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التحقق من قابلية تشكيل التيتانيوم النقي تجاريا بواسطة الموجات فوق الصوتية وباستخدام نظرية الرسم وطريقة المصفوفة

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الخلاصة

أي نوع من عمليات تشكيل المعادن يتأثر بشكل كبير من مادة العمل تحت التصنيع. ويقترح هذا البحث تقنية تستند على نظرية الرسم وطريقة مصفوفة لتقييم قابلية التيتانيوم للتشكل في تصنيع الآلات بالموجات فوق الصوتية. وتحديد مختلف سمات الآلات جنبا إلى جنب مع أهميتها النسبية وتحليلها من خلال تطوير وظيفة رياضية، وقد تم من خلال توظيف نظرية المخططات وطريقة المصفوفة. تم تطوير الدوغراف والذي يزود بالتوضيح المرئي لصفة المعتبرة مع التفاعلات النسبية. ويمثل هذا الدوغراف كذلك التمثيل المنفرد باستخدام تعبير المصفوفة. يتم الحصول على مؤشر قابلية التشكيل الدائم لجميع العمليلت التجريبية أيضا من شكل المصفوفة على أساس بيان صفة الدوغراف. والمزيج من كل الصفات لأي عملية قطع يجعل هذا الأسلوب متنوعا للغاية. وتبين النتائج بأن التشغيل التجريبي لهذه التركيبة يتألف مادة التيتانيوم، وحجم 500، وإمدادات الطاقة من 300 وات وهي بذلك تعطي نتائج قابلية التشكيل الأمثل.

Investigation of the machinability of commercially pure titanium in ultrasonic machining using graph theory and matrix method

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ABSTRACT

Any type of machining operation is well influenced by the machinability of work material under processing. This work proposes a graph theory and matrix method based technique for the assessment of machinability of titanium in ultrasonic machining. Identification of various machining attributes along with their relative importance has been considered and analyzed by developing a mathematical function by employing graph theory and matrix method. An attribute digraph is developed, which provides a visual illustration of considered attributes with their relative interactions. This digraph is further represented by using matrix expression. A permanent machinability index for all the experimental runs is also obtained from matrix form demonstration based on attribute digraph. The combination of all the attributes for any machining operation makes this method quite versatile. The results reveal that an experimental run having the combination consisting tool material of titanium, grit size of 500, and power supply of 300 W yields optimized results for machinability.

Keywords: Attribute diagraph; graph theory and matrix method; permanent machinability index; titanium; ultrasonic machining.

INTRODUCTION

Ultrasonic Machining (USM) is a mechanical type non-traditional manufacturing process capable to machine both the metallic and non-metallic materials possesses hardness above 40HRC such as composite materials, glasses, ceramics, quartz etc. (Kumar, 2013). USM has been variously termed ultrasonic cutting, ultrasonic drilling, slurry drilling, and ultrasonic abrasive machining (Kumar, 2013).

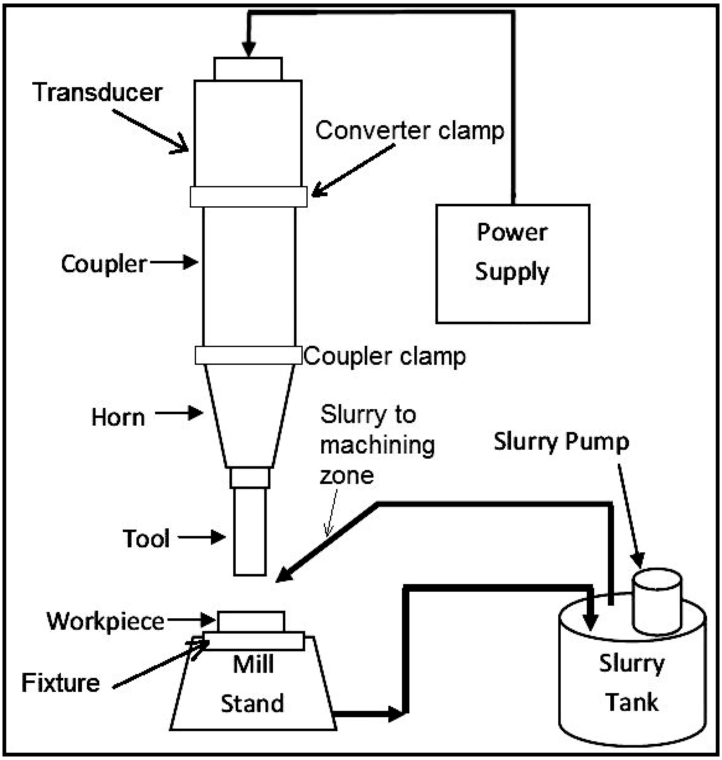

Fig. 1. Schematic representation of USM set-up

Detailed illustration of USM set-up is depicted in Figure 1. In this machining process, a transducer is employed to convert electrical energy of high frequency into longitudinal mechanical vibrations, which is further transmitted to horn. This causes the tool to vibrate at high frequency; usually greater than 20 kHz and amplitude of 12-50 μm. The range of power rating varies from 50 to 3000 W and tool is provided with controlled static load in order to feed it in the longitudinal direction. A mixture of abrasive material such as boron carbide, silicon carbide, and aluminum oxide suspended in water or some appropriate carrier medium is termed as abrasive slurry, which is pumped continuously across the gap between the tool end face and work surface. The work material is observed to be removed by the phenomenon of micro-chipping caused due to impact of tool vibrations on abrasive particles present in slurry.

LITERATURE REVIEW

Kumar & Khamba, (2008) investigated the effects of different input variables on the machining characteristics of titanium in ultrasonic machining. An improvement of approx 35% was reported for the material removal rate (MRR) response at optimal process setting. Jadoun, *et al*. (2009) optimized the process parameters for production accuracy in ultrasonic drilling of alumina-based ceramic composites using Taguchi method. Lalchhuanvela, *et al.* (2012) studied the effect of different USM process parameters and developed empirical response surface methodology (RSM) model for material removal rate and surface roughness while machining alumina ceramic. The results reported that, the higher level of every input parameter gives higher MRR. Surface roughness decreases with decrease in grit size and power rating. Kumar & Khamba, (2010) examined the effect of process parameters (material of tool, abrasive, power rating, and grain size) on MRR in ultrasonic machining of titanium. Results reported that, higher value of MRR was achieved while using a harder tool material (cemented carbide) along with hard abrasive material (boron carbide), higher rating of power supply (400 W), and coarse grit size (220).

Kataria, *et al.* (2015a) investigated the hole quality in ultrasonic machining of WC-Co composite material, and results revealed that power rating and grit size were leading parameters, which affects the hole accuracy. Dvivedi & Kumar, (2007) carried out an investigation on surface quality in ultrasonic machining using Taguchi method. Results reported that concentration of slurry, and grit size were found as most significant parameters affecting the surface roughness. Jadoun, *et al.* (2006) optimized the process parameters for cutting ratio (MRR/TWR) in ultrasonic machining using Taguchi method. Cutting rate was observed to increase with an increment in power rating. Kataria *et al.* (2015b) carried out an experimental investigation with a view to optimize machining characteristics in ultrasonic machining of WC-Co composite material. It was observed that work material with higher cobalt content resulted into lower rate of machining and tool wear.

Tong & Su, (1997) explained a multi-response robust design problem using a multiple-attribute decision-making (MADM) method. They considered the quality loss of each response and then adopted a MADM method - a technique for order preference by similarity to ideal solution (TOPSIS) to optimize the multi-response robust design problem. There are several common methodologies for MADM-simple additive weighting (SAW), TOPSIS, analytical hierarchy process (AHP), graph theory & matrix approach, and data envelopment analysis (DEA) etc.

Jangra, *et al.* (2011) utilized digraph and matrix based method to evaluate the machinability of tungsten carbide composite with wire electrical discharge machining (WEDM). In their research work machinability was evaluated in terms of MRR. Kataria & Kumar, (2014) employed MADM techniques to perform multi-response optimization in turning operation of AISI O1 tool steel. Rao & Gandhi, (2002) carried out the machinability evaluation of work materials using digraph and matrix method. It was reported that proposed method offers a simple and effective solution for the machinability evaluation in any machining operation of work materials. Table 1 depicts a review of various investigations carried out by numerous researchers.

Table 1. A review on optimization techniques applied by various investigators and their findings

S. No.	Investigators (Year)	Work Material	Process Parameters	Optimization Technique	Results/Findings
1.	S.K.Gauri, R.Chakravorty, S.Chakraborty (2011)	Titanium	Tool material, Power rating, Grit Size	PCA-based GRA, PCA-based TOPSIS, WPC	Both WPC and PCA-based TOPSIS methods provide better results for Multi-response optimization.
2.	Jatinder Kumar, J. S. Khamba, S. K. Mohapatra (2008)	Pure Titanium (ASTM Grade-I)	Tool material, Abrasive type, Grit size, Power rating, Slurry conc.	Taguchi method based single response optimization	Optimized Setting: Tool - Titanium alloy, Slurry- Alumina, Grit size-500, Power-100 W, concentration- 30%.
3.	S. K. Gauri, R. Chakravorty, S. Chakraborty (2011)	Titanium	Tool material, Abrasive type, Slurry Conc., Power rating, Grit Size	PCA-based GRA, PCA-based TOPSIS, WPC	WPC and PCA- based TOPSIS are providing better solutions for multiple response optimization.
4.	Jatinder Kumar, J. S. Khamba (2010)	Pure Titanium (ASTM Grade-I)	Tool material, Abrasive, Grit size, Power rating	Taguchi method based single response optimization	Optimized Setting: cemented carbide, Abrasive- Boron carbide, Grit size- 220, Power- 400 W (80%)
5.	Rupinder Singh, J. S. Khamba (2008)	Titanium (ASTM Gr.2), Titanium alloy (ASTM Gr.5)	Tool material, Abrasive type, Slurry conc., Grit size, Power rating, Slurry temp.	Taguchi method based single response optimization	Ultrasonic power rating at 450 W, with S.S tool and 500-grit-size slurry yielded best results for TWR.
6.	Vinod Kumar, J. S. Khamba (2009)	Stellite 6 (Cobalt alloy)	Tool material, Abrasive, Slurry conc., Grit size, Power rating	Taguchi method based multi response optimization	Optimized Setting: Titanium tool, boron carbide abrasive, 30% slurry conc., 220 mesh grit size, 125 W (25%) power input
7.	R. Singh, J.S. Khamba (2007)	Titanium (ASTM Gr.2), Titanium alloy (ASTM Gr.5)	Work material, Tool material, Abrasive, Power rating	Taguchi method based single response optimization	Best results have been obtained with SS tool and boron carbide slurry.
8.	A. Dvivedi, P. Kumar (2007)	Titanium (ASTM Gr.2), Titanium (ASTM Gr.5)	Work material, Grit size, Slurry conc., Power rating, Tool material	Taguchi method based single response optimization	Optimized Setting: Workpiece- ASTM Gr.2, Grit size- 320, Slurry conc.- 25%, Power rating- 40%, Tool- HSS
9.	Jatinder Kumar, J.S. Khamba (2008)	Pure Titanium (ASTM grade-I)	Tool material Abrasive Grit size Power rating	Taguchi method based single response optimization	Optimized Setting: titanium alloy (ASTM Gr.5), Abrasive- Alumina, Grit size- 500, Power- 100 W (20%) (For both response)

10.	Jatinder Kumar, J.S. Khamba (2009)	HCS, HSS, Aluminium, Titanium, Carbide, Glass	Work material, Tool material, Abrasive, Grit size, Tool geometry	Taguchi method based single response optimization	For TWR, tool geometry (35.37%), abrasive material (19.95), grit size (20.12), are the significant factors. For MRR, three factors are significant- work material (79%), tool geometry (10.86%) and grit size (5.50%). For SR, only work material is most significant.
11.	Vinod Kumar, J. S. Khamba (2009)	Stellite 6 (Cobalt alloy)	Tool material, Abrasive, Slurry conc., Grit size, Power rating	Taguchi Based Approach	Optimized Setting: titanium(ASTM Gr. 2), Abrasive- Al_2O_3, Slurry Conc.- 25%, Grit size- 500, Power- 125 W (25%)
12.	Jatinder Kumar, J.S. Khamba (2010)	Pure Titanium (ASTM grade-I)	Tool material, Abrasive type, Grit size, Power rating	Utility Concept for multi-response optimization	Optimized Setting: titanium alloy, Abrasive- Boron carbide, Grit size- 500, Power- 400W (80%)
13.	Vinod Kumar, J. S. Khamba (2008)	Tungsten-Carbide	Tool material, Abrasive type, Slurry conc., Grit size, Power rating	Taguchi Based Approach	MRR and the SR of the work piece are directly proportional to the abrasive slurry concentration and grit size.

SCOPE OF PRESENT RESEARCH

In view of reported literature, for the evaluation of machinability of pure titanium in ultrasonic machining, it is essential to examine a variety of factors and their effects on the machinability. Thus, there is need of scientific/mathematical tool possessing capability for analyzing the effects of various factors along with their effect on machinability. MADM approaches also offer a solution to the above discussed problem in an effective way (Rao & Gandhi, 2002). Graph theory and matrix based approach is one of the methodologies available, which can offer elucidation to the above discussed problem, as this method incorporates the relative importance. It also visually represents the attributes and their inter-dependencies by utilizing digraph. This method is well organized and logical that has been proven and validated in umpteen fields of science and technology (Jangra, *et al.* 2011). In this present work, graph theory and matrix based approach is proposed to evaluate the machinability of titanium in terms of tool wear rate, material removal rate, and surface roughness of machined surface, during ultrasonic machining. The analysis and quantification of various attributes, which affect the machinability, is made by using the machinability attribute digraph and matrix method. A permanent machinability index is utilized for the evaluation of machinability of titanium.

METHODOLOGY AND DESCRIPTION OF GRAPH THEORY AND MATRIX METHOD

Graph theory and matrix method is a systematic and logical concept, which was proved to be useful for analyzing and modeling wide range of applications in engineering and numerous other areas (Rao & Gandhi, 2002; Rao & Padmanabhan, 2007). The digraph is employed in order to visually represent the attributes with their relative importance, which affects the machinability. The matrix further describes attribute digraph into mathematical form. The machinability index is determined by using mathematical representation i.e. permanent function. Therefore, this article aims to present graph theory and matrix based method for the study of machinability of titanium in ultrasonic machining. The fundamental steps followed for implementing the above said approach are stated below.

Selection of attributes

In this primary step, various attributes, which affect the machinability of titanium for ultrasonic machining are identified, and the experimental design that satisfies the operation requirements is finalized. The attributes values (T_i) and relative importance (u_{ij}) are obtained using Tables 2 and 3.

Machinability attributes digraph representation

A digraph is utilized to illustrate the factors which affect the machinability, and interdependencies among them in terms of *edges* and *nodes*. A set of directed edges $R=\{u_{ij}\}$ and a set of nodes $Q=\{T_i\}$, with $i=1, 2, ..., X$ are consisted in digraph. A node *Ti* represents the i^{th} machinability attribute and edges represent their relative importance. The number of nodes (*X)* considered are same to the number of machinability attributes considered for the machining operation. Three important attributes namely (1) material removal rate, (2) tool wear rate, and (3) surface roughness are selected for the evaluation of machinability work considered. The machinability attribute diagraph for the present work is represented as shown in Figure 2.

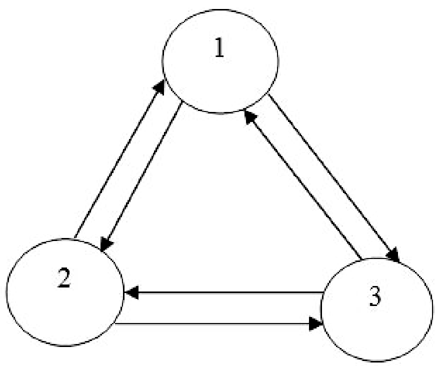

Fig. 2. Machinability attributes digraph for the ultrasonic machining operation (attributes: 1. Material removal rate; 2. Tool wear rate; and 3. Surface roughness)

Matrix representation of the machinability attributes digraph

The digraph is further needed to be represented in matrix form (H) called as machinability evaluation matrix or variable permanent matrix for permanent machinability index (VPM_M). This is $X \times X$ matrix and considers all of the attributes (i.e., T_i) with their relative importance (i.e., u_{ij}). The matrix shown in Equation (1) is expressed as per the machinability evaluation digraph (Figure 2).

The inheritance of the three important attributes is represented by diagonal elements T1, T2, and T3, and interdependencies among them are shown by off-diagonal elements of matrix for each attributes (Rao, 2007; Paramasivam & Senthil, 2009).

For the considered machinability attributes digraph, the matrix H, is illustrated as:

VPM_M=H

$$= \begin{array}{c} \text{Attributes} \\ 1 \\ 2 \\ 3 \\ 4 \\ \ldots \\ \ldots \\ X \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 & \ldots & \ldots & X \\ T_1 & u_{12} & u_{13} & u_{14} & \ldots & \ldots & u_{1X} \\ u_{21} & T_2 & u_{23} & u_{24} & \ldots & \ldots & u_{2X} \\ u_{31} & u_{32} & T_3 & u_{34} & \ldots & \ldots & u_{3X} \\ u_{41} & u_{42} & u_{43} & T_4 & \ldots & \ldots & u_{4X} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ u_{X1} & u_{X2} & u_{X3} & u_{X4} & \ldots & \ldots & T_X \end{bmatrix} \quad (1)$$

Representation of this variable permanent matrix for permanent machinability index (VPM_M) for the considered machinability attributes digraph (as shown in Figure 2) is given below:

VPM_M= H

$$= \begin{array}{c} \text{Attributes} \\ \text{MRR} \\ \text{TWR} \\ \text{SR} \end{array} \begin{array}{c} \begin{array}{ccc} \text{MRR} & \overrightarrow{\text{TWR}} & \text{SR} \end{array} \\ \begin{bmatrix} T_1 & u_{ij} & u_{ik} \\ u_{ji} & T_2 & u_{jk} \\ u_{ki} & u_{kj} & T_3 \end{bmatrix} \end{array} \quad (2)$$

Variable permanent function representation

The permanent of this matrix *H*, i.e., per (*H*), is defined as the permanent machinability function. Machinability evaluation represents machinability attributes of various experimental runs as considered for this present work. In addition, this approach leads to prevent any loss of information as it does not carry any negative sign in the expression (Rao, 2007).

The 'variable permanent machinability function' is expressed in sigma form as:

per(H)=

$$\prod_{i=1}^{X} T_i + \sum_{i,j,..,X}(u_{ij}u_{ji})T_kT_l...T_X + \sum_{i,j,....,X}(u_{ij}u_{jk}u_{ki} + u_{ik}u_{kj}u_{ji})T_lT_m...T_X$$

$$+\{\sum_{i,j,..,X}(u_{ij}u_{ji})(u_{kl}u_{lk})T_mT_n...T_X + \sum_{i,j,..,X}(u_{ij}u_{jk}u_{kl}u_{li})(u_{il}u_{lk}u_{kj}u_{ji})T_mT_n...T_X\}$$

$$+[\sum_{i,j,...,X}(u_{lm}u_{ml})(u_{ij}u_{jk}u_{ki} + u_{ik}u_{kj}u_{ji})T_nT_o..T_X$$

$$+\sum_{i,j,...X}(u_{ij}u_{jk}u_{kl}u_{lm}u_{mi} + u_{im}u_{ml}u_{lk}u_{kj}u_{ji})T_nT_o..T_X]$$

$$+[(\sum_{i,j,...,X}(u_{mn}u_{nm})(u_{ij}u_{jk}u_{kl}u_{li} + u_{il}u_{lk}u_{kj}u_{ji})T_o....T_X$$

$$+\sum_{i,j,...,X}(u_{ij}u_{jk}u_{ki} + u_{ik}u_{kj}u_{ji})(u_{lm}u_{mn}u_{nl} + u_{ln}u_{nm}u_{ml})T_o...T_X$$

$$+\sum_{i,j,...,X}(u_{ij}u_{ji})(u_{kl}u_{lk})(u_{mn}u_{nm})T_o...T_X$$

$$+\sum_{i,j,...,X}(u_{ij}u_{jk}u_{kl}u_{lm}u_{mn}u_{ni} + u_{in}u_{nm}u_{ml}u_{lk}u_{kj}u_{ji})T_o...T_X)] + ... \quad (3)$$

Evaluation of permanent machinability index

The permanent machinability function defined in Equation (3) is employed for evaluation of the permanent machinability index. The '*permanent machinability index*' is expressed as numerical value of permanent machinability function.

All the quantitative values of T_i are desirable to be normalized on the same scale as qualitative values, i.e. 0 to10. For *beneficial machinability attributes,* assignment of 0 and 10 is for smaller range value (T_{is}) and bigger range value (T_{ib}), respectively. Other intermediate values T_{ii} of the attributes could also be assigned in the scale from 0 to 10, as shown in Eqn. (4).

$$T_i = \{10/T_{ib}\} * T_{ii} \qquad \text{for } T_{is} = 0 \qquad (4)$$

$$T_i = \{10/(T_{ib} - T_{is})\} * (T_{ii} - T_{is}) \qquad \text{for } T_{is} > 0$$

Table 2. Quantification of factors affecting machinability of Titanium (Rao, 2007)

S. No.	Qualitative measure of factors affecting machinability of Titanium	Assigned value of machinability factors (T_i)
1.	Exceptionally low	0
2.	Extremely low	1
3.	Very low	2
4.	Below average	3
5.	Average	4
6.	Above average	5
7.	Moderate	6
8.	High	7
9.	Very high	8
10.	Extremely high	9
11.	Exceptionally high	10

For *non-beneficial machinability attributes,* assignment of 0 and 10 is for bigger range value (T_{is}) and smaller range value (T_{ib}), respectively. Other intermediate values T_{ii} of the attributes could also be assigned in the scale from 0 to 10, as shown in Eqn. (5).

$$T_i=10\{ 1-(T_{ii}/T_{ib})\} \quad \text{for } T_{is}=0 \tag{5}$$

$$T_i= \{10/(T_{ib} - T_{is})\} * (T_{ib}-T_{ii}) \quad \text{for } T_{is} > 0$$

Relative interdependency between two attributes (i.e. u_{ij}) for considered machining operation is also assigned as a value over the range from 0 to 10, and is arranged into six categories. The interdependency between two attributes can be distributed on the scale 0 to 10 as given below:

$$u_{ij}= 10-u_{ji} \tag{6}$$

Table 3. Relative importance of machinability attributes (u_{ij}) (Rao, 2007)

S. No.	Category description	Interdependencies of attributes	
		u_{ij}	$u_{ji}=10-u_{ij}$
1.	Two attributes are of equal importance	5	5
2.	One attribute is slightly more important than the other	6	4
3.	One attribute is more important than the other	7	3
4.	One attribute is much more important than the other	8	2
5.	One attribute is extremely more important than the other	9	1
6.	One attribute is exceptionally more important than the other	10	0

The experimental runs are then arranged in descending/ascending order as per the computed values of permanent machinability index. The experimental run possessing the *highest value* of permanent machinability index is chosen as the *best alternative* for the problem under consideration.

Identification and comparison of different available alternatives

Let V_{ij} represent the total value of the terms of j^{th} sub-grouping of the i^{th} grouping of the variable permanent machinability function. For the case of no sub-grouping, then the condition will be; $V_{ij}=V_i$, i.e., total value of terms of the i^{th} grouping. The identification set for an experimental run for the considered machining process is:

$$/V_1/V_2/V_3/V_4/V_{51}+V_{52}/V_{61}+V_{62}/ \tag{7}$$

A comparison between any two experimental run can also be made by using Equation (8). (Rao & Gandhi, 2002; Rao & Padmanabhan, 2006). On the basis of dissimilarity of performance, the dissimilarity coefficient (C_d) for any two experimental runs is proposed as;

$$C_d = (1/B) \sum_{i,j} \Phi_{ij} \tag{8}$$

where;

$$B = \text{max. of } [\sum_{i,j} |V_{ij}| \text{ and } \sum_{i,j} |V'_{ij}|]$$

The values of the terms for the variable permanent machinability function (V_{ij} and V'_{ij}) for two experimental run under the evaluation and comparison, and $\Phi_{ij} = |V_{ij} - V'_{ij}|$. The similarity coefficient is also expressed as;

$$C_s = 1 - C_d \tag{9}$$

Coefficients of similarity and dissimilarity

The calculation is being preformed for similarity and dissimilarity coefficients as per Equations (8) and (9). All the possible combinations are considered and evaluated in this step.

APPLICATION OF THE METHOD FOR EVALUATION OF MACHINABILITY IN USM OF TITANIUM

The graph theory and matrix method has been applied to evaluate the machinability of pure titanium for drilling operation using ultrasonic machining process. The experimental results obtained for surface roughness, TWR, and MRR have been taken from previously published research work (Kumar, *et al*. 2008) shown in Table 4. The experimentation work was conducted with full factorial design (24 experiments run) in randomized form to reduce the effect of noise factors and error. All the values indicated are averages of three samples for each run as each experiment was replicated twice.

Work material of Titanium (ASTM grade I) was used in the experimentation and alumina abrasives were used for slurry preparation. Experiments were conducted with Sonic Mill-AP 500W set up manufactured by Sonic Mill, Albuquerque.

Table 4. Experimental results of machining titanium work material (Kumar, *et al*. 2008)

Run	Tool Material	Grit Size	Power Rating (W)	MRR (mg/min.)	TWR (mg/min.)	SR (R_a) µm
1	TI	220	400	1.70	1.67	1.25
2	HCS	320	400	2.41	4.83	1.12
3	HCS	500	100	0.25	1.00	0.75
4	TI	220	100	0.71	0.67	1.12
5	TI	320	100	0.40	0.43	0.78
6	HCS	220	200	1.62	3.66	0.77
7	TI	320	200	0.63	0.31	0.96
8	HCS	500	300	0.52	1.61	0.87
9	TI	500	100	0.17	0.18	0.81
10	HCS	500	200	0.50	1.16	0.72
11	HCS	220	300	1.00	2.50	0.99
12	TI	500	200	0.13	0.16	0.83
13	TI	320	400	0.65	0.69	0.81
14	HCS	320	200	0.42	1.30	0.93
15	TI	500	400	0.45	0.48	0.80
16	HCS	320	100	0.55	1.52	0.97
17	HCS	320	300	0.70	1.64	0.61
18	HCS	500	400	1.68	3.37	0.69
19	TI	220	200	1.04	0.86	0.98
20	HCS	220	100	1.94	4.27	0.92
21	TI	320	300	0.37	0.37	0.77
22	HCS	220	400	3.50	7.00	1.50
23	TI	500	300	0.87	0.73	0.35
24	TI	220	300	0.50	0.45	1.20

The several steps included in graph theory and matrix based approach are given as follows:

Selection of attributes and normalization of the experimental results

The selection of machinability attributes and normalization of various experimental results has been carried out as described below:

- The machinability attributes are identified and the attributes considered are surface roughness (SR), tool wear rate (TWR), and material removal rate (MRR). Tool wear rate and surface roughness are considered as non-beneficial attributes whereas material removal rate is as beneficial attribute. The value of these machinability attributes are normalized for the different experimental runs using Equations (4) and (5), and are shown in Table 5. The interaction of attributes (i.e. u_{ij}) is also assigned values in the range from 0 to10, based on Table 3 and Equation (6). These values are presented in Table 6.

Table 5. Machinability attribute values (T_i) for the problem considered

Experimental Run	MRR	TWR	SR
1	4.7	7.8	2.2
2	6.8	3.2	3.3
3	0.4	8.8	6.5
4	1.7	9.3	3.3
5	0.8	9.6	6.3
6	4.4	4.9	6.4
7	1.5	9.8	4.7
8	1.2	7.9	5.5
9	0.1	9.9	6
10	1.1	8.5	6.8
11	2.6	6.6	4.4
12	0	10	5.8
13	1.5	9.2	6
14	0.9	8.3	5
15	0.9	9.5	6.1
16	1.2	8	4.6
17	1.7	7.8	7.7
18	4.6	5.3	7
19	2.7	9	4.5
20	5.4	4	5
21	0.7	9.7	6.4
22	10	0	0
23	2.2	9.2	10
24	1.1	9.6	2.6

Table 6. Relative importance of machinability attributes (u_{ij})

Attributes	MRR	TWR	SR
MRR	-	8	6
TWR	2	-	5
SR	4	5	-

- Figure 3 is depicting a permanent machinability attribute digraph which includes the considered machinability attribute and their interrelations.

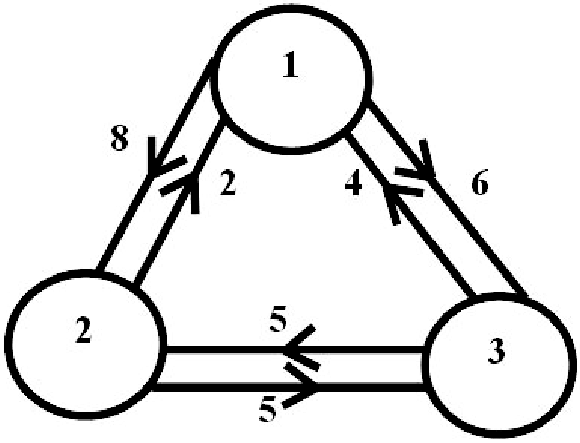

Fig. 3. Digraph illustrating machinability attributes with their relative importance for the considered problem (attributes: 1. MRR; 2. TWR; and 3. SR)

Representation of permanent machinability attributes matrix and variable permanent machinability function (VPF)

The permanent machinability attribute matrix H for the considered ultrasonic machining example is written as Equation (10).

$$H = \begin{array}{c} \text{Attributes} \\ \text{MRR} \\ \text{TWR} \\ \text{SR} \end{array} \begin{array}{c} \begin{array}{ccc} \text{MRR} & \text{TWR} & \text{SR} \end{array} \\ \begin{bmatrix} T_1 & u_{ij} & u_{ik} \\ u_{ji} & T_2 & u_{jk} \\ u_{ki} & u_{kj} & T_3 \end{bmatrix} \end{array} \tag{10}$$

The variable permanent machinability function (VPF) for the above matrix H, Equation (10), is

$$\text{per}(H) = \prod_{i=1}^{3} T_i + \sum_{i,j,k} (u_{ij} u_{ji}) T_k + \sum_{i,j,k} (u_{ij} u_{jk} u_{ki} + u_{ik} u_{kj} u_{ji}) \tag{11}$$

Computation of the numerical value of permanent machinability index

The value of permanent machinability index is calculated by utilizing the data of Table 10 and Table 11. The values of T_i and u_{ij} for each experimental run are also used for this index calculation. The permanent machinability index values of the different experimental runs in descending order are given in Table 7.

Table 7. Values of permanent machinability in descending order

Experimental Run	Permanent Machinability Index
23	858.200
18	744.860
6	687.984
19	684.850
17	675.002
13	657.100
1	640.552
20	639.00
7	636.990
10	623.880
15	620.255
5	619.584
21	616.156
2	591.408
4	590.673
11	589.304
8	579.740
3	568.080
9	562.040
16	559.760
14	559.050
12	552.800
24	546.956
22	470.000

From the various values of permanent machinability index for different experimental runs, it can be found that **23rd** experimental run gives the best machinability index followed by 18th experimental run.

This particular result obtained is found to be well consistent with that investigated by Kumar, *et al.* 2008 as the experimental values for MRR, TWR and SR were optimal at this particular setting in ultrasonic machining of pure titanium. Similar results were also reported by Kumar & Khamba, (2008), where optimized results included titanium tool material and grit size of 500, while performing ultrasonic machining of pure titanium. These research outcomes suggest that a power level of 100-300 W and grit size of 500 with titanium as tool material reported as optimized parametric setting for machinability evaluation.

Selection of best alternative among available alternatives can only be made, when all the alternatives must be provided with similar conditions of process variables. For this research work MADM (multi-attribute decision making) technique has been utilized in the form of graph theory and matrix method. As the experimentation was performed (Kumar, *et al.* 2008) with full factorial design, it is logical to apply this method as all the feasible combinations of process parameters have been tried for conducting the experiments. Hence, the best alternative (machining solution) can directly be chosen on the basis of machinability index value.

Identification and comparison of experimental runs conducted for the present study

The experimental runs are further compared on the basis of identification sets (using Equation (7)) for the problem considered. The combination of all these four groupings results into machinability index. Similarity/dissimilarity coefficients are also calculated for the different experimental runs using Equations (8) and (9). Table 9 depicts the values of similarity coefficient (C_s) for different experimental runs, as this method has the capability to perform calculation for the similarity/dissimilarity coefficient for all the possible combinations of available alternatives. This feature enables decision maker to know and confirm the extent to which two alternatives are similar or dissimilar to each other.

Table 8. Values of different groupings (V_i) of the permanent machinability function of the problem taken under consideration

Experimental Runs	Grouping			
	I	II	III	IV
1	80.652	0	339.90	220
2	71.808	0	299.60	220
3	22.880	0	325.20	220
4	52.173	0	318.50	220
5	48.384	0	351.20	220
6	137.984	0	330.00	220
7	69.090	0	347.90	220
8	52.140	0	307.60	220
9	5.940	0	336.10	220
10	63.580	0	340.30	220
11	75.504	0	293.80	220
12	0	0	332.80	220
13	82.800	0	354.30	220
14	37.350	0	301.70	220
15	52.155	0	348.10	220
16	44.160	0	295.60	220
17	102.102	0	352.90	220
18	170.660	0	354.20	220
19	109.350	0	355.50	220
20	108.000	0	311.00	220
21	43.456	0	352.70	220
22	0	0	250.00	220
23	202.400	0	435.80	220
24	27.456	0	299.50	220

Table 9. Values of similarity coefficient (C_s) for different experimental runs

Exp. No.	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0.92	0.89	0.92	0.97	0.93	0.99	0.99	0.91	0.97	0.92	0.86	0.97	0.87	0.97	0.87	0.95	0.86	0.94	0.99	0.96	0.73	0.75	0.85
2		0.96	0.99	0.95	0.86	0.93	0.98	0.95	0.95	0.99	0.93	0.90	0.95	0.95	0.95	0.88	0.79	0.86	0.93	0.96	0.79	0.69	0.92
3			0.96	0.92	0.83	0.89	0.98	0.99	0.91	0.96	0.97	0.86	0.98	0.92	0.99	0.84	0.76	0.83	0.89	0.92	0.83	0.66	0.96
4				0.95	0.86	0.93	0.98	0.95	0.95	0.99	0.94	0.90	0.95	0.95	0.95	0.88	0.79	0.86	0.92	0.96	0.80	0.69	0.93
5					0.90	0.97	0.94	0.91	0.99	0.95	0.89	0.94	0.90	0.99	0.90	0.92	0.83	0.90	0.97	0.99	0.76	0.72	0.88
6						0.93	0.84	0.82	0.91	0.86	0.80	0.96	0.81	0.90	0.81	0.98	0.92	0.99	0.93	0.90	0.68	0.80	0.80
7							0.91	0.88	0.98	0.93	0.87	0.97	0.88	0.97	0.88	0.94	0.86	0.93	0.99	0.97	0.74	0.74	0.86
8								0.97	0.93	0.98	0.95	0.88	0.96	0.93	0.97	0.86	0.78	0.85	0.91	0.94	0.81	0.68	0.94
9									0.90	0.95	0.98	0.86	0.99	0.91	0.99	0.83	0.75	0.82	0.88	0.91	0.84	0.65	0.97
10										0.94	0.89	0.95	0.90	0.99	0.90	0.92	0.84	0.91	0.98	0.99	0.75	0.73	0.88
11											0.94	0.90	0.95	0.95	0.95	0.87	0.79	0.86	0.92	0.96	0.80	0.69	0.93
12												0.84	0.98	0.89	0.98	0.82	0.74	0.80	0.86	0.89	0.85	0.64	0.99
13													0.85	0.94	0.85	0.97	0.88	0.96	0.97	0.93	0.71	0.76	0.83
14														0.90	0.99	0.83	0.75	0.82	0.87	0.91	0.84	0.65	0.98
15															0.90	0.92	0.83	0.90	0.97	0.99	0.76	0.72	0.88
16																0.83	0.75	0.82	0.88	0.90	0.84	0.65	0.97
17																	0.90	0.98	0.94	0.91	0.70	0.79	0.81
18																		0.92	0.85	0.83	0.63	0.87	0.73
19																			0.93	0.89	0.69	0.80	0.79
20																				0.96	0.74	0.74	0.86
21																					0.76	0.72	0.88
22																						0.55	0.86
23																							0.64

CONCLUSIONS

In this present work, the evaluation of machinability of work material in any machining process is highlighted using graph theory and matrix method based approach. This method consists of machinability attribute digraph, matrix representation of attributes, permanent machinability function and permanent machinability index. On the basis of the study, following conclusions can be drawn.

1. A graph theory and matrix method based methodology is proposed and validated for the machinability evaluation of titanium work material in ultrasonic machining. The versatile nature of this method makes it more suitable for numerous applications of machining.

2. In the present work, various attributes, which define the machinability of titanium in ultrasonic machining are identified. Results reveal that the 23rd experimental run gives highest value of permanent machinability index. This experimental run is consisting of the combination of titanium as tool material, silicon carbide as abrasive, fine grit size (mesh 500) and moderate level of power rating (300W).

3. The suggested graph theory and matrix method based approach can be utilized for any machining problem related to optimization of multiple, correlated responses of interest, under the influence of several input parameters.

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