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في اتجاه تطوير علاقات ارتباطية عالمية للخانق السطحي – تحسين شامل وتقنيات حسابية دقيقة

***محمد الرمح، **فيصل العدواني و* سعد العاطفي**
*قسم تكنولوجيا هندسة البترول، كلية الدراسات التكنولوجية، الهيئة العامة للتعليم التطبيقي والتدريب، الكويت
**قسم هندسة البترول، كلية الهندسة والبترول، جامعة الكويت، الكويت

الخلاصة

يعتبر خانق رأس البئر جزءً لا يتجزأ من أنظمة الإنتاج. تحمي خوانق رأس البئر المعدات السطحية من الضربات القوية الناتجة عن معدل التدفق العالي، تضرر الطبقة وتكوين الماء المخروطي. هذه الخوانق تتحكم أيضاً في معدل انخفاض ضغط المكمن. وفقاً لذلك، تم تجميع بنك من المعلومات للتدفق الحرج عبر صمامات الخانق لتطوير نموذج أداء عالمي للخانق. تم تطوير ثلاثة نماذج للتنبؤ بمعدل التدفق ثنائي الحالة خلال الخوانق. النموذج الأول والثاني تم قياسهما بواسطة نماذج الذكاء الاصطناعي وهما الشبكة العصبية الاصطناعية ونموذج أقل مربع لدعم ناقلات الآلة. النموذج الثالث تم تطويره باستخدام خوارزمية التحسين الأمثل العالمي. النماذج الثلاثة تتفوق نتائجها على جميع النماذج الحالية، والنتائج تسلط الضوء بوضوح على دقة وتفوق نماذج الذكاء الاصطناعي على النماذج التجريبية الحالية.

Toward the Development of a Universal Choke Correlation – Global Optimization and Rigorous Computational Techniques

Muhammad Alrumah*,***, Faisal Aladwani** and Saad Alatefi*

*Department of Petroleum Engineering Technology, College of Technological Studies, Public Authority for Applied Education and Training, P.O. Box 42325, Shuwaikh 70654, Kuwait
**Petroleum Engineering Department, Specialty of Drilling Engineering, College of Engineering and Petroleum, Kuwait University, P.O. Box 5969 Safat 13060, Kuwait
***Corresponding Author: mk.alrumah@paaet.edu.kw

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ABSTRACT

Wellhead choke is an integral part of production systems. Wellhead chokes act as regulators to protect surface facilities from high rate slugs, formation damage, and water coning. These chokes also control the decline rate of reservoir pressure. Accordingly, a diverse data bank of critical flow across wellhead chokes has been compiled to develop a universal choke performance model. Three models were developed to predict liquid flow rate in two-phase flow through chokes. The first two models are measured by computational intelligence paradigms such as the Artificial Neural Network and the Least Square Support Vector Machine. The third model was developed using a global optimization simplex algorithm. The three models outperformed all existing models, and the results clearly highlight the accuracy and superiority of the intelligence models over the existing empirical correlations.

Keywords: choke correlation; vector machine; artificial intelligence; global optimization; computational technique.

INTRODUCTION

The production of crude oil in almost all wells must observe the two-phase flow through chokes in the wellhead. Wellhead chokes are essential in designing production schemes due to their ability to regulate the flow according to different requirements such as meeting the optimum flow rate according to the production plan scheme, preventing the damage of producing formation, reducing or delaying water coning, and preventing flow rate fluctuation. Therefore, it is very important to have a representative accurate method to predict the flow rate for the installed choke. The situation is very complicated when it comes to a two-phase flow due to the presence of both oil and gas. Production engineers are looking for a method that simplifies the problem by developing an easier technique that calculates surface flow rate using readily available data that has a direct relation to hydrocarbon flow across the wellhead choke.

Flow rate is a function of various parameters, but for practical purposes, empirical correlations have been developed to use the readily available field data to calculate the flow rate across wellhead choke. These correlations have limitations because they are best applied to data with a similar range for which they are developed. In addition, large errors are expected when the correlations are used with data that are out of range of the original correlations for some field applications. The presence of the two-phase flow across the wellhead choke adds more complexity to an already complicated problem. The use of compositional choke flow model might lead to eliminating some of the complexity to the composition variation during production, but seldom does a detailed composition exist for a wellhead since it requires a detailed analysis that might add additional operational costs to the well. Furthermore, if a detailed composition exists, then it must be thermodynamically modelled using equation of states method to determine flow and transport properties. The complexity of the two-phase flow is reduced by decreasing the affecting parameters. However, this act sacrifices accuracy. This complexity can be captured using artificial intelligence techniques like the artificial neural network (ANN) or the Least Square Support Vector Machine (LSSVM). These methods prove their ability to capture the complexity of a behaviour and produce a model that can predict the behaviour within an acceptable accuracy.

LITERATURE REVIEW

One of the earliest studies for two-phase flow through wellhead chokes was developed by Tangren et. al. (Tangren, R.F., Dodge, C.H. and Seifert, 1949), and the results indicated a constrained for volumetric gas oil ratio less than one. Gilbert (Gilbert, 1954) was the first one to develop an empirical correlation for two-phase flow through chokes. The correlation calculates the liquid flow rate as a function of three operational parameters. These parameters include the choke diameter, gas-liquid ratio, and wellhead pressure as shown in the following equation.

$$q_L = \frac{(p_{WH})^D \ (d_c)^C}{A \ (R_{GL})^B} \tag{1}$$

A, B, C, and D are constant coefficients that are determined based on the actual field data used by Gilbert. Many authors provided several modifications to Gilbert's correlation using their own field data. Subsequently, many of these authors updated the coefficients of their correlations to produce accurate results based on their data. Table 1 shows various coefficients used by different authors following the empirical correlation developed by Gilbert (Gilbert, 1954). Ros (Ros, 1960) developed a model for subcritical flow through chokes and restrictions. Poettmann and Beck (Poettmann, F.H and Beck, 1963) developed a set of charts that depicted the development process by adapting Ros's analysis. Following that, several authors (Achong, 1961; Al-Attar and Abdul-Majeed, 1988; Alrumah and Bizanti, 2007; Mesallati, Bizanti, and Mansouri, 2000; Owolabi et al., 1991; Pilehvari, 1981; Sadeq, 2012) developed a set of constants similar to Gilbert equation using field data. Al-Khalifa and Al-Marhoun (Al-Khalifa and Al-Marhoun, 2013) developed two neural network models for two-phase flow through chokes. The parameters used in both models include wellhead pressure, temperature, choke diameter, oil flow rate, oil specific gravity, gas specific gravity, and producing gas-oil ratio. The first model predicts choke diameter, and the second predicts the oil flow rate.

Gorjaei et al. (Gholgheysari Gorjaei et al., 2015) used the least squared support vector machine technique with particle swarm optimization to develop a model that predicts the oil flow rate using three input parameters. These three input parameters included wellhead pressure, choke diameter, and producing gas-oil ratio. AlAjmi et. al. (AlAjmi et al., 2015) employed artificial neural networks to develop a model to predict oil flow rate for multiphase flow through chokes. These models are for both critical and subcritical flow conditions. The prediction ability of the existing models relies on their data range in which they are developed for rather than the data size.

Table 1. Regression Coefficients for Various Models.

Author	A	B	C	D
Gilbert, 1954	1.00E+01	5.46E-01	1.89E+00	1.00E+00
Baxendell, 1958	9.56E+00	5.46E-01	1.93E+00	1.00E+00
Ros, 1960	1.74E+01	5.00E-01	2.00E+00	1.00E+00
Achong, 1961	3.82E+00	6.50E-01	1.88E+00	1.00E+00
Pilehvari, 1981	4.67E+01	3.13E-01	2.11E+00	1.00E+00
Al-Attar and Abdul-Majeed, 1988	6.15E+01	4.71E-01	1.63E+00	8.31E-01
Owolabi et al., 1991	3.57E+01	2.89E-01	1.83E+00	1.00E+00
Bizanti and Mansouri, 2000	1.77E+01	9.46E-01	1.43E+00	1.68E+00
Alrumah and Bizanti, 2007	1.88E+02	6.32E-02	1.95E+00	9.66E-01
Sadeq, 2012	5.25E-05	-1.01E-01	7.04E-01	-6.90E-01

STUDY OBJECTIVES

The purpose of this research study is to develop new models for predicting wellhead choke performance. The new model covers a wide range of field operational parameters that were not covered within existing models. In addition, the new models (Artificial Neural Networks and Least Squares Support Vector Machines) are computationally intelligent, and both techniques are kernel based learning methods, which handle various nonlinear complexities within input parameters; this is not easily captured with existing conventional regression models (Gharbi and Elsharkawy, 2003; Hegeman et al., 2009).

LEAST SQUARE SUPPORT VECTOR MACHINE

Least square support vector machine (LSSVM) is an extension of the support vector machine (SVM). SVM is the classification and pattern recognition technique introduced in 1992 (Boser et al., 1992). The advantage of SVM method over ANN is that ANN training functions such as the classical multilayer perceptron might suffer from various local minima, which requires increasing the number of neurons to capture a correct pattern. On the other hand, SVM based on kernel functions (i.e., polynomial, radial basis functions, splines, etc.) will determine the optimum unique solution and number of neurons that is determined from a convex optimization technique such as simplex. Recently, the LSSVM method was used for nonlinear classification and function approximation applicable in large dimensional parameters (Boser et al., 1992). The training data set is used to determine optimum LSSVM parameters, while the testing data set is used to validate the developed LSSVM model. Both training and testing datasets like those used in the development of the ANN were utilized in this study. The two main LSSVM parameters that must be tuned are the regularization parameter (γ) and the squared bandwidth (σ^2). The tuning parameters are determined using a standard simplex optimization algorithm from the training data set. Both mean absolute percentage error (MAPE) and correlation coefficient (R^2) were used to test the performance of the developed LSSVM model. A cross plot between actual and calculated data using LSSVM shows a functional agreement between the calculated rates with the actual rates for both training and testing data set. The LSSVM is measured using a radial basis function kernel to determine the optimum LSSVM parameters (γ, σ2). The model is initialized with arbitrary random values, and the optimum values are then determined using a simplex optimization algorithm. Optimum results show that $\gamma = 519$ and $\sigma^2 = 0.3$.

ARTIFICIAL NEURAL NETWORK

Artificial neural networks are information processing systems that imitate the process of the human's brain in learning and making decisions. The human's brain learns from the information that is received, then analyzes the information, establishes relations and connections between the information, and then builds its own correlations and predictive models, which are a result of a learning process. These built models and correlations are used to predict future outputs or possible outcomes based on new information. Inspired from the learning process of the human's brain, the idea of the artificial neural networks came to introduce a new way of building models for solving problems. An artificial neural network consists of neurons and weights. The process of building a neural network is by introducing a data set consisting of independent and dependent variables (inputs and outputs), and this set is divided into two groups: training and testing. The training group will be fed to the neural network to learn and establish the relationship between the input and output. During the learning process, the weights are changed to obtain the desired output. After building the network, the input from the test data is introduced to the network to estimate the output based on the learning process. The estimated output can be compared to the actual output and to calculate the error for evaluating the network. The best developed model was a feedforward network with two hidden layers (Figure 1). The first layer has 39 neurons and the second layer has 23 neurons.

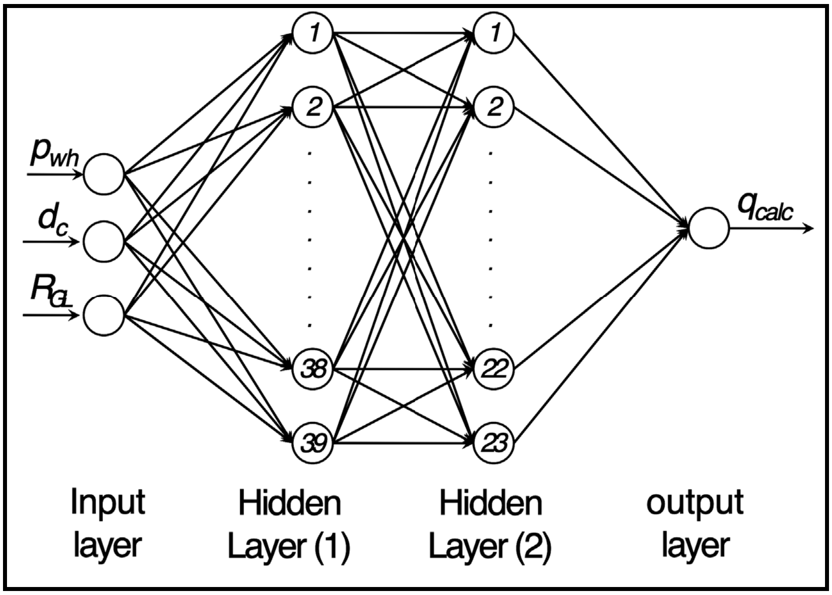

Figure 1. The structure of the ANN model.

SIMPLEX OPTIMIZATION ALGORITHM

The simplex optimization method is a direct search global optimization method for real world problems (Nelder and Mead, 1965; Wright, 2012). The method simply defines a set of different solutions for given problems. These methods used the operations of reflection, expansion, contraction, and shrinkage, which are the best solutions to solve the problems faced. In this paper, the developed model's parameters vary from other existing models using a large databank. The algorithm will find the optimum solution by minimizing the error between the actual and calculated datasets.

DATA DESCRIPTION

The dataset is composed of a total of 1,111 samples collected from the literature (Al-Attar and Abdul-Majeed, 1988; Alrumah and Bizanti, 2007; Ashford and Pierce, 1975; Gholgheysari Gorjaei et al., 2015; Mesallati, Bizanti, and Mansouri, 2000; Poettmann, F.H and Beck, 1963). Each sample is composed of a choke diameter (in units of /64th in), well head pressure, and gas oil ratio. Table 2 shows a descriptive statistic for the dataset used in this study. As shown in Table 2, this study covers a wide range of field data.

Table 2. Descriptive Statistics.

Parameter	Mean	SE Mean	St. Dev.	Minimum	Maximum
d_c (/64th in)	37.504	0.487	16.216	4.2	90.51
p_{wh} (psi)	1186.4	27.9	928.8	85.3	4374
R_{GL} (SCF/STB)	1337.6	50	1666.9	17	18579.3
q_L (STB/D)	3178	94.1	3134.1	10.5	14480.8

PERFORMANCE OF EXISTING MODELS

The models in Table 1 were used to predict flow rate through wellhead chokes for the dataset used in this study. Results of the existing models will be compared with each other to choose the best model. The best model will be used as a benchmark to compare with the models developed in this study (ANN, LSSVM, and SIMPLEX).

MODELS PERFORMANCE METRICS

The three developed models in this study were validated against the dataset. The dataset points are divided into two groups. The training group (778 data points) was used to derive the proposed models, while the remaining 333 data points were used to validate the proposed models. Several accuracy measurements were made available to validate the performance of the developed model against the database. Mean Absolute Percentage Error (MAPE), Pearson's correlation coefficient (R^2), and Root Mean Square Error (RMSE) were used as statistical error measurements for validation purposes.

For "n" data points, both the actual flow rate (q_{act}) and the calculated flow rate (q_{calc}) were used to calculate MAPE. The formula is shown as follows.

$$\text{MAPE} = 100\left(\frac{1}{n}\right)\sum\left(\frac{|q_{act} - q_{calc}|}{q_{act}}\right) \qquad (2)$$

MAPE has an advantage over other error statistical measurements since it expresses errors as percentages relative to the experimental data points reflected over the total number of data points. This reflection is without any distortion from either positive or negative deviation from the mean, which could result in a mean average error.

The coefficient of determination (R^2) used widely in regression analysis models is defined as

$$R^2 = \left(\frac{\sum (q_i - \bar{q})_{act} (q_i - \bar{q})_{calc}}{\left(\sqrt{(q_i - \bar{q})^2_{act}}\right)\left(\sqrt{(q_i - \bar{q})^2_{calc}}\right)} \right)^2 \quad (3)$$

R^2 ranges from 0 to 1. In this case, 1 denotes a perfect fit of the developed model to the experimental data. The closer the values to 1 indicates the model precision as it relates to accuracy.

The relative mean squared error (RMSE) is a statistical accuracy measurement, which is an alternative to the popular mean square error (MSE). The advantage of RMSE over MSE is that it severely punishes data points with large errors. The calculation is shown as follows.

$$RMSE = \sqrt{\frac{\sum (q_{act} - q_{calc})^2}{n}} \quad (4)$$

In addition to the above numerical statistical parameters, two graphical representations of model performance used to qualitatively determine the effectiveness of the developed model over the experimental data points. The first graph is a cross plot, which is a plot of experimental data vs. calculated data. This graph shows the effectiveness of the developed model. The second graph is a plot of the relative deviation error (RDE) vs. experimental data, and it shows the standard accuracy for several developed models. RDE is defined as

$$RDE = 100\left(\frac{q_{act} - q_{calc}}{q_{act}}\right) \quad (5)$$

RESULTS OF EXISTING MODELS

The existing models using Eq. (1), with their coefficients, as shown in Table 1 were applied to the dataset. Statistical results are shown in Table 3. In addition, Table 4 shows the ranking of the existing models according to the three performance indicators (MAPE, RMSE, and R^2). For example, Gilbert's model has the highest MAPE and RMSE, but it has the second lowest coefficient of determination (R^2). As shown in Table 4, Gilbert (Gilbert, 1954) shows the best performance in terms of all three performance indicators. The second best model is Baxendell (Baxendell, 1958). According to the results (Table 4), Gilbert is chosen as the base model to be compared with the new developed models in this study.

Table 3. Performance of Existing Models.

Model	MAPE	RMSE	R^2
Gilbert, 1954	29.95%	2957	0.529
Baxendell, 1958	33.31%	3019	0.528
Ros, 1960	32.33%	3041	0.528
Achong, 1961	36.18%	3008	0.521
Pilehvari, 1981	134.44%	8009	0.525
Al-Attar and Abdul-Majeed, 1988	96.47%	5285	0.527
Owolabi et al., 1991	45.11%	3305	0.533
Bizanti and Mansouri, 2000	45.22%	3116	0.507
Alrumah and Bizanti, 2007	62.38%	4215	0.517
Sadeq, 2012	653.45%	5896	0.032

Table 4. Performance Rank for Existing Models.

Model	Performance Rank				Final Rank
	MAPE	RMSE	R^2	Average	
Gilbert, 1954	1	1	2	1.33	1
Baxendell, 1958	3	3	3	3.00	2
Ros, 1960	2	4	4	3.33	3
Achong, 1961	4	2	7	4.33	5
Pilehvari, 1981	9	10	6	8.33	9
Al-Attar and Abdul-Majeed, 1988	8	8	5	7.00	7
Owolabi et al., 1991	5	6	1	4.00	4
Bizanti and Mansouri, 2000	6	5	9	6.67	6
Alrumah and Bizanti, 2007	7	7	8	7.33	8
Sadeq, 2012	10	9	10	9.67	10

RESULTS AND ANALYSIS OF THE DEVELOPED MODELS

In this study, the dataset used to generate a new set of coefficients following Eq. (1) determined the best coefficients. A Nelder-Mead simplex optimization algorithm was applied to determine the optimum values that yield the minimum MAPE between actual oil rates (q_{act}) and the calculated flow rate (q_{calc}) following Eq. (1). The training data group was used to determine the optimum coefficient values (A, B, C, and D) following Eq. (1), utilizing the simplex algorithm. After determining the optimum coefficients, the model is applied against the testing group, which was not used in determining the optimum coefficients. The optimum coefficients following Eq. (1) are shown in Table 5.

Table 5. Regression Coefficients from Simplex Algorithm.

	A	B	C	D
Simplex algorithm	11.262	0.525	1.918	0.987

Table 6 shows the performance of the new developed models for both the training and testing groups. In addition, the results in Table 6 show the superiority of LSSVM prediction method over both ANN and the new model using the simplex-mead algorithm. Also, the table shows the performance of the best model among existing correlations. Results show that the intelligence computational technique performed better than the developed model that used the simplex algorithm. This finding is due to the flexibility of the intelligent technique in capturing the relationship for real life complex problems.

Table 6. Comparison Between Developed Model for both Training and the Testing dataset.

	Training				Testing			
	Gilbert	ANN	LSSVM	SIMPLEX	Gilbert	ANN	LSSVM	SIMPLEX
MAPE (%)	30.60%	21.26%	15.41%	30.64%	27.05%	22.06%	21.45%	26.77%
R^2 (-)	0.8662	0.9763	0.9824	0.8675	0.8829	0.9292	0.9477	0.8847
RMSE (BOPD)	1174.76	485.01	418.41	1162.66	1075.84	863.98	719.63	1066.76

Figure 2 shows the comparison of the MAPE between the new developed models for both training and testing groups. As shown in the figure, the LSSVM shows the best performance over the other two models with a MAPE of 15.41% (training) and 21.45% (testing). In terms of model accuracy, LSSVM outperformed the other two models as shown in Figure 3, where it has the highest coefficient of determination value of 0.9824 (training) and 0.9477 (testing). The previous R^2 results clearly depicted the cross plot figure where the actual rates are plotted against the calculated rates using the three developed models in this study for both the training and testing groups.

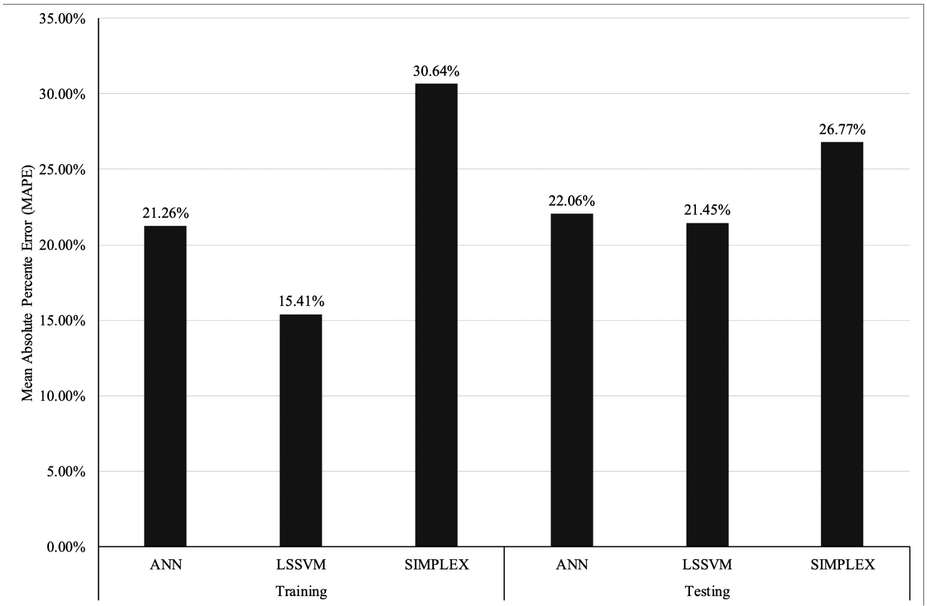

Figure 2. MAPE Comparison for Developed Models for both Training and Testing Datasets.

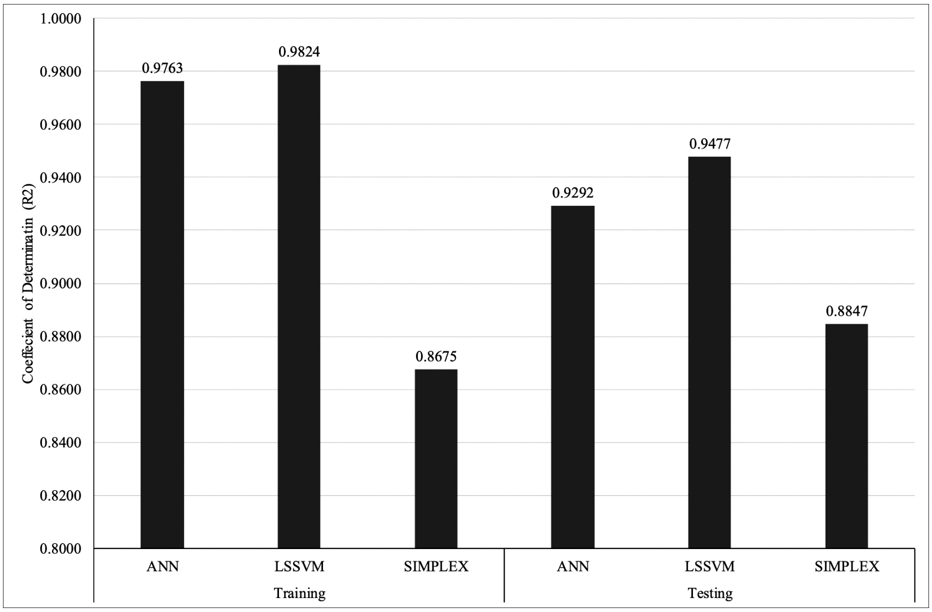

Figure 3. R^2 Comparison for Developed Models for both Training and Testing Datasets.

Figure 4, Figure 5, and Figure 6 show the cross plot of the new developed models. The unit slope line plotted in each figure represents how each model fits the actual rate values, the closeness of the data points to the unit slope line, and the better representation of the model to the actual values. Another graphical representation is the plot of the relative deviation error (RDE) for each model. This plot identifies error distribution for each model relative to the actual rates. Figure 7, Figure 8, and Figure 9 show the relative deviation plot for the three developed models versus the actual flow rate. The closer the data is to the zero line, the better the model is. The smaller deviation among the real and calculated variables is shown clearly in both ANN and LSSVM.

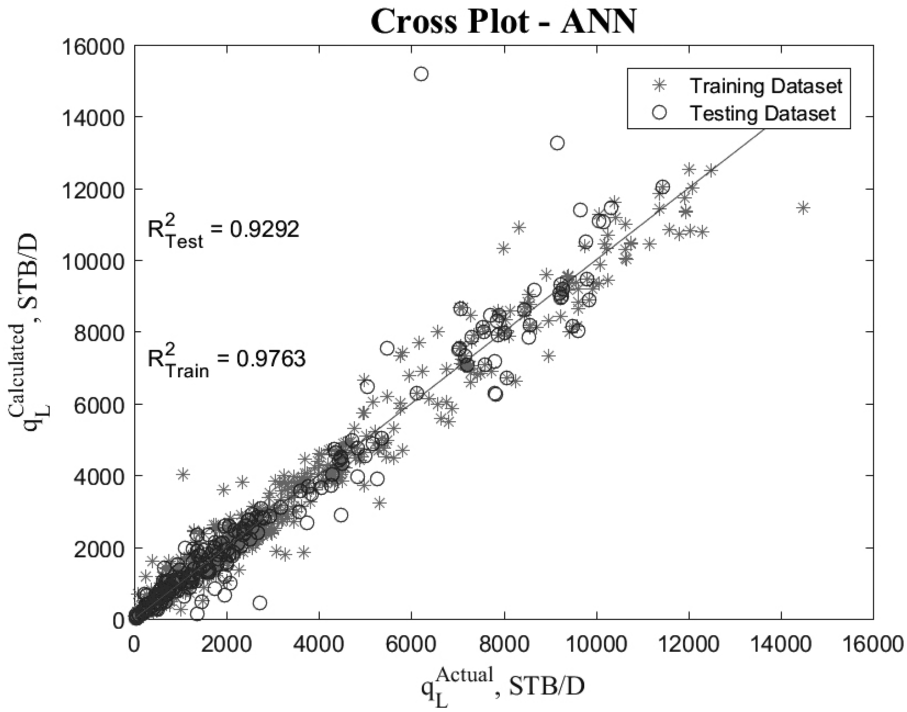

Figure 4. Cross plot for the ANN Developed Model (Training and Testing).

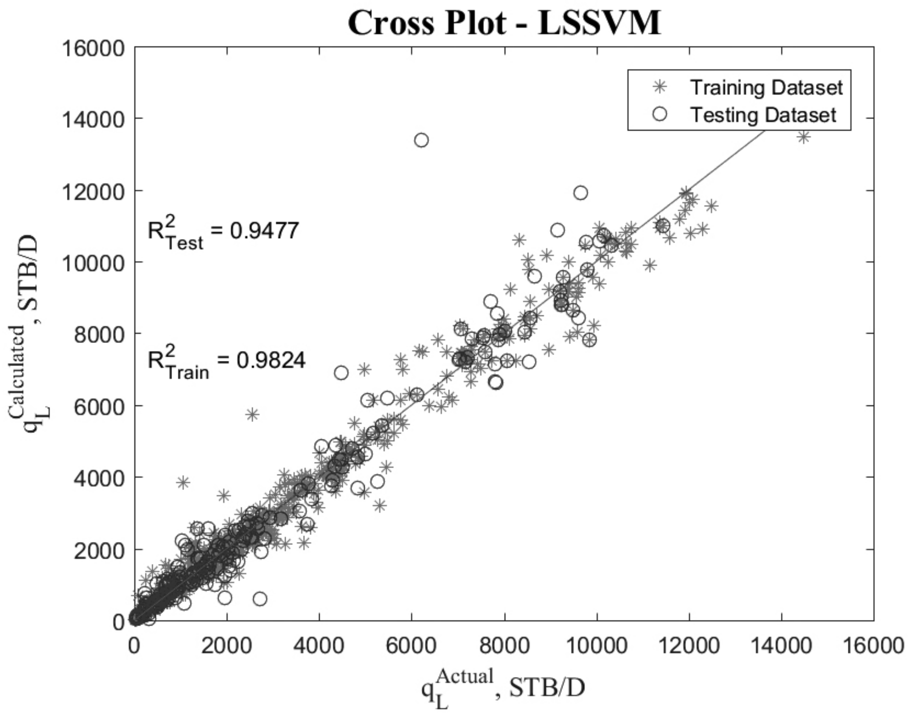

Figure 5. Cross plot for the LSSVM Developed Model (Training and Testing).

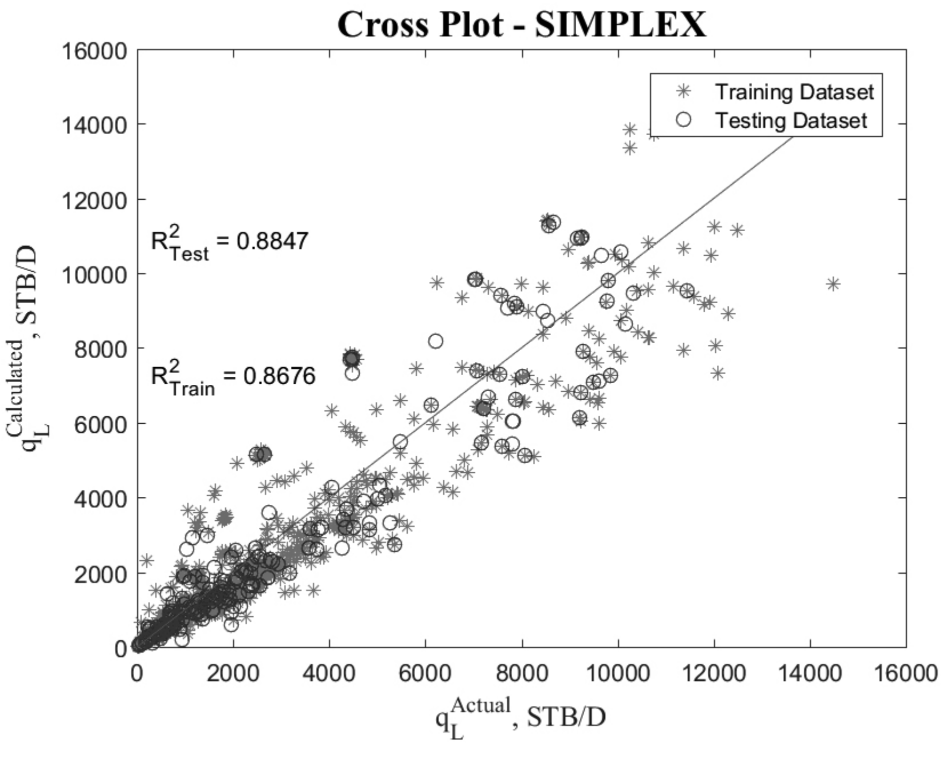

Figure 6. Cross plot for the SIMPLEX Developed Model (Training and Testing).

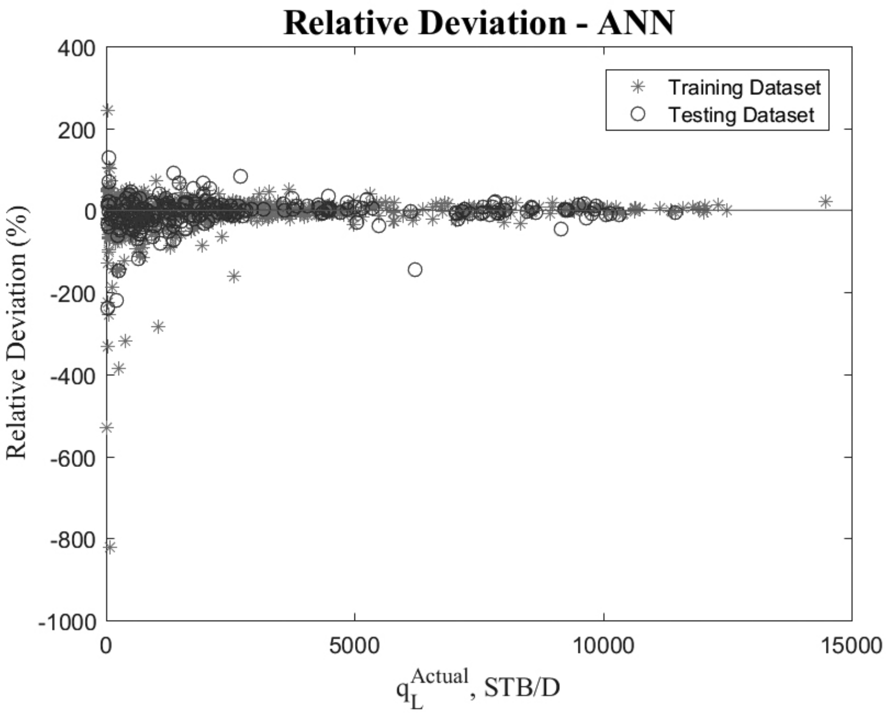

Figure 7. Relative Deviation plot for the ANN Developed Model (Training and Testing).

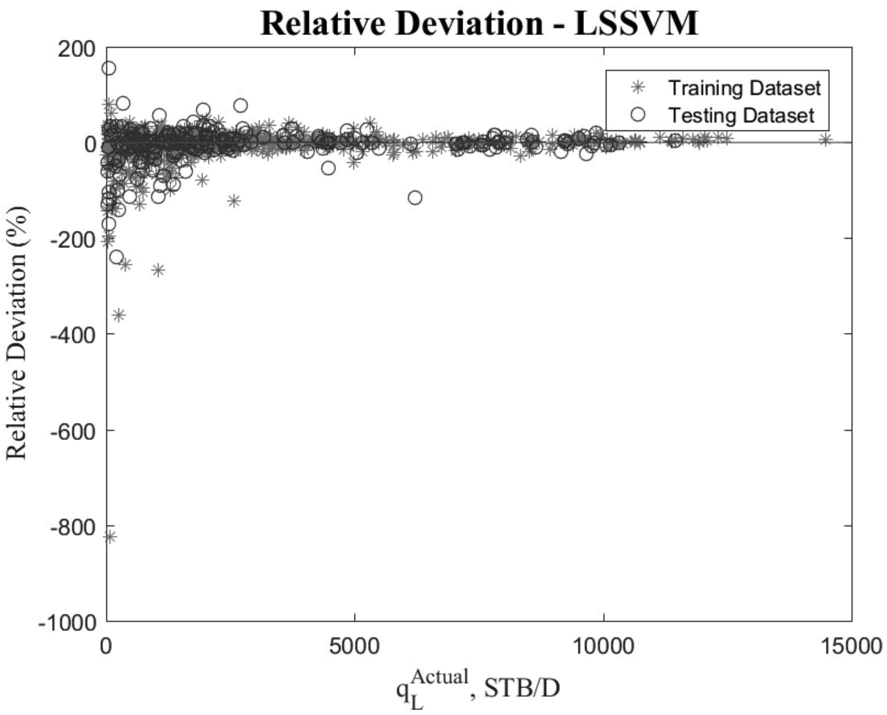

Figure 8. Relative Deviation plot for the LSSVM Developed Model (Training and Testing).

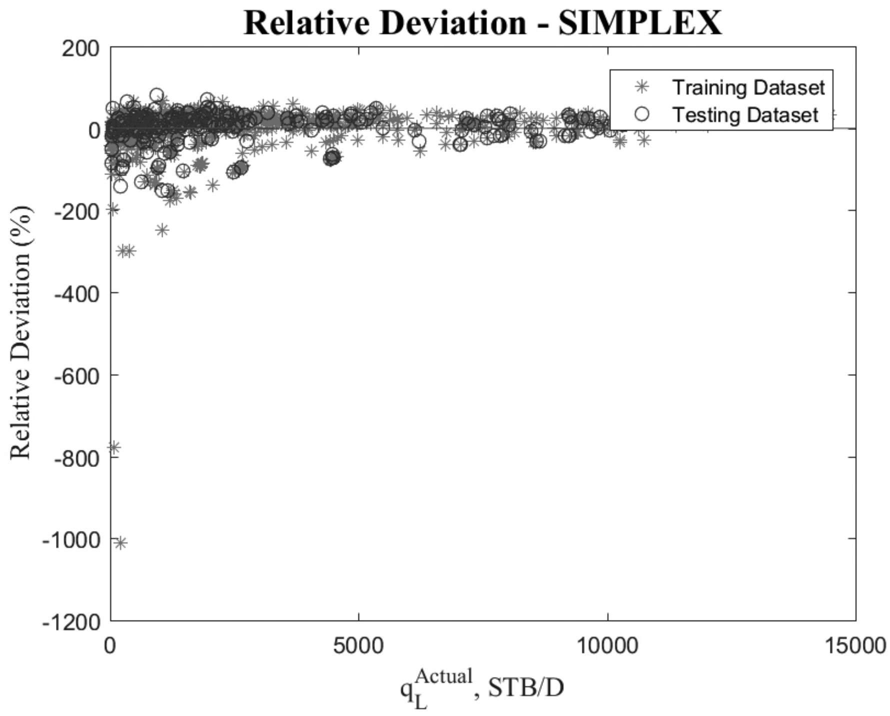

Figure 9. Relative Deviation plot for the SIMPLEX Developed Model (Training and Testing).

CONCLUSIONS

- A wide range of field data (1111 data points) from six different reservoirs and various regions was used to determine the best model describing critical choke flow rate given operational parameter input data points.
- The best model according to the three-error metric criterion is the least square support vector machine. The artificial neural network model is rated second, and the developed model using the global optimization simplex algorithm is last in ranking.
- A new set of constants following Eq. (1) generated and showed an enhanced performance from similar set of constants from the literature.
- A comprehensive review of the existing models reveals that the correlation developed by Gilbert (Gilbert, 1954) shows the best performance.

Nomenclature

A: Area

ANN: Artificial neural network

D: Diameter

LSSVM: Least squares support vector machines

MAPE: Mean average percent error

n: Total number of data points

p: Pressure

q: Volume Flow Rate

R: Ratio

R^2: Coefficient of determination

RDE: Relative deviation error

RMSE: Root mean square error

Subscript

act: Actual

c: Choke

calc: Calculated

G: Gas

L: Liquid

WH: Well head

Superscript

A: Correlation Parameter

B: Correlation Parameter

C: Correlation Parameter

D: Correlation Parameter

Greek Letters

γ: Least Square Support Vector Machine Regularization Parameter

σ: Least Square Support Vector Machine Bandwidth Parameter

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