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بيان تحليلي لنمو الشقوق في الأقواس الأفقية لأسطح نصف مغمورة

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الخلاصة

تظهر هذة المقالة حل تحليلي لحساب خصائص نمو التشقق في الأقواس الأفقية للأسطح نصف المغمورة مع تشقق سطحي يقع في الحدود بين القوس الأفقي وعمود محمل بواسطة الضغط. نتائج النموذج المقترح يمكن تطبيقها في مجال الكسر من شقوق ثابتة إلى إنهيار بلاستيكي. وطريقة الحل واضحة والبيان التحليلي متوفر ومناسب في التطبيقات الهندسية العملية. نتائج التحليل تدل أن فتحة رأس الشقوق في الجزء المنشق يزداد بإنسيابية عندما يحمل بضغط خفيف بينما يزداد بحدة عندما يحمل بضغط عالي. وزيادة فإن الحمل المسبب لبداية الإنشقاق والحمل الكلي يختلفان بطريقة دراماتيكية مع الزاوية الأولى وضغط التحمل مقارنة بالقيمة الحرجة ل (CTOD) و (CTOA) ومعامل يونج.

Analytical solution for crack growthing of semi-submersible platform's horizontal brace

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ABSTRACT

The article presents an analytical solution to calculate the crack growth characteristics of semi-submersible platform's horizontal brace with a circumferential through–crack lies at the boundary between the horizontal brace and column loaded by tension. The results of the proposed model may be applied in the full range of fracture from stationary crack to plastic collapse and the solution process is clear and the analytical solution is found which is especially suitable to solve problems in practical engineering application. The analysis results indicate that the crack tip opening displacement (CTOD) on the cracked section increases smoothly when loaded with small tension but increases sharply when loaded with larger tensions. Furthermore, the crack initiation load and the ultimate load vary dramatically with the initial angles and yield stress compared with critical CTOD, critical CTOA and Young's Modulus.

Key words: Analytical solution; circumferential through-crack; crack growth; horizontal brace of semi-submersible platform.

INTRODUCTION

Semi-submersible platforms, one of the most widely used for deep-water applications due to their reuse ability and mobility, have gained popularity in recent decades with on-going development of deep-water oil and gas exploration. The horizontal brace is one of the main structures in semi-submersible platforms which serves as the supporting structure especially when the platform encounter horizontal tension load in ocean engineering. Although the safety design standards for this kind of structures are quite strict, cracks inescapably are initiated during their service life. The presence of such cracks at critical locations can compromise the safety of the whole structure such as the catastrophic fracture failure of Alexander L. Kielland in 1980, which resulted from a single circumferential crack at the boundary of its horizontal brace (Almar et al., 1984).

To ensure the safety and reliability of the semi-submersible platforms which would be subject to very harsh marine environment during their service life, analysis of the mechanical characteristics of a cracked horizontal brace is necessary. The problem of brace structure with a crack has been investigated by a number of authors (Sanders, 1987; Brighenti, 2000; Andrea et al., 2007). In most of the articles cited, the crack has been assumed to be a surface crack or present far away from boundary. However, a circumferential through-crack in a horizontal brace usually lies at the boundary where stress concentration in high such as the crack-induced total losses of the semi-submersible platforms of Sedco in 1967, Alexander L. Kielland in 1980 and Ocean Ranger in 1982 (Maier, 1985; Inge and Odd, 2005; Colin et al., 2014). As for the method of computational analysis, the most popularly applied method to analyse the characteristics of cracked brace structures is the finite element method whose effectiveness has been accepted by the engineering community. Nevertheless, the finite element method should be carried out for every member and structure system with local defective elements else the nonlinear calculations will be inefficient and would require spending significant amount of resources. Therefore, the theoretical analysis is still necessary.

The present article aims to investigate an analytical method calculating the mechanical characteristics of semi-submersible platform's horizontal brace having a circumferential through-crack at the boundary.

GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

The horizontal braces of the semi-submersible platform are of medium length which means that the radius of the cross section is significantly smaller than the length of the horizontal braces. From the viewpoint of the shell theory, the semi-submersible platform's horizontal braces belong to the mid-long cylindrical shell category. Regarding the length of circumferential through-cracks on the brace structures, historical records show that such cracks are very long (Reason; 1997). For instance, in the disastrous event of semi-submersible platform Alexander L. Kielland (Figure 1), the circumferential through-crack near the boundary of the horizontal brace had propagated to almost 67% of circumferential length of the brace before fracture (Moan, 2007). Above all, the characteristic equation of the horizontal brace could be expressed under a semi-membrane state, in which, characteristic functions vary more slowly in the axial direction than that in the circumferential direction.

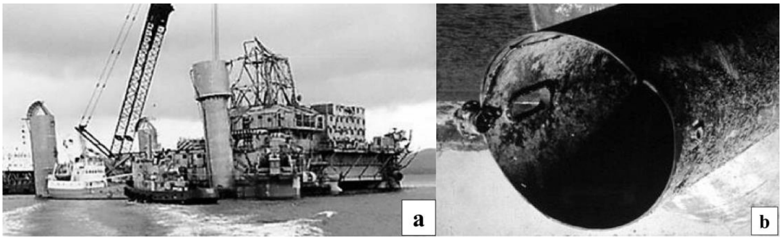

Fig. 1 a, the salvaged Alexander L. Kielland; b, fracture of the horizontal brace

In this paper, a horizontal brace of a semi-submersible platform with a circumferential through-crack present at the boundary between the horizontal brace and the column of the semi-submersible platform is considered. The column is assumed to be considerably stiffer than the horizontal brace and is subjected to a tension load as illustrated in Figure 2, where a coordinate system and load direction have also been shown.

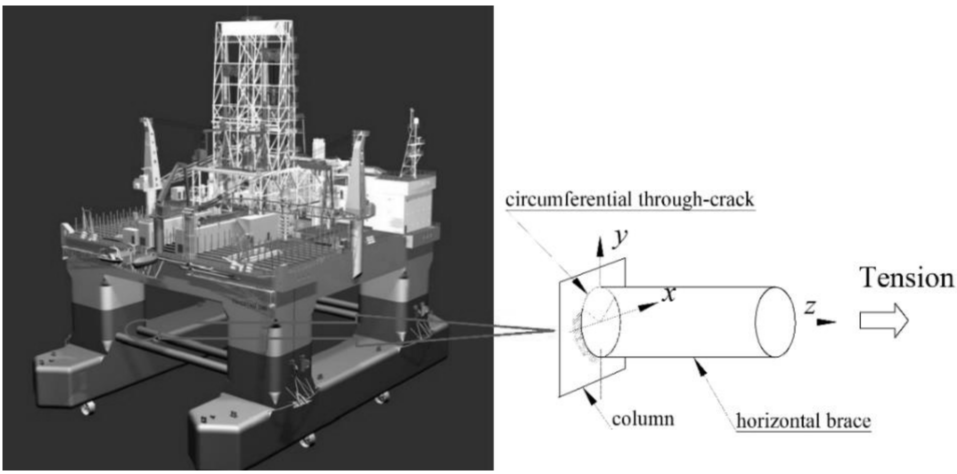

Fig. 2 A circumferential through-crack lies at boundary of a semi-submersible platform

Under a semi-membrane state (Sanders, 1987) the characteristic equation satisfied by each of the two complex characteristic functions Φ and φ, which are related to each other by $\partial^2\Phi/\partial z^2=\varepsilon^2\varphi$ is

$$\frac{\partial^2}{\partial\theta^2}\left(\frac{\partial^2\Omega}{\partial\theta^2}+\Omega\right)-i\varepsilon^{-2}\frac{\partial^2\Omega}{\partial z^2}=0 \tag{1}$$

Here, ε is a small parameter given by $\varepsilon^2= (h/R)\ [12(1-\mu^2)]^{-1/2}$ where h and R are the thickness and radius respectively of the horizontal brace of the semi-submersible platform and μ is Poisson's ratio. Then, the expression of dimensionless complex displacements u, v, and w, stress functions χ_z, χ_θ, and ζ can be given in terms of Φ and φ by

$$\begin{cases}\varepsilon^2 u=\dfrac{\partial}{\partial z}\left(\dfrac{\partial^2\Phi}{\partial\theta^2}\right);\varepsilon^2\upsilon=-\dfrac{\partial^3\Phi}{\partial\theta^3};\omega=-\dfrac{\partial^2\Phi}{\partial\theta^2}+i\varphi\\ \varepsilon^2\chi_z=-i\dfrac{\partial}{\partial z}\left(\dfrac{\partial^2\Phi}{\partial\theta^2}\right);\varepsilon^2\chi_\theta=-i\dfrac{\partial^3\Phi}{\partial\theta^3};\zeta=-i\dfrac{\partial^2\Phi}{\partial\theta^2}+\varphi\end{cases} \tag{2}$$

The complete solution Φ_c to the problem presented in this article can be expressed as $\Phi_c=\Phi_s+\Phi_b$, where Φ_s is the solution obtained by the existence of the circumferential through crack at the boundary and Φ_b is the elementary solution composed of simple axial tension solutions, rigid-body motion solutions (for which stresses disappear) and null solutions (for which displacements disappear). The elementary solution for a cylindrical shell was derived by Sanders (1972) as

$$\frac{\partial^2\Phi_b}{\partial\theta^2}=\left\{0.5i\left[1+i\varepsilon^2(2+\upsilon)\right]\theta^2+\left(1+i\varepsilon^2\upsilon\right)\left(0.5\varepsilon^2z^2-i\right)\right\}\sigma_T+ia-\varepsilon bz+iccos\theta+\varepsilon dzcos\theta \quad (3)$$

Here, a, b, c, d are constants unknown and σ_T is dimensionless load parameter corresponding to tension load T which can be defined as $\sigma_T=T/(2\pi Rh\sigma_F)$, where σ_F is yield stress of the material.

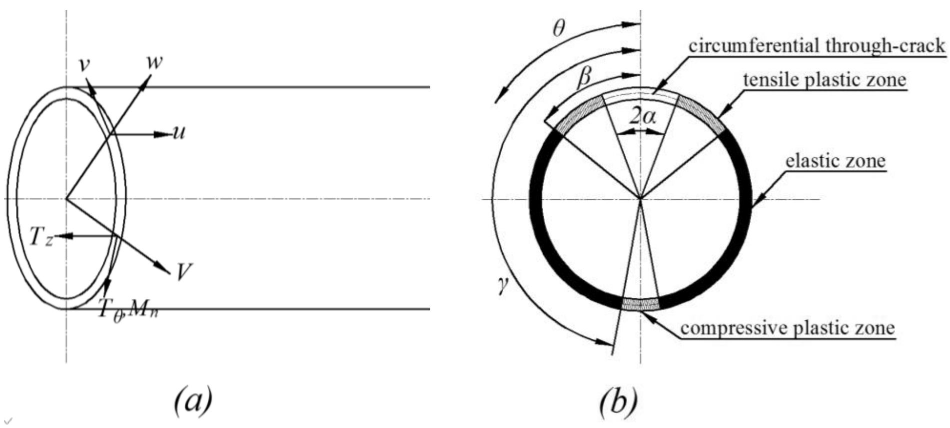

Fig. 3 (a) boundary conditions on the horizontal brace; (b) circumferential through-crack section of the horizontal brace

Since there is a static-geometric analogy between displacements and stress functions and Sanders had derived the expression for the stress functions in terms of integrals of the effective Kirchhoff edge resultants, the treatment of the boundary condition is thus simplified (Sanders, 1980). The displacements (u, v, and w) and Kirchhoff edge load (T_z, T_θ, V and M_n) are used to analyze the boundary conditions of the cracked section of horizontal brace, shown as in Fig 3 (a). This section is composed of circumferential through-crack, tensile plastic zone, elastic zone and compressive zone, shown as in Fig 3 (b). The opening angle of the circumferential through-crack is 2α, while the angle β and γ mark the beginning of tensile plastic zone and compressive plastic zone of the brace away from the crack, respectively.

On the free surface of the crack ($0\leq\theta<\alpha$) there is no acting force and the boundary conditions may be written as $T_z=T_\theta=V=M_n=0$. The tensile stress equal to the yield stress σ_F acting on the tensile plastic zone ($\alpha\leq\theta<\beta$) of the crack section in axial direction of the brace and the plastic deformation exist in this zone. Additionally, there should be no circumferential displacement, considering the rigid constraint of the crack on this section. Therefore, the boundary conditions in this zone lead to $v=V=M_n=0$ and $Tz=1$. Contrary to the tensile plastic zone, the compressive stress equal to the yield stress σ_F acting on the compressive plastic zone ($\gamma \leq \theta < \pi$) of the cracked section and the boundary condition can be obtained as $v=V=M_n=0$ and $Tz=-1$. Additionally, the boundary condition of elastic zone ($\beta \leq \theta < \gamma$) of the cracked section can be obtained as $u=v=w=\partial w/\partial z=0$. The expression for the boundary values of the stress functions in terms of prescribed edge load Tz, T_θ, V and M_n acting on the edge $z=0$ derived by Sanders (1972) which could simplify the treatment of boundary conditions as follows,

$$\varepsilon^2 \chi_\theta = sin\theta \int^\theta \left(T_z + \varepsilon^2 M_n\right) sin\eta d\eta + cos\theta \int^\theta \left(T_z + \varepsilon^2 M_n\right) cos\eta d\eta - \int^\theta T_z d\eta \tag{4}$$

$$\varepsilon^2 \chi_z = sin\theta \int^\theta \left(T_\theta sin\eta + \varepsilon^2 V cos\eta\right) d\eta + cos\theta \int^\theta \left(T_\theta cos\,\eta - \varepsilon^2 V sin\,\eta\right) d\,\eta - \int^\theta T_\theta d\,\eta \tag{5}$$

The boundary condition mentioned above can be obtained in terms of the characteristic functions Φ and φ following Equations (2), (4) and (5) as

$$\begin{cases} \mathrm{R}\left\{ i\dfrac{\partial}{\partial z}\left(\dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right)\right\} = 0 \\ \mathrm{R}\left\{ i\dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right\} = 0 \end{cases} ;\left(0 \leq \theta < \alpha\right) \tag{6}$$

$$\begin{cases} \mathrm{R}\left\{ \dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right\} = 0 \\ \mathrm{R}\left\{ i\dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right\} = -\cos\left(\theta-\alpha\right) - 0.5\left(\theta-\alpha\right)^2 + 1 \end{cases} ;\left(\alpha \leq \theta < \beta\right) \tag{7}$$

$$\begin{cases} \mathrm{R}\left\{ \dfrac{\partial}{\partial z}\left(\dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right)\right\} = 0 \\ \mathrm{R}\left\{ \dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right\} = 0 \end{cases} ;\left(\beta \leq \theta < \gamma\right) \tag{8}$$

$$\begin{cases} \mathrm{R}\left\{ \dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right\} = 0 \\ \mathrm{R}\left\{ i\dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right\} = -\pi\sigma_T\theta - cos\theta - G_1 cos\theta + 0.5\left(\pi-\theta\right)^2 - 1 + G_2 \end{cases} ;\left(\gamma \leq \theta \leq \pi\right) \tag{9}$$

The symbol *R{}* denotes the real part of the expression in brackets and the subscript *c* means that the expressions are in terms of the complete characteristic functions. With Equations (3), (6), (7), (8), and (9), the particular integral Φs at the boundary que to the existence of the crack turns into

$$\begin{cases} \mathrm{R}\left\{ i\dfrac{\partial}{\partial z}\left(\dfrac{\partial^2\Phi_s}{\partial\theta^2}\right)\right\} = -\varepsilon b_I + \varepsilon d_I\cos\theta \\ \mathrm{R}\left\{ i\dfrac{\partial^2\Phi_s}{\partial\theta^2}\right\} = a_R + c_R\cos\theta + \left(1-0.5\theta^2\right)\sigma_T \end{cases};\left(0\le\theta<\alpha\right) \tag{10}$$

$$\begin{cases} \mathrm{R}\left\{ \dfrac{\partial^2\Phi_s}{\partial\theta^2}\right\} = a_I + c_I\cos\theta \\ \mathrm{R}\left\{ i\dfrac{\partial^2\Phi_s}{\partial\theta^2}\right\} = a_R + c_R\cos\theta + \left(1-0.5\theta^2\right)\sigma_T - \cos\left(\theta-\alpha\right) - 0.5\left(\theta-\alpha\right)^2 + 1 \end{cases};\left(\alpha\le\theta<\beta\right) \tag{11}$$

$$\begin{cases} \mathrm{R}\left\{ \dfrac{\partial^2\Phi_s}{\partial\theta^2}\right\} = a_I + c_I\cos\theta \\ \mathrm{R}\left\{ \dfrac{\partial}{\partial z}\left(\dfrac{\partial^2\Phi_s}{\partial\theta^2}\right)\right\} = \varepsilon b_R - \varepsilon d_R\cos\theta \end{cases};\left(\beta\le\theta<\gamma\right) \tag{12}$$

$$\begin{cases} \mathrm{R}\left\{ \dfrac{\partial^2\Phi_s}{\partial\theta^2}\right\} = a_I + c_I\cos\theta \\ \mathrm{R}\left\{ i\dfrac{\partial^2\Phi_s}{\partial\theta^2}\right\} = a_R + G_2 + \left(c_R - G_1\right)\cos\theta + \left[\pi\left(\pi-\theta\right) + \left(0.5\theta^2-1\right)\right]\sigma_T - \cos\theta + 0.5\left(\pi-\theta\right)^2 - 1 \end{cases};\left(\gamma\le\theta\le\pi\right) \tag{13}$$

Here, the subscripts R and I refer to the real and imaginary parts of these constants and G_I is real constant of integration. Furthermore, any solution to Equation (1) satisfies the conditions

$$\int_0^\pi \left.\frac{\partial^2\Phi_s}{\partial\theta^2}\right|_{z=0} d\theta = \int_0^\pi \left.\frac{\partial^2\Phi_s}{\partial\theta^2}\right|_{z=0} \cos\theta d\theta = 0 \tag{14}$$

$$\frac{\partial}{\partial z}\left(\frac{\partial^2\Phi_s}{\partial\theta^2}\right) = -i^{\frac{3}{2}}\varepsilon\frac{\partial^2}{\partial\theta^2}\left(\frac{\partial^2\Phi_s}{\partial\theta^2} + \frac{1}{2}\Phi_s\right) \tag{15}$$

Now substitute $\partial^2\Phi_s/\partial\theta^2\ (0, \theta) = F\ (\theta)$ into Equations (14) and (15) to get

$$\begin{cases} \int_0^\pi \left(F'' + F\right) d\theta = 0 \\ \int_0^\pi F''\cos\theta d\theta = 0 \end{cases} \tag{16}$$

$$\left.\frac{\partial}{\partial z}\left(\frac{\partial^2\Phi_s}{\partial\theta^2}\right)\right|_{z=0} = -i^{\frac{3}{2}}\varepsilon\left(F'' + \frac{1}{2}F\right) \tag{17}$$

Additionally, by using Equations (16) and (17), the boundary condition of Equations (10), (11), (12) and (13), can be expressed in the equivalent form as follows,

$$F_R = -\left(a_R + 2\sqrt{2}b_I\right) - \left(c_R + 2\sqrt{2}d_I\right)cos\theta + Scos\left(\theta/\sqrt{2}\right) - \left(0.5\theta^2 - 1\right)\sigma_T; 0 \le \theta < \alpha \tag{18}$$

$$F_R = a_I + c_I cos\theta; \alpha \le \theta \le \pi \tag{19}$$

$$F_I = -\left(a_R + c_R cos\theta\right) - \left(0.5\theta^2 - 1\right)\sigma_T; 0 \le \theta < \alpha \tag{20}$$

$$F_I = -\left(a_R + c_R cos\theta\right) - \left(0.5\theta^2 - 1\right)\sigma_T + cos\left(\theta - \alpha\right) + 0.5\left(\theta - \alpha\right)^2 - 1; \alpha \le \theta \le \beta \tag{21}$$

$$F_I = 2\sqrt{2}b_R - a_I + \left(2\sqrt{2}b_R - c_I\right)cos\theta + 2\sqrt{2}Pcos\left[\left(\theta - \beta\right)/\sqrt{2}\right] + 2\sqrt{2}Qcos\left[\left(\theta - \beta\right)/\sqrt{2}\right]; \beta \le \theta < \gamma \tag{22}$$

$$F_I = -\left[a_R + G_2 + \left(c_R - G_1\right)cos\theta\right] - \left[\pi\left(\pi - \theta\right) + \left(0.5\theta^2 - 1\right)\right]\sigma_T + cos\theta - 0.5\left(\pi - \theta\right)^2 + 1; \gamma \le \theta \le \pi \tag{23}$$

Here, S, P, Q, G_2 are real constants of integration. Adding the undetermined β and γ there exis totally fifteen unknown constants in the above equation. The continuity of the displacements and stress functions at the cracked section implies that F_R, F_R', F_R'' at α and F_I, F_I', F_I'', F_I''' at β and γ should be continuous (F_I, F_I', F_I'' at α are continuous from simply checking), from which eleven equations can be obtained. Four more equations can be found for the real parts and the imaginary parts equal to zero in Equation (16). There are thus fifteen conditions to determine the fifteen constants and all the constants can be finally determined by means of algebraic methods.

The extensions of the plastic zones β and γ with given α and σ_T can be determined by the following simultaneous transcendental equations

$$\begin{aligned}&\left[\left(sin\beta + \beta cos\beta\right)\sin x + \sqrt{2}\left(\pi - \gamma + \beta cos\beta \cos x\right)sin\beta\right]\sigma_T - \left[sin\beta - sin\alpha + \left(\beta - \alpha\right)cos\beta\right]\sin x \\ &+\sqrt{2}\left[\pi - \gamma - \left(\beta - \alpha\right)\cos x\right]sin\beta = 0\end{aligned} \tag{24}$$

$$\begin{aligned}&\left\{\left[\left(\pi - \gamma\right)cos\gamma - sin\gamma\right]\sin x - \sqrt{2}\left[\left(\pi - \gamma\right)\cos x + \beta\right]sin\gamma\right\}\sigma_T - \left[sin\gamma - \left(\pi - \gamma\right)cos\gamma\right]\sin x \\ &-\sqrt{2}\left[\left(\pi - \gamma\right)\cos x - \beta + \alpha\right]sin\gamma = 0\end{aligned} \tag{25}$$

The dimensionless crack tip opening displacement (CTOD) subjected to the tension load can be obtained as

$$\mathrm{CTOD} = 0.25\sqrt{2}\varepsilon^{-1}\left[-\mathrm{A} + \mathrm{C}\cos\alpha - \left(0.5\alpha^2 + 1\right)\sigma_T\right] \tag{26}$$

where

$$\mathrm{A} = -\sqrt{2}\left\{\left[(\pi - \gamma)(1 - \sigma_T)\cos x - \beta\sigma_T + \beta - \alpha\right]\cot x + (\pi - \gamma)(1 - \sigma_T)\sin x\right\} - 0.5\beta^2\sigma_T + 0.5(\beta - \alpha)^2$$

$$\mathrm{C} = \left[-\beta\sigma_T + sin\left(\beta - \alpha\right) + \beta - \alpha\right]/\sin\beta$$

For tearing after the crack growth started, the crack tip opening angle (CTOA) based on Dugdale model defined by Sanders (1987) is

$$\psi = \frac{\sigma_F}{E}\left[\frac{d\delta}{d\alpha} - \left.\frac{\partial u}{\partial \theta}\right|_{\theta=\alpha}\right] \tag{27}$$

The crack growth hypothesis set by Budiansky (1985) is that the crack opening shape in the neighbourhood of the crack tip remains invariant as the crack grows which means that the CTOA remains constant in the process of the stable growth of the crack. According to the Equation (27), the Equation (26) can be put in the following form,

$$\frac{\sqrt{2}\sigma_F}{4E\varepsilon}\left[\frac{dC}{d\alpha}\cos\alpha - \frac{dA}{d\alpha} - \left(\frac{1}{2}\alpha^2+1\right)\frac{d\sigma_T}{d\alpha}\right] + \frac{\sigma_F}{E\varepsilon}\left(\frac{1}{4}S\sin\frac{\alpha}{\sqrt{2}} + \frac{2}{\sqrt{2}}\alpha\sigma_T\right) = \psi_c \tag{28}$$

Where ψ_c is the CTOA reflecting tearing resistance.

In the absence of such a zone where the compressive plastic zone of the crack section does not appear ($\gamma=\pi$), the tearing equation is restated as in the following form by Equation (28)

$$\left(r_1 + g_1 L_1\right)\frac{d\sigma_T}{d\alpha} + g_1 W_1 + I + H = 2\sqrt{2}\frac{E\varepsilon}{\sigma_F}\psi_c \tag{29}$$

where

$$r_1 = -\beta cos\alpha/sin\beta + 0.5\beta^2 - \sqrt{2}\beta cotx + 0.5\alpha^2 + 1$$

$$g_1 = cos\alpha\left[\left(\beta cos\beta - sin\beta\right)\sigma_T + sin\alpha + sin\beta - \left(\beta-\alpha\right)cos\beta\right]/sin^2\beta + \left(\beta - \alpha - \beta\sigma_T\right)cot^2 x$$
$$+\sqrt{2}\left(1-\sigma_T\right)cotx + cosx/sin^2 x$$

$$L_1 = \frac{\left(sin\beta + \beta cos\beta\right)\sin x + \sqrt{2}\left(\beta cos\beta\cos x\right)sin\beta}{\left[0.5\sqrt{2}\left(sin\beta + \beta cos\beta\right)cosx + 2cos\beta sinx\right]\sigma_T - 0.5\sqrt{2}\left[sin\beta + sin\alpha + \left(\beta-\alpha\right)cos\beta\right]cosx - 2cos\beta sinx}$$

$$W_1 = \frac{\left(cos\alpha + cos\beta\right)sinx + \sqrt{2}sin\beta cosx}{0.5\sqrt{2}\left[sin\beta + sin\alpha + \left(\beta-\alpha\right)cos\beta\right]cosx - \left[0.5\sqrt{2}\left(sin\beta + \beta cos\beta\right)cosx + 2cos\beta sinx\right]\sigma_T + 2cos\beta sinx}$$

$$I = -cos\alpha\left[1 + cos\left(\beta-\alpha\right)\right]/sin\beta + \left(\beta-\alpha\right) - \sqrt{2}cotx$$

$$H = 0.5\sqrt{2}Ssin\left(\alpha/\sqrt{2}\right) + 2\alpha\sigma_T + 2\sigma_T sin\alpha$$

When the compressive plastic zone appears ($\gamma<\pi$), the parameters of β, γ and σ_T could be regarded as the functions of α and the tearing equation becomes

$$\left(r_2 + g_2 L_2 + JM\right)\frac{d\sigma_T}{d\alpha} + I + g_2 W_2 + JN + H = 2\sqrt{2}\frac{E\varepsilon}{\sigma_F}\psi_c \tag{30}$$

where

$$r_2 = -\beta cos\alpha / sin\beta + 0.5\beta^2 - \sqrt{2}\beta cotx - \sqrt{2}(\pi-\gamma)/sinx + 0.5\alpha^2 + 1$$

$$g_2 = cos\alpha\left[(\beta cos\beta - sin\beta)\sigma_T + sin\alpha + sin\beta - (\beta-\alpha)cos\beta\right]/sin^2\beta + (\beta-\alpha-\beta\sigma_T)cot^2x + \sqrt{2}(1-\sigma_T)cotx + cotx/sinx - (\pi-\gamma)(\sigma_T+1)$$

$$J = \left[\sqrt{2}(1+\sigma_T)sinx + (\pi-\gamma)(\sigma_T+1)cosx - \beta(1-\sigma_T) + \alpha\right]/sin^2x$$

$$W_2 = -\left\{\left[(cos\alpha + cos\beta)sinx - \sqrt{2}sin\beta cosx\right]K_2 + \sqrt{2}sin\gamma K_3\right\}/\left[K_1K_2 - K_3K_4\right]$$

$$L_2 = (B_2K_3 - B_1K_2)/(K_1K_2 - K_3K_4)$$

$$M = [B_1K_4 - B_2K_1]/[K_1K_2 - K_3K_4]$$

$$N = \left\{\sqrt{2}sin\gamma K_1 + \left[(cos\alpha + cos\beta)sinx - \sqrt{2}sin\beta cosx\right]K_2\right\}/\left[K_1K_2 - K_3K_4\right]$$

$$K_1 = \left[0.5\sqrt{2}(sin\beta + \beta cos\beta)cosx + 2cos\beta sinx + \sqrt{2}(\pi-\gamma)cos\beta\right]\sigma_T - 0.5\sqrt{2}\left[sin\beta + sin\alpha + (\beta-\alpha)cos\beta\right]cosx - 2cos\beta sinx + \sqrt{2}(\pi-\gamma)cos\beta$$

$$K_2 = \left\{-0.5\sqrt{2}\left[(\pi-\gamma)cos\gamma - sin\gamma\right]cosx - 2cos\gamma sinx - \sqrt{2}\beta cos\gamma\right\}\sigma_T + 0.5\sqrt{2}\left[sin\gamma - (\pi-\gamma)cos\gamma\right]cosx - 2cos\gamma sinx + \sqrt{2}(\beta-\alpha)cos\gamma$$

$$K_3 = \left[0.5\sqrt{2}(sin\beta + \beta cos\beta)cosx - \beta sin\beta sinx + \sqrt{2}sin\beta\right]\sigma_T - 0.5\sqrt{2}\left[sin\beta - sin\alpha + (\beta-\alpha)cos\beta\right]cosx + (\beta-\alpha)sin\beta sinx - \sqrt{2}sin\beta$$

$$K_4 = \left\{-0.5\sqrt{2}\left[(\pi-\gamma)cos\gamma - sin\gamma\right]cosx - (\pi-\gamma)cos\gamma sinx - \sqrt{2}sin\gamma\right\}\sigma_T + 0.5\sqrt{2}\left[sin\gamma - (\pi-\gamma)cos\gamma\right]cosx - (\pi-\gamma)sin\gamma sinx + \sqrt{2}sin\gamma$$

SOLUTIONS

The results of mathematical model proposed in this article to calculate the mechanical characteristic of semi-submersible platform's horizontal brace with a circumferential through-crack at the boundary between horizontal brace and column loaded by tension may be applied in the full range of fracture, including the crack growth from elasticity to full plasticity and the solution process is simple as the analytical solution is found.

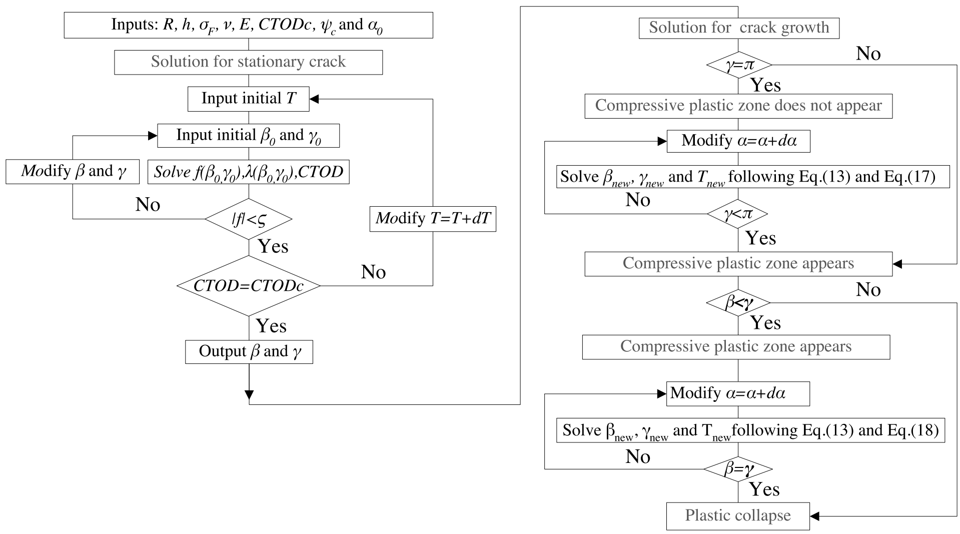

Fig. 4 Flow scheme of solutions of stationary crack and crack growth

At the beginning of the solution, basic parameters including parameters of the horizontal brace and initial angle of the crack are input for the solutions. An initial tension load should be assigned to start the calculation for stationary crack. As explicit solution for plastic zones of the cracked section of horizontal brace is hard to find from Equations (24) and (25), assuming initial β_0 and initial γ_0 is necessary. Two more parameters, f and λ, are defined based on Equations (24) and (25):

$$f = \left[B_1(\beta_0,\gamma_0)\sigma_T - N_1(\beta_0,\gamma_0)\right]^2 + \left[B_2(\beta_0,\gamma_0)\sigma_T - N_2(\beta_0,\gamma_0)\right]^2$$

$$\lambda = 10^4 f(\beta_0,\gamma_0) \Big/ \left\{\left[f(\beta_0+0.01,\gamma_0)/\beta_0 - f(\beta_0,\gamma_0)/\beta_0\right]^2 + \left[f(\beta_0,\gamma_0+0.01)/\gamma_0 - f(\beta_0,\gamma_0)/\gamma_0\right]^2\right\}$$

To guarantee the precision of the solution, verification is needed to see if $|f|<\varsigma$ is satisfied, where ς is a small specified quantity. If $|f|<\varsigma$ is not satisfied, the assumed initial β_0 should be modified as $\beta=\beta_0-\lambda[f(\beta_0+0.01,\gamma_0)-f(\beta_0,\gamma_0)]/(0.01\beta_0)$ and the assumed initial γ_0 should be modified as $\gamma=\gamma 0-\lambda[f(\beta_0,\gamma_0+0.01)-f(\beta_0,\gamma_0)]/(0.01\gamma_0)$.

After $|f|<\varsigma$ is satisfied, the solution for plastic zones of the cracked horizontal brace is complete. Then, the output of β and γ can be input into Equation (26) to determine the CTOD. If the crack growth condition is satisfied when the CTOD extends to the critical CTOD, the solution for stationary crack is done, or else the tension load input should be modified as $T=T+dT$.

The output of calculations of the stationary crack may be used to start the solution for crack growth. When the parameter of γ which marks the beginning of compressive plastic zone of the cracked section of horizontal brace equal π, modify the angle of crack α as $\alpha=\alpha+d\alpha$ to solve the new characteristics of the cracked section following Equations (24), (25) and (29) until the compressive plastic zone appears. Similarly, modify the angle of crack as $\alpha=\alpha+d\alpha$ following Equations (24), (25) and (30) until the cracked section is in full plastic ($\beta=\gamma$). Thus, the solutions of the mechanical characteristics of semi-submersible platform's horizontal brace having a circumferential through-crack at the boundary are obtained and the detailed process is shown in Figure 4.

EXAMPLES

To illustrate the previous model with some examples, a typical semi-submersible platform with circumferential through-crack at the boundary between the horizontal brace and column is selected with parameters of the horizontal brace including the radius $R=0.915m$, thickness $h=0.0308m$, Poisson's ratio $\mu=0.3$, Young's Modulus $E=210$GPa and yield stress of the material $\sigma_F=400MPa$.

As shown in Figure5, a crack angle has been chosen as α=π/6 to analyze the characteristic variation of stationary crack. The CTOD on the circumferential through-cracked section increase smoothly when the horizontal brace of the semi-submersible platform is subjected to small tension loads and increases sharply when subjected to larger tensions (Figure 5a). The elastic zone of the horizontal brace's cracked section, whose variation tendency is contrary to the variation tendency of plastic zone (Figure 5a), decrease with the tension load until the cracked section is in the full plastic condition.

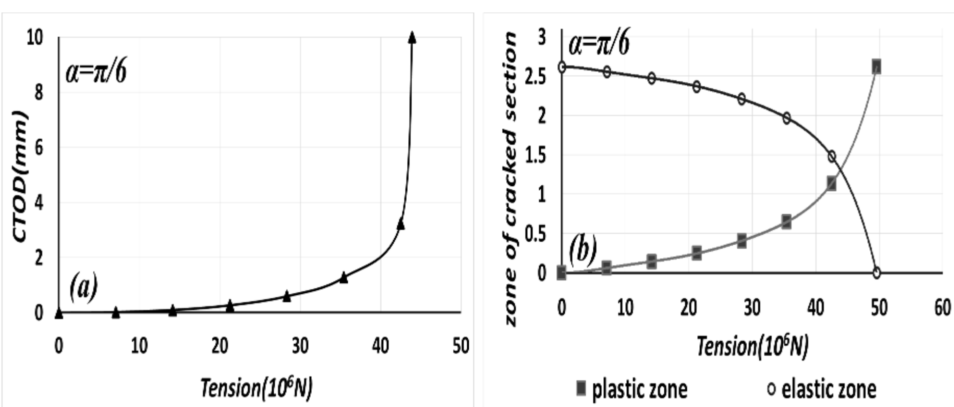

Fig. 5 (a) variation of the CTOD on the cracked section with different tension loads; (b) zones of the cracked section's variation tendency with tension

The relationship between tension and crack growth where P_0 indicates the initial angle of the crack, P_1 indicates the point crack growth started and P_2 indicates ultimate carrying capacity of the horizontal brace, as shown in Figure 6a, applied the full range of fracture from stationary crack to plastic collapse. The angle of the circumferential through-crack remains unchanged with tension load increase until crack growth started and the crack propagate quickly after the carrying capacity of horizontal brace reaches the ultimate load.

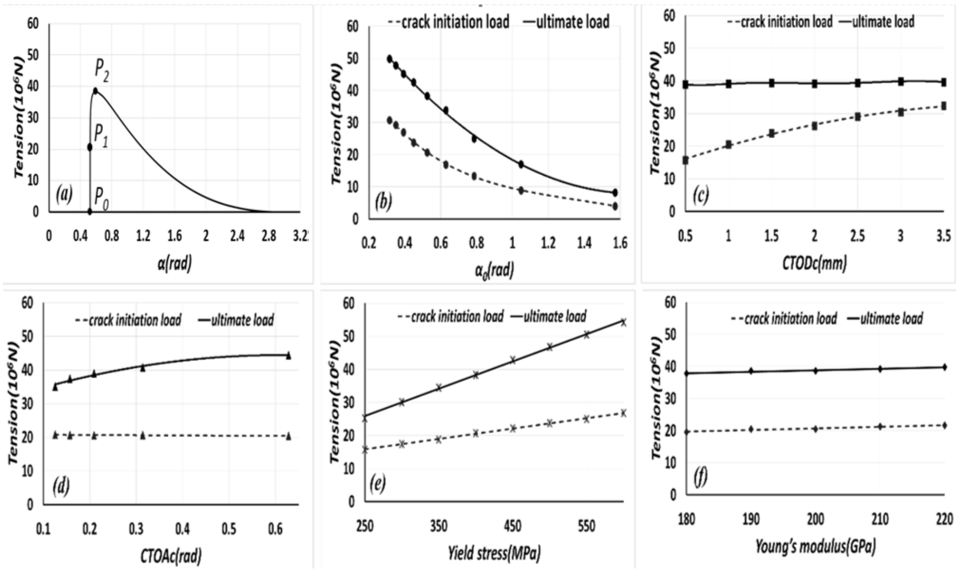

Fig. 6 (a) relationship between tension load and crack growth; (b) crack initiation load and ultimate load under different initial angles of the circumferential through-crack; (c) crack initiation load and ultimate load under different critical crack tip opening displacement; (d) crack initiation load and ultimate load under different critical crack tip opening angle; (e) crack initiation load and ultimate load under different yield stress of the material; (f) crack initiation load and ultimate load under different Young's Modulus of the material

Also, the crack initiation load where crack growth started and the ultimate load under different initial angles and material parameters of the horizontal brace including critical tip opening displacement (CTODc), critical tip opening angle (CTOAc), yield stress and Young's Modulus are shown from Figure 6b to Figure 6f. Both the crack initiation load and the ultimate load of the horizontal brace decrease with the initial angle of the circumferential through-crack, as shown in Figure 6b, because the initial angle had the value of bearing area of the cracked section of the horizontal brace. The crack initiation load increase with the critical crack tip displacement while the ultimate load remains almost the same, as shown in Figure 6c. Contrary to the variation of crack initiation load and ultimate load under different critical crack tip displacement, as shown in Figure 6d, the crack initiation load remains almost the same while the ultimate load increases with the critical crack tip angle. Both the crack initiation load and ultimate load are larger when the material has a bigger yield stress, as shown Figure 6e. Additionally, the Young's Modulus of the material has little effect on the crack initiation load and ultimate load of the horizontal brace with a circumferential through-crack at the boundary, as shown in Figure 6f.

CONCLUSIONS

(1) The crack tip opening displacement as well as the plastic zone of the cracked section increase smoothly when the horizontal brace is subjected to small tension while it increases sharply when loaded with larger tensions. The elastic zone whose variation tendency is contrary to the variation tendency of plastic zone decreases with the tension load until the cracked section is in the full plastic condition.

(2) The angle of the crack remain unchanged with tension increase until crack growth started and the crack propagate quickly after the carrying capacity of horizontal brace reaches the ultimate load.

(3) The crack initiation load and the ultimate load vary dramatically with the initial angles and yield stress compared with critical CTOD, critical CTOA and Young's Modulus.

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