $S_2(\tilde{A}_i^U), S_2(\tilde{A}_i^L), S_3(\tilde{A}_i^U), S_3(\tilde{A}_i^L), S_4(\tilde{A}_i^U)$ *and* $S_4(\tilde{A}_i^L)$ is used as a penalty score in the simple ranking equation.

BSC - INTERVAL TYPE-2 FUZZY AHP-TOPSIS HYBRID METHOD

The steps of the proposed BSC-interval type-2 fuzzy AHP-TOPSIS hybrid method are 11, and they are as follows:

X: Set of alternatives

F: Set of criteria

$X = \{x_1, x_2, \dots, x_i\}$

$F = \{f_1, f_2, \dots, f_i\}$

Assume that p is the number of decision makers $(D_1, D_2, \dots, D_p)$.

The set of criteria, F, is divided into two separate sets as F_1 *and* F_2. F_1 represents a set of benefit criteria, while F_2 represents a set of cost criteria. In this case:

$F_1 \cap F_2 = \emptyset$

$F_1 \cup F_2 = F$

Step 1. This is the initializing step for algorithm. Here, criteria and alternatives are stated. Criteria are determined by using Balanced Score Card method. According to the criteria, hieratical AHP structure was created and a survey was applied for decision making team. Finally, consistencies were checked for AHP decisions.

Consistency ratios are calculated based on the data obtained through questionnaires completed by each decision maker, by creating pairwise comparison matrices. First of all, consistency indicator is calculated as equation 15.

$$\text{Consistency indicator } = \frac{\lambda_{\max} - \text{n}}{\text{n}-1} \tag{15}$$

Here, n represents number of criteria and λ_{max} is the Eigenvalue where, $\lambda_{max} \in \mathbb{R}$ and $\lambda_{max} \geq n$

For the randomness indicator, included in the calculation of the consistency ratio, the main criterion and subcriteria for each decision maker were calculated with reference to the "Randomness Indicator" table of Saaty (Saaty, 1980). Consistency ratio calculation is given in equation 16.

$$\text{Consistency Ratio } = \frac{\text{Consistency indicator}}{\text{Randomness Indicator}} \tag{16}$$

Consistency ratios must be smaller than 0.1 (Saaty, 1980). This ratio must be calculated for each comparison matrix.

Step 2. For p^{th} decision maker, the type-2 fuzzy pairwise comparison matrix $\tilde{\tilde{A}}_p$ and group decision matrix $\bar{A}$ are created (Kahraman et al., 2014).

$$\tilde{\tilde{A}}_p = \begin{bmatrix} 1 & \tilde{\tilde{a}}_{12}^p & \cdots & \tilde{\tilde{a}}_{1n}^p \\ \tilde{\tilde{a}}_{21}^p & 1 & \cdots & \tilde{\tilde{a}}_{2n}^p \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{a}}_{n1}^p & \tilde{\tilde{a}}_{n2}^p & \cdots & 1 \end{bmatrix} = \begin{bmatrix} 1 & \tilde{\tilde{a}}_{12}^p & \cdots & \tilde{\tilde{a}}_{1n}^p \\ 1/\tilde{\tilde{a}}_{12}^p & 1 & \cdots & \tilde{\tilde{a}}_{2n}^p \\ \vdots & \vdots & \ddots & \vdots \\ 1/\tilde{\tilde{a}}_{1n}^p & 1/\tilde{\tilde{a}}_{2n}^p & \cdots & 1 \end{bmatrix}, \text{ where } \bar{A} = \left(\tilde{\tilde{a}}_{ij}\right)_{nxn}$$

For group decision-making, the geometric mean of p decision maker is calculated as equation 17.

$$\bar{A} = \left[\tilde{\tilde{a}}_{ij}^1 \otimes \tilde{\tilde{a}}_{ij}^2 \otimes \dots \otimes \tilde{\tilde{a}}_{ij}^p\right]^{\frac{1}{p}} = \sqrt[p]{\tilde{\tilde{a}}_{ij}^1 \otimes \tilde{\tilde{a}}_{ij}^2 \otimes \dots \otimes \tilde{\tilde{a}}_{ij}^p} \tag{17}$$

where

$$1/\tilde{\tilde{a}} = \left(\left(\frac{1}{a_{14}^U}, \frac{1}{a_{13}^U}, \frac{1}{a_{12}^U}, \frac{1}{a_{11}^U}; H_1(a_{12}^U), H_2(a_{13}^U)\right), \left(\frac{1}{a_{24}^L}, \frac{1}{a_{23}^L}, \frac{1}{a_{22}^L}, \frac{1}{a_{21}^L}; H_1(a_{22}^L), H_2(a_{23}^L)\right)\right) \tag{18}$$

The trapezoidal linguistic terms to be used in the interval type-2 fuzzy AHP are given in Table 1.

Table 1. Linguistic terms expressing the weight of each criterion of interval type-2 fuzzy sets (Kahraman et al., 2014).

Definition and interval type 2 fuzzy scales of the linguistic variables.	
Absolutely Strong (AS)	((7,8,9,9;1,1), (7.2,8.2,8.8,9;0.8,0.8))
Very Strong (VS)	((5,6,8,9;1,1), (5.2,6.2,7.8,8.8;0.8,0.8))
Fairly Strong (FS)	((3,4,6,7;1,1), (3.2,4.2,5.8,6.8;0.8,0.8))
Slightly Strong (SS)	((1,2,4,5;1,1), (1.2,2.2,3.8,4.8;0.8,0.8))
Exactly Equal (E)	((1,1,1,1;1,1), (1,1,1,1;1,1)

Step 3. The geometric mean of each line is calculated, and then the normalization process is applied to the fuzzy weights (Kahraman et al., 2014).

The geometric mean of each line, $\tilde{\tilde{r}}_i$ is calculated as follows:

$$\tilde{\tilde{r}}_i = [\tilde{\tilde{a}}_{i1} \otimes \ldots . \otimes \tilde{\tilde{a}}_{in}]^{1/n} \tag{19}$$

$$\sqrt[n]{\tilde{\tilde{a}}_{ij}} = \left(\left(\sqrt[n]{a^U_{ij1}}, \sqrt[n]{a^U_{ij2}}, \sqrt[n]{a^U_{ij3}}, \sqrt[n]{a^U_{ij4}}; H^U_1(a_{ij}), H^U_2(a_{ij})\right),\right.$$

$$\left.\left(\sqrt[n]{a^L_{ij1}}, \sqrt[n]{a^L_{ij2}}, \sqrt[n]{a^L_{ij3}}, \sqrt[n]{a^L_{ij4}}; H^L_1(a_{ij}), H^L_2(a_{ij})\right)\right)$$

The fuzzy weight of the i^{th} criterion is calculated as follows:

$$\tilde{\tilde{w}}_i = \tilde{\tilde{r}} \otimes [\tilde{\tilde{r}} \oplus \ldots \oplus \tilde{\tilde{r}}_i \oplus \ldots \oplus \tilde{\tilde{r}}_n]^{-1} \tag{20}$$

$$\frac{\tilde{\tilde{a}}_{ij}}{\tilde{\tilde{b}}_{ij}} = \left(\frac{a^U_1}{b^U_4}, \frac{a^U_2}{b^U_3}, \frac{a^U_3}{b^U_2}, \frac{a^U_4}{b^U_1}, min\left(H^U_1(a), H^U_1(b)\right), min\left(H^U_2(a), H^U_2(b)\right)\right),$$

$$\left(\frac{a^L_1}{b^L_4}, \frac{a^L_2}{b^L_3}, \frac{a^L_3}{b^L_2}, \frac{a^L_4}{b^L_1}, min\left(H^L_1(a), H^L_1(b)\right), min\left(H^L_2(a), H^L_2(b)\right)\right)$$

Step 4. The global weights for each subcriterion are calculated according to

$$\widetilde{\widetilde{w'}}_{ij} = \tilde{\tilde{w}}_{ij} \otimes \tilde{\tilde{w}}_i, where\ 1 \le i \le m. and\ 1 \le j \le n. \tag{21}$$

Here $\widetilde{\widetilde{w'}}_{ij}$ represents the global weights of criteria.

Step 5. For p. decision maker, a decision matrix of Y_p and normalized decision matrix $\bar{Y}$ are created (S.-M. Chen & Lee, 2010a).

$$Y_P = \left(\tilde{\tilde{f}}_{ij}^{P}\right)_{mxn} = \begin{array}{c} \\ \tilde{\tilde{f}}_{11}^{P} \\ \tilde{\tilde{f}}_{21}^{P} \\ \vdots \\ \tilde{\tilde{f}}_{m1}^{P} \end{array} \begin{array}{c} \begin{array}{cccc} X_1 & X_2 & \cdots & X_n \end{array} \\ \begin{bmatrix} \tilde{\tilde{f}}_{11}^{P} & \tilde{\tilde{f}}_{12}^{P} & \cdots & \tilde{\tilde{f}}_{1n}^{P} \\ \tilde{\tilde{f}}_{21}^{P} & \tilde{\tilde{f}}_{22}^{P} & \cdots & \tilde{\tilde{f}}_{2n}^{P} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{f}}_{m1}^{P} & \tilde{\tilde{f}}_{m2}^{P} & \cdots & \tilde{\tilde{f}}_{mn}^{P} \end{bmatrix} \end{array}, \text{ where } \bar{Y} = \left(\tilde{\tilde{f}}_{ij}\right)_{m\times n} \tag{22}$$

Here, $\tilde{\tilde{f}}_{ij}$ is an interval type-2 fuzzy set where

$$\tilde{\tilde{f}}_{ij} = \left(\frac{\tilde{\tilde{f}}_{ij}^{1} \oplus \tilde{\tilde{f}}_{ij}^{2} \oplus \ldots \oplus \tilde{\tilde{f}}_{ij}^{k}}{p}\right), 1 \le i \le m, 1 \le j \le n, 1 \le k \le p \tag{23}$$

and p represents the number of decision makers. Equation 22 is represented the initial matrix for type-2 fuzzy TOPSIS methodology. Here, rows are referred alternatives and columns are indicated criteria where comes from type-2 fuzzy AHP. The linguistic terms used in the application of the fuzzy numbers to be used in the interval type-2 fuzzy TOPSIS method are shown in Table 2.

Table 2. Linguistic terms expressing the weight of each criterion of interval type-2 fuzzy sets (S.-M. Chen & Lee, 2010a).

Linguistic terms demonstrating the weight of each criterion for interval type-2 fuzzy TOPSIS method	
Very Low (VL)	((0,0,0,0.1;1,1), (0,0,0,0.05;0.9,0.9))
Low (L)	((0,0.1,0.1,0.3;1,1), (0.05,0.1,0.1,0.2;0.9,0.9))
Medium Low (ML)	((0.1,0.3,0.3,0.5;1,1), (0.2,0.3,0.3,0.4;0.9,0.9))
Medium (M)	((0.3,0.5,0.5,0.7;1,1), (0.4,0.5,0.5,0.6;0.9,0.9))
Medium High (MH)	((0.5,0.7,0.7,0.9;1,1), (0.6,0.7,0.7,0.8,0.9,0.9))
High (H)	((0.7,0.9,0.9,1.0;1,1), (0.8,0.9,0.9,0.95;0.9,0.9))
Very High (VH)	((0.9,1.0,1.0,1.0;1,1), (0.95,1.0,1.0,1.0;0.9,0.9))

Step 6. A weighted decision matrix, $\bar{Y}_w$, is formed (S.-M. Chen & Lee, 2010a).

$$\bar{Y}_w = \left(\tilde{\tilde{v}}_{ij}\right)_{mxn} = \begin{array}{c} \\ f_1 \\ f_2 \\ \vdots \\ f_m \end{array} \begin{array}{c} \begin{array}{cccc} X_1 & X_2 & \cdots & X_n \end{array} \\ \begin{bmatrix} \tilde{\tilde{v}}_{11} & \tilde{\tilde{v}}_{12} & \cdots & \tilde{\tilde{v}}_{1n} \\ \tilde{\tilde{v}}_{21} & \tilde{\tilde{v}}_{22} & \cdots & \tilde{\tilde{v}}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{v}}_{m1} & \tilde{\tilde{v}}_{m2} & \cdots & \tilde{\tilde{v}}_{mn} \end{bmatrix} \end{array} \tag{24}$$

$$\tilde{\tilde{v}}_{ij} = \tilde{\tilde{w}}_{ijg} \otimes \tilde{\tilde{f}}_{ij}, 1 \leq i \leq m. and\ 1 \leq j \leq n. \tag{25}$$

Step 7. The ranking value, $Rank(\tilde{\tilde{v}}_{ij})$, of the interval type-2 fuzzy set $\tilde{\tilde{v}}_{ij}$ is calculated using Equation 12 $(1 \leq j \leq n)$. The sorted weighted decision matrix $\bar{Y}_w^*$ is calculated according to Equation 26 (S.-M. Chen & Lee, 2010a).

$$\bar{Y}_w^* = \left(Rank(\tilde{\tilde{v}}_{ij})\right)_{m\times n} \text{ where } 1 \leq i \leq m\ and\ 1 \leq j \leq n \tag{26}$$

Step 8. Positive ideal solution $x^+ = (v_1^+, v_2^+, \cdots, v_m^+)$ and negative ideal solution $x^- = (v_1^-, v_2^-, \cdots, v_m^-)$ are found (S.-M. Chen & Lee, 2010a).

F_1; set of benefit criteria

F_2; set of cost criteria

$$v_i^+ = \begin{cases} max\{Rank(\tilde{\tilde{v}}_{ij})\}, & if\ \ f_i \in F_1 \\ min\{Rank(\tilde{\tilde{v}}_{ij})\}, & if\ \ f_i \in F_2 \end{cases}, v_i^- = \begin{cases} min\{Rank(\tilde{\tilde{v}}_{ij})\}, & if\ \ f_i \in F_1 \\ max\{Rank(\tilde{\tilde{v}}_{ij})\}, & if\ \ f_i \in F_2 \end{cases}, 1 \leq i \leq m. \tag{27}$$

Step 9. The distance $d^+(x_j)$ between each alternative (x_j) and the positive ideal solution x^+ is calculated (S.-M. Chen & Lee, 2010a).

$$d^+(x_j) = \sqrt{\sum_{i=1}^{m}\left(Rank(\tilde{\tilde{v}}_{ij}) - v_i^+\right)^2}, where\ 1 \leq j \leq n. \tag{28}$$

The distance $d^-(x_j)$ between each alternative (x_j) and the negative ideal solution x^- is calculated.

$$d^-(x_j) = \sqrt{\sum_{i=1}^{m}\left(Rank(\tilde{\tilde{v}}_{ij}) - v_i^-\right)^2}, where\ 1 \leq j \leq n. \tag{29}$$

Step 10. The proximity coefficient $C(x_j)$ of x_j is calculated by using Eq. (28) and (29) (S.-M. Chen & Lee, 2010a).

$$C(x_j) = \frac{d^-(x_j)}{d^+(x_j)+d^-(x_j)} \tag{30}$$

Step 11. $C(x_j)$ values are sorted where $1 \leq j \leq n$. According to the largest $C(x_j)$ values, the alternative x_j values are sorted (S.-M. Chen & Lee, 2010a).

APPLICATION

In this section, an application for strategy selection problem is presented. The proposed method is applied for a public transportation company. A selected company wanted to improve its performance, competition abilities, and market share. To do that, the company needed to determine strategies and investment on them. Here, it is aimed to state the most suitable investment options for the company.

First of all, a decision-making group is constituted with high level and operational managers. This group is authorized with the management of company and taking decisions. Criteria and alternatives are the result of meetings with decision-making group. On the other hand, fuzzy AHP and TOPSIS surveys are also applied to this decision-making group. The proposed methodology has assisted a group of managers to make strategic decisions.

Step 1. In our example problem, there are seven decision makers, four main criteria, 12 subcriteria, and 12 alternatives. Decision makers are denoted as $D_1, D_2, D_3, D_4, D_5, D_6$ and D_7, alternatives donated as a_1, a_2, a_3, a_4, a_5, $a_6, a_7, a_8, a_9, a_{10}, a_{11}$ and a_{12}, main criteria donated as $k_{a1}, k_{a2}, k_{a3}, k_{a4}$, subcriteria donated as $k_1, k_2, k_3, k_4, k_5, k_6$, $k_7, k_8, k_9, k_{10}, k_{11}$ and k_{12}. Main criteria and subcriteria used in example problem are given in Table 3.

Table 3. The hierarchy of the theme assessment model.

Goal	Main Criteria	Subcriteria
Choosing Investment Strategy	Customer (k_{a1})	Customer Satisfaction (k_1)
		New Customer (k_2)
		Customer Loyalty (k_3)
	Finance (k_{a2})	Cost (k_4)
		Income (k_5)
		Financial Sustainability (k_6)
	Internal Process (k_{a3})	Quality (k_7)
		Productivity (k_8)
		Activity (k_9)
	Learning and Development (k_{a4})	Employee Qualification (k_{10})
		Information System Competency (k_{11})
		Motivation Authorization and Adaptation (k_{12})

The alternatives are as follows:

- ✓ Passenger and Driver Safety (a_1)
- ✓ New Product and Service Development (a_2)
- ✓ Innovation (a_3)
- ✓ Effective and Efficient Processes (a_4)
- ✓ Teamwork and Communication (a_5)
- ✓ Holistic Leadership (a_6)
- ✓ Agility in Service (a_7)
- ✓ Strong Financial Structure (a_8)
- ✓ Sustainable Services (a_9)
- ✓ Environmental Protection (a_{10})
- ✓ Profitability (a_{11})
- ✓ Service Quality (a_{12})

AHP surveys were applied for decision group. Here, main criteria and subcriteria were compared within their own group. After that, consistency ratios were calculated. According to equation 16, all AHP comparison tables are consistent.

Step 2. In this step, fuzzy pairwise comparison matrices were composed. As an example, D_1's pairwise comparisons for main criteria are shown in Table 4. The linguistic terms used in here were given in Table 1.

Table 4. The pairwise comparison matrix of the main criteria and for D_1

Main Criteria	k_{a1}	k_{a2}	k_{a3}	k_{a4}
k_{a1}	E	QS	VS	QS
k_{a2}	1/QS	E	SS	1/SS
k_{a3}	1/VS	1/SS	E	1/SS
k_{a4}	1/QS	SS	SS	E

For the initial group decision matrices, the geometric means of the main criteria are calculated. The initial matrices for the main criteria are given in Table 5.

Table 5. Initial group decision matrices.

	$\boldsymbol{k_{a1}}$	$\boldsymbol{k_{a2}}$	$\boldsymbol{k_{a3}}$	$\boldsymbol{k_{a4}}$
$\boldsymbol{k_{a1}}$	(1,1,1,1;1,1) (1,1,1,1;1,1)	(2.36,3.48,5.57,6.59;1,1) (2.59,3.69,5.36,6.39;0.8,0.8)	(4.07,4.76,5.90,6.34;1,1) (4.21,4.89,5.77,6.26;0.8,0.8)	(4.27,5.47,7.31,7.98;1,1) (4.52,5.70,7.10,7.85;0.8,0.8)
$\boldsymbol{k_{a2}}$	(0.15,0.18,0.29,0.42;1,1) (0.6,0.19,0.27,0.39;0.8,0.8)	(1,1,1,1;1,1) (1,1,1,1;1,1)	(1.26,1.84,2.97,3.73;1,1) (1.38,1.95,2.85,3.55;0.8,0.8)	(1.32,1.92,3.02,3.73;1,1) (1.45,2.03,2.90,3.56;0.8,0.8)
$\boldsymbol{k_{a3}}$	(0.16,0.17,0.21,0.25;1,1) (0.16,0.17,0.20,0.24;0.8,0.8)	(0.27,0.340.54,0.79;1,1) (0.28,0.35,0.51,0.72,0.8,0.8)	(1,1,1,1;1,1) (1,1,1,1;1,1)	(0.79,1.22,2.00,2.51;1,1) (0.89,1.30,1.92,2.39;0.8,0.8)
$\boldsymbol{k_{a4}}$	(0.13,0.14,0.18,0.23;1,1) (0.13,0.14,0.18,0.22;0.8,0.8)	(0.27,0.33,0.52,0.76;1,1) (0.28,0.35,0.49,0.69;0.8,0.8)	(0.40,0.50,0.82,1.26;1,1) (0.42,0.52,0.77,1.13;0.8,0.8)	(1,1,1,1;1,1) (1,1,1,1;1,1)

Step 3. Using Table 4 and Equation 17, the group decisions are calculated as shown in Table 6.

Table 6. Linear geometric averages of the main criteria.

$\boldsymbol{k_{a1}}$	(2.53,3.09,3.94,4.27;1,1) (2.65,3.19,3.85,4.21;0.8,0.8)
$\boldsymbol{k_{a2}}$	(0.71,0.89,1.27,1.56;1,1) (0.75,0.93,1.22,1.49;0.8,0.8)
$\boldsymbol{k_{a3}}$	(0.43,0.51,0.69,0.84;1,1) (0.45,0.53,0.67,0.80;0.8,0.8)
$\boldsymbol{k_{a4}}$	(0.34,0.39,0.53,0.69;1,1) (0.35,0.40,0.51,0.64;0.8,0.8)

The normalized criteria weights are calculated with equation 19 as given in Table 7.

Table 7. Normalized main criteria

k_{a1}	(0.34,0.48,0.81,1.07;1,1) (0.41,0.55,0.83,1.00;0.8,0.8)
k_{a2}	(0.10,0.14,0.26,0.39;1,1) (0.12,0.16,0.26,0.35;0.8,0.8)
k_{a3}	(0.06,0.08,0.14,0.21;1,1) (0.07,0.09,0.14,0.19;0.8,0.8)
k_{a4}	(0.05,0.06,0.11,0.17;1,1) (0.05,0.07,0.11,0.15;0.8,0.8)

Step 4. By using Table 7 and Equation 21, the global weights of sub criteria are calculated as given in Table 8.

Table 8. Global weight of the subcriteria.

k_1	(0.10,0.21,0.60,1.08;1,1) (0.13,0.25,0.58,0.95;0.8,0.8)
k_2	(0.04,0.08,0.22,0.41;1,1) (0.05,0.09,0.21,0.36;0.8,0.8)
k_3	(0.05,0.09,0.24,0.43;1,1) (0.06,0.10,0.23,0.38;0.8,0.8)
k_4	(0.01,0.03,0.09,0.19;1,1) (0.02,0.03,0.08,0.16;0.8,0.8)
k_5	(0.01,0.02,0.06,0.13;1,1) (0.01,0.02,0.05,0.11;0.8,0.8)
k_6	(0.02,0.06,0.22,0.49;1,1) (0.03,0.07,0.21,0.41;0.8,0.8)
k_7	(0.01,0.01,0.05,0.10;1,1) (0.01,0.02,0.04,0.09;0.8,0.8)
k_8	(0.01,0.01,0.03,0.07;1,1) (0.01,0.01,0.03,0.06;0.8,0.8)
k_9	(0.01,0.03,0.12,0.26;1,1) (0.02,0.04,0.11,0.22;0.8,0.8)
k_{10}	(0.01,0.02,0.06,0.13;1,1) (0.01,0.02,0.06,0.11;0.8,0.8)
k_{11}	(0.01,0.01,0.04,0.11;1,1) (0.01,0.02,0.04,0.08;0.8,0.8)
k_{12}	(0.01,0.01,0.04,0.09;1,1) (0.01,0.02,0.04,0.08;0.8,0.8)

Step 5. A decision matrix is established for each decision maker and criteria. An example table for customer satisfaction criterion is given in Table 9. Decision matrices are denoted by $Y_1, Y_2, Y_3, Y_4, Y_5, D_6$ and Y_7, alternatives by $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}$ and a_{12}. $\bar{Y}$ is the average decision matrix. The linguistic terms used in the TOPSIS method are given in Table 2.

Table 9. Initial decision table part for TOPSIS.

	Alternatives	**Decision-Makers**						
		D1	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**
Criterion: Customer Satisfaction	**A1**	VH	VH	VH	VH	VH	VH	VH
	A2	M	ML	H	ML	H	MH	MH
	A3	MH	ML	MH	L	H	M	H
	A4	L	M	MH	ML	M	ML	ML
	A5	ML	H	M	L	L	M	VL
	A6	MH	MH	M	L	ML	L	VL
	A7	M	MH	VH	H	MH	MH	M
	A8	VL	M	MH	M	M	ML	ML
	A9	M	H	MH	MH	M	H	H
	A10	MH	MH	MH	ML	M	VH	MH
	A11	H	H	H	H	H	H	H
	A12	VH	VH	MH	VH	VH	H	H

Based on Eq. (22) and Eq. (23), the normalized group decision matrix $\bar{Y}$ was constructed. Some of, $\bar{Y}$ values were calculated as follows:

$$\tilde{\tilde{f}}_{11} = \big((0.9, 1, 1, 1;\ 1, 1), (0.95, 1, 1, 1;\ 0.9, 0.9)\big)$$

$$\tilde{\tilde{f}}_{12} = \big((0.41, 0.61, 0.61, 0.78;\ 1, 1), (0.51, 0.61, 0.61, 0.7;\ 0.9, 0.9)\big)$$

$$\tilde{\tilde{f}}_{13} = \big((0.4, 0.58, 0.58, 0.75;\ 1, 1), (0.49, 0.58, 0.58, 0.67;\ 0.9, 0.9)\big)$$

$$\tilde{\tilde{f}}_{14} = \big((0.15, 0.32, 0.32, 0.52;\ 1, 1), (0.24, 0.32, 0.32, 0.42;\ 0.9, 0.9)\big)$$

$$\tilde{\tilde{f}}_{15} = \big((0.2, 0.34, 0.34, 0.51;\ 1, 1), (0.27, 0.34, 0.34, 0.42;\ 0.9, 0.9)\big)$$

$$\tilde{\tilde{f}}_{16} = \big((0.27, 0.42, 0.42, 0.61;\ 1, 1), (0.35, 0.42, 0.42, 0.52;\ 0{,}9, 0{,}9)\big)$$

$$\tilde{\tilde{f}}_{17} = \big((0.52, 0.71, 0.71, 0.87;\ 1, 1), (0.62, 0.71, 0.71, 0.79;\ 0.9, 0.9)\big)$$

$$\tilde{\tilde{f}}_{18} = \big((0.22, 0.4, 0.4, 0.58;\ 1, 1), (0.31, 0.4, 0.4, 0.49;\ 0.9, 0.9)\big)$$

$\tilde{\tilde{f}}_{19} = ((0.52, 0.72, 0.72, 0.88; 1, 1), (0.62, 0.72, 0.72, 0.80; 0.9, 0.9))$

$\tilde{\tilde{f}}_{110} = ((0.47, 0.65, 0.65, 0.82, 1, 1), (0.56, 0.65, 0.65, 0.74; 0.9, 0.9))$

$\tilde{\tilde{f}}_{111} = ((0.78, 0.92, 0.92, 0.98; 1, 1), (0.85, 0.92, 0.92, 0.95; 0.9, 0.9))$

$\tilde{\tilde{f}}_{112} = ((0.78, 0.92, 0.92, 0.98; 1, 1), (0.85, 0.92, 0.92, 0.95; 0.9, 0.9))$

Step 6. Based on Eq. (24) and (25), a weighted decision matrix $\overline{Y}_w$ is constructed. The weighted decision matrix is demonstrated as follows:

$\tilde{\tilde{v}}_{11} = ((0.09,0.21,0.60,1.08; 1,1), (0.13,0.25,0.58,0.95; 0.72,0.72))$

$\tilde{\tilde{v}}_{12} = ((0.04,0.08,0.21,0.41; 1,1), (0.05,0.09,0.21,0.35; 0.72,0.72))$

$\tilde{\tilde{v}}_{13} = ((0.04,0.09,0.24,0.43; 1,1), (0.05,0.10,0.23,0.38; 0.72,0.72))$

$\tilde{\tilde{v}}_{14} = ((0.01,0.02,0.06,0.16; 1,1), (0.01,0.02,0.05,0.12; 0.72,0.72))$

$\tilde{\tilde{v}}_{15} = ((0.01,0.01,0.04,0.12; 1,1), (0.01,0.02,0.04,0.09; 0.72,0.72))$

$\tilde{\tilde{v}}_{16} = ((0.01,0.04,0.16,0.43; 1,1), (0.02,0.05,0.16,0.33; 0.72,0.72))$

$\tilde{\tilde{v}}_{17} = ((0.01,0.01,0.04,0.10; 1,1), (0.01,0.02,0.04,0.08; 0.72,0.72))$

$\tilde{\tilde{v}}_{18} = ((0.00,0.00,0.01,0.04; 1,1), (0.00,0.00,0.01,0.03; 0.72,0.72))$

$\tilde{\tilde{v}}_{19} = ((0.01,0.02,0.08,0.21; 1,1), (0.01,0.03,0.07,0.16; 0.72,0.72))$

$\tilde{\tilde{v}}_{110} = ((0.00,0.01,0.03,0.10; 1,1), (0.01,0.01,0.03,0.07; 0.72,0.72))$

$\tilde{\tilde{v}}_{111} = ((0.00,0.01,0.03,0.09; 1,1), (0.01,0.01,0.03,0.07; 0.72,0.72))$

$\tilde{\tilde{v}}_{112} = ((0.00,0.01,0.03,0.08; 1,1), (0.00,0.01,0.03,0.06; 0.72,0.72))$

Step 7. Based on Eq. (26), the ranking value of $\mathrm{Rank}(\tilde{\tilde{v}}_{11})$ of the $\tilde{\tilde{v}}_{11}$,which is the fuzzy set of the interval type-2, is calculated as follows:

Here, $(1 \leq i \leq 12)$ and $1 \leq j \leq 12)$.

$$\mathrm{Rank}(\tilde{\tilde{v}}_{11}) = M_1(\tilde{v}_{11}^U) + M_1(\tilde{v}_{11}^L) + M_2(\tilde{v}_{11}^U) + M_2(\tilde{v}_{11}^L) + M_3(\tilde{v}_{11}^U) + M_3(\tilde{v}_{11}^L) - \frac{1}{4}\Big(S_1(\tilde{v}_{11}^U) +$$

$$S_1(\tilde{v}_{11}^L) + S_2(\tilde{v}_{11}^U) + S_2(\tilde{v}_{11}^L) + S_3(\tilde{v}_{11}^U) + S_3(\widetilde{A}_i^L) + S_4(\tilde{v}_{11}^U) + S_4(\tilde{v}_{11}^L)\Big) + H_1(\tilde{v}_{11}^U) + H_1(\tilde{v}_{11}^L) +$$

$$H_2(\tilde{v}_{11}^U) + H_2(\tilde{v}_{11}^L)$$

$$= 0.15 + 0.19 + 0.40 + 0.42 + 0.84 + 0.76 - \frac{1}{4}(0 + 0 + 0.06 + 0.06 + 0.22 + 0.19 +$$

$$0.39 + 0.32) + 1 + 0.72 + 1 + 0.72 = 5.89$$

By the same token, other ranking values are; $\text{Rank}(\tilde{\tilde{v}}_{12}) = 4.35$, $\text{Rank}(\tilde{\tilde{v}}_{13}) = 4.43$, $\text{Rank}(\tilde{\tilde{v}}_{14}) = 3.70$, $\text{Rank}(\tilde{\tilde{v}}_{15}) = 3.64$, $\text{Rank}(\tilde{\tilde{v}}_{16}) = 4.14$, $\text{Rank}(\tilde{\tilde{v}}_{17}) = 3.62$, $\text{Rank}(\tilde{\tilde{v}}_{18}) = 3.50$, $\text{Rank}(\tilde{\tilde{v}}_{19}) = 3.78$, $\text{Rank}(\tilde{\tilde{v}}_{110}) = 3.60, \text{Rank}(\tilde{\tilde{v}}_{111}) = 3.60, \text{Rank}(\tilde{\tilde{v}}_{112}) = 3.56$

Based on Eq. (10) the sorted weighted decision matrix, $\overline{Y}_w^*$, values are calculated.

Step 8. Based on Eq. (27), the positive ideal solution is $a^+ = (v_1^+, v_2^+, \cdots, v_m^+)$ and the negative ideal solution is $a^- = (v_1^-, v_2^-, \cdots, v_m^-)$.

$$a^+ = (v_1^+, v_2^+, \ldots, v_m^+) = \Big(\max\big(\text{Rank}(\tilde{\tilde{v}}_{11}), \text{Rank}(\tilde{\tilde{v}}_{12}), \text{Rank}(\tilde{\tilde{v}}_{13}), \text{Rank}(\tilde{\tilde{v}}_{14}), \text{Rank}(\tilde{\tilde{v}}_{15}),$$

$$\text{Rank}(\tilde{\tilde{v}}_{16}), \text{Rank}(\tilde{\tilde{v}}_{17}), \text{Rank}(\tilde{\tilde{v}}_{18}), \text{Rank}(\tilde{\tilde{v}}_{19}), \text{Rank}(\tilde{\tilde{v}}_{110}), \text{Rank}(\tilde{\tilde{v}}_{111}), \text{Rank}(\tilde{\tilde{v}}_{112})\big)\Big)$$

$$= (5.89, 4.35, 4.43, 3.77, 3.64, 4.33, 3.62, 3.57, 3.84, 3.63, 3.60, 3.62)$$

$$a^- = (v_1^-, v_2^-, \ldots, v_m^-) = \Big(\min\big(\text{Rank}(\tilde{\tilde{v}}_{11}), \text{Rank}(\tilde{\tilde{v}}_{12}), \text{Rank}(\tilde{\tilde{v}}_{13}), \text{Rank}(\tilde{\tilde{v}}_{14}), \text{Rank}(\tilde{\tilde{v}}_{15}), \text{Rank}(\tilde{\tilde{v}}_{16})$$

$$\text{Rank}(\tilde{\tilde{v}}_{17}), \text{Rank}(\tilde{\tilde{v}}_{18}), \text{Rank}(\tilde{\tilde{v}}_{19}), \text{Rank}(\tilde{\tilde{v}}_{110}), \text{Rank}(\tilde{\tilde{v}}_{111}), \text{Rank}(\tilde{\tilde{v}}_{112})\big)\Big)$$

$$= (4.35, 3.78, 3.84, 3.61, 3.53, 3.88, 3.55, 3.50, 3.70, 3.53, 3.52, 3.51)$$

Step 9. By using Eq. (28), the distance $d^+(a_j)$ between each alternative (a_j) and the positive ideal solution a^+ and by using Eq. (29), the distance $d^-(a_j)$ between each alternative (a_j) and the negative ideal solution a^- can be calculated as shown below where, $1 \leq j \leq 12$.

$$d^+(a_1) = \sqrt{\textstyle\sum_{i=1}^{12}\big(\text{Rank}(\tilde{\tilde{v}}_{i1}) - v_i^+\big)^2} = 0.23$$

$$d^-(a_1) = \sqrt{\textstyle\sum_{i=1}^{12}\big(\text{Rank}(\tilde{\tilde{v}}_{i1}) - v_i^-\big)^2} = 1.77$$

$$d^+(a_2) = \sqrt{\textstyle\sum_{i=1}^{12}\big(\text{Rank}(\tilde{\tilde{v}}_{i2}) - v_i^+\big)^2} = 0.97$$

$$d^-(a_2) = \sqrt{\textstyle\sum_{i=1}^{12}\big(\text{Rank}(\tilde{\tilde{v}}_{i2}) - v_i^-\big)^2} = 0.89$$

$$d^+(a_3) = \sqrt{\textstyle\sum_{i=1}^{12}\big(\text{Rank}(\tilde{\tilde{v}}_{i3}) - v_i^+\big)^2} = 1.05$$

$$d^-(a_3) = \sqrt{\textstyle\sum_{i=1}^{12}\big(\text{Rank}(\tilde{\tilde{v}}_{i3}) - v_i^-\big)^2} = 0.79$$

Step 10. By using Eq. (30), the proximity coefficient $C(a_j)$ of a_j can be calculated as:

$$C(a_1) = \frac{d^-(a_1)}{d^+(a_1) + d^-(a_1)} = \frac{1{,}77}{0{,}23 + 1{,}77} = 0{,}89$$

$C(a_2) = 0.48$, $C(a_3) = 0.43$, $C(a_4) = 0.18$, $C(a_5) = 0.08$, $C(a_6) = 0.15$, $C(a_7) = 0.57$, $C(a_8) = 0.26$,

$C(a_9) = 0.63$, $C(a_{10}) = 0.49$, $C(a_{11}) = 0.75$, $C(a_{12}) = 0.76$ where, $1 \leq j \leq 12$.

Step 11. The shorted alternatives are given in Table 10.

Table 10. Ranking table.

1	0.89	**A1**	Passenger and Driver Safety
2	0.76	**A12**	Service Quality
3	0.75	**A11**	Profitability
4	0.63	**A9**	Sustainable Services
5	0.57	**A7**	Agility in Service
6	0.49	**A10**	Environmental Protection
7	0.48	**A2**	New Product and Service Development
8	0.43	**A3**	Innovation
9	0.26	**A8**	Strong Financial Structure
10	0.18	**A4**	Effective and Efficient Processes
11	0.15	**A6**	Holistic Leadership
12	0.08	**A5**	Teamwork and Communication

Here, company needed to define important strategies. According to Table 10, top-managerial level of company decided to focus on top three strategies. The most suitable strategies are, respectively, “Passenger and Driver Safety” (a_1), “Service Quality” (a_{12}), and “Profitability” (a_{11}).

The main purpose of company is passenger transportation. For this reason, “Passenger and Driver Safety” is the most important strategy. On the other hand, public transportation is a part of service sector. Therefore, “Service Quality” is second important strategy that also has strong relationship between profitability and safety. Finally, profitability is another important strategy that the company needs to focus on.

Selected alternative strategies were agreed with all members of decision-making group. At the end of the decision process, a group consensus was shown up. As a result, these feedbacks indicate that our method yields good results.

CONCLUSION

The strategy selection process, which is extremely important for firms, is a very complicated process. It can be examined in several main criteria, subcriteria, and alternatives. Due to the nature of the problem, linguistic expressions are more successful than crisp numbers. In the literature, there are several variants of fuzzy sets used for this purpose. The literature and past experiences both show that type-2 fuzzy sets represent more flexible and smarter solutions in terms of the representation of the weighting qualities in decision-making problems.

In this paper, Balanced Score Card, group decision-making, interval type-2 fuzzy AHP method, and interval type-2 TOPSIS method are integrated together to form a new model for decision-making. Criteria and subcriteria identified by Balanced Score Card and interval type-2 fuzzy AHP-TOPSIS method were applied to the group decision-making process for the selection of the best alternative. In this study, a new road map for strategy selection and an integrated model for interval type-2 fuzzy AHP-TOPSIS method were suggested.

In future works, the same problems could be solved with different decision-making tools, or the suggested interval type-2 AHP-TOPSIS model could be applied to other problems. In addition, triangular type-2 membership function could be used instead of the trapezoid type-2 membership function. Comparisons between these two membership functions could be explored.

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