On developing linear profile methodologies: a ranked set approach with engineering application

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ABSTRACT

Linear profiles are quite popular in establishing relationships among different variables associated with each other in an ongoing process. Control charting methodologies for these linear profiles are used to monitor and improve the performance of a process. The commonly used Phase I methodologies of linear profiles are mostly based on simple random selection procedures. In this study, we intend to improve the existing Phase I profile methods by considering different ranked set strategies including ranked set sampling (RSS), median RSS (MRSS) and extreme RSS (ERSS). The profile monitoring is considered in terms of three main parameters namely slope, intercept and error variance for efficient detection of any assignable cause(s). We have used probability to signal as a performance measure in our study. A real-life application of the proposed methods is also presented in this study using real data from electrical engineering related to a grid-connected photovoltaic system.

Keywords: control charts; profiles monitoring; phase I methods; probability to signal; solar power monitoring.

1. INTRODUCTION

The control chart is a primary tool used for the monitoring of process parameters and applied in different manufacturing and industrial processes (cf. Yen et al. (2018)). In many manufacturing processes, an inspection of a product is studied on the samples drawn by using the simple random sampling (SRS) method. In SRS, samples are drawn at random with equal probability, but in some processes, samples are expensive and difficult to obtain. For such instance, McIntyre (1952) proposed a new sampling methodology known as ranked set sampling (RSS) which provides more efficient, precise and accurate estimates as compared to the SRS mechanism. Recently, many researchers have used RSS and its modified forms such as; median RSS (MRSS) and extreme RSS (ERSS), in the monitoring of process parameters. For more details see, Abujiya et al. (2015a, 2015b).

In many applied situations, characteristic (quantitative or qualitative) of a product is considered by the functional relationship between two or more variables and termed as profiles which may be linear or non-linear. Profile monitoring is an assessment to observe the constancy of profile parameters in order to detect unusual behavior in the profile parameters. There are several practical situations that are possessing the abovementioned characteristics such as: in drug manufacturing studies see, Kulasekera (1995); in the area of automotive engineering and semiconductor manufacturing see, Kang and Albin (2000b) and Mahmoud and Woodall (2004).

Earlier, the idea of regression control chart was designed by Mandel (1969) and the multivariate control charting structures for regression-adjusted variables were discussed by Hawkins (1991, 1993), Mestek et al. (1994) and

Stover and Brill (1998) and Hauck et al. (1999). However, Kang and Albin (2000a) proposed two schemes; bivariate Hotelling's T^2 chart for detecting any change in regression coefficients and $EWMA/R$ chart for the detection of the unusual change in the error variance. The simultaneous inspection of shifts in simple linear profiles (SLP) parameters (e.g., slope, intercept and standard deviation of error term) was proposed by Kim et al. (2003) while retrospective study for the parameters of multiple linear regression was designed by Mahmoud and Woodall (2004).

Noorossana et al. (2004a) designed a study to improve the performance of linear profiles by considering the structures of $MCUSUM$ and R charts while the issue of non-normality in SLP was addressed by Noorossana et al. (2004b). Moreover, monitoring of SLP based on the change point model was designed by Mahmoud et al. (2007), Yeh and Zerehsaz (2013) and Zou et al. (2006). Further, a comparative study on the methods initiated by Croarkin and Varner (1982) and Kim et al. (2003) was developed by Gupta et al. (2006). In phase II, enhancement of linear profile monitoring using integrated MCUSUM and χ^2 charts were designed by Noorossana and Amiri (2007) while a comprehensive review on profile monitoring was provided by Woodall (2007). Niaki et al. (2007) established a study related to generalized linear test and made comparisons with multivariate Hotelling's T^2 and $EWMA/R$ control charts. The recursive residuals and mixed model in linear profiles was studied by Zou et al. (2007) and Jensen et al. (2008). Soleimani et al. (2009) planned a study to eliminate the problem of within autocorrelation in the proposal of Jensen et al. (2008) while monitoring the SLP based on likelihood ratio test was discussed by Zhang et al. (2009). Saghaei et al. (2009) expands the study of Kim et al. (2003) by incorporating the CUSUM structure while a new approach based on small sample size was discussed by Mahmoud et al. (2010). Phase II studies for the SLP under the random effect model was studied by Noorossana et al. (2015) and Abbas et al. (2019). Riaz and Touqeer (2015) proposed run rule schemes to enhance the performance of both linear and multiple linear profile methodologies and Aslam et al. (2018) discussed several sampling plans based on EWMA structure for the regression estimator. Riaz et al. (2017) examined the linear profile parameters by using the EWMA-3 approach under ranked set samplings while Taghipour et al. (2017) proposed phase I mechanism for linear profile under within autocorrelated multivariate model. The Bayesian approach of SLP using DEWMA control chart structure was designed by Abbasi et al. (2018). The max and sum of square structures for the simultaneous monitoring of SLP parameters are discussed by Mahmood et al. (2018) and progressive statistics based SLP study was designed by Saeed et al. (2018). Moreover, Riaz et al. (2019) explored SLP monitoring under modified successive sampling.

Recently, Mahmood et al. (2019) have designed a Phase I study originated by Kim et al. (2003) under different ranked set strategies while other Phase I methods proposed by Mestek et al. (1994), Stover and Brill (1998) and Mahmoud and Woodall (2004) were not discussed under different ranked set strategies in the best of our knowledge. Therefore, this study is purely designed to investigate the performance of the aforementioned Phase I methodologies under different ranked set samplings.

The rest of the article is arranged as follows: Section 2 provides the description of linear profile methods under different RSS schemes. Section 3 comprises the performance of modified methods and their comparison. Section 4 consists of illustrative example and Section 5, finally, provides the summary and conclusions of our study.

2. PROFILE METHODOLOGIES UNDER RSS SCHEMES

In this section, we discuss the structure of RSS strategies which are further used to enhance the monitoring of SLP parameters. Moreover, we provide the theoretical background of linear profile methodology under RSS.

2.1. RSS schemes

The concept of RSS was introduced by McIntyre (1952). The procedure of RSS is outlined as select n sets having n random samples in each set. Rank (ascending) the samples within each set according to the concomitant variable or variable of interest. For actual RSS samples, choose the 1^{st} smallest sample from 1^{st} set, the 2^{nd} smallest sample from 2^{nd} set and continue this procedure until the largest sample is selected from the n^{th} set. This cycle may have repeated r time until nr samples have been observed. These nr samples are the observations of the RSS data set. Further, more mathematical modifications in RSS are made by Takahasi and Wakimoto (1968).

The median RSS (MRSS) is a modified version of RSS which was introduced by Muttlak (1997). In MRSS, we observed n sets having of n random samples. Arrange (ascending) the samples within each set according to the concomitant variable or variable of interest. If the size of the set is even, select first half from the smallest ranked $(n/2)^{th}$ order and remaining half from the smallest ranked $((n+2)/2)^{th}$ order. When the size of the set is odd, select the median of the sets (i.e., $((n+1)/2)^{th}$ smallest rank). This cycle may have repeated r time until nr samples have been observed. These nr samples are the observations of MRSS data set.

Samawi et al. (1996) proposed another sampling strategy named as extreme RSS (ERSS). The mechanism of ERSS is defined as, select n sets having n random samples in each set and sort the samples within each set according to concomitant variable or variable of interest. For odd set size, pick the largest sample from last $((n-1)/2)$ sets, smallest sample from the 1^{st} $((n-1)/2)$ sets and median of the rest of the sets. when set size is even, pick the smallest sample from 1^{st} $(n/2)$ sets and largest sample from remaining $(n/2)$ sets for an actual measurement. This cycle may have repeated r time until nr samples have been observed. These nr samples are the observations of ERSS data set.

2.2. Linear profile methodologies under RSS schemes

In many industrial processes, the study variable may be associated with the other explanatory variable(s). The monitoring of the study variable when it is linearly associated with an explanatory variable is known as simple linear profile (SLP). In this subsection, we provide the theoretical foundation of SLP under RSS schemes (later expressed by $\mathbb{R}$). In the stated study, we have considered different choices of $\mathbb{R}$, such as RSS, MRSS and ERSS to compare the methods given in Mahmoud and Woodall (2004). For the inferences of simple linear profiles under RSS strategies see Samawi and Ababneh (2001), and Alodat et al. (2010). A traditional model for the SLP is defined as follows:

$$Y_{ij} = \alpha + \beta X_i + \varepsilon_{ij}\ ;\ i = 1,2,..,n\ ;j = 1,2,\dots,k; \tag{1}$$

and the model (1) under RSS schemes is expressed as follows:

$$Y_{[i]jl} = \alpha + \beta X_{(i)} + \varepsilon_{[i]jl}\ ;\ i = 1,2,3,..,n\ ;j = 1,2,3,\dots,k;\ \ l = 1,2,3,..,r \tag{2}$$

where α represents intercept, β denotes slope, ε is the error term, n represents sample size, k is the size of subgroups and r denotes the number of cycles. Further, it is noted that in the whole study a single cycle is considered. The least square estimates for α and β under $\mathbb{R}$ schemes are defined as:

$$\hat{\beta}_{[\mathbb{R}]} = \frac{S_{xy_{[\mathbb{R}]}}}{S_{xx}} = \frac{\sum_{l=1}^{r}\sum_{j=1}^{k}\sum_{i=1}^{n}(X_{(i)} - \bar{X})Y_{[i]jl}}{\sum_{i=1}^{n}(X_{(i)} - \bar{X})^2},$$

$$\hat{\alpha}_{[\mathbb{R}]} = \bar{Y}_{[\mathbb{R}]} - \hat{\beta}_{[\mathbb{R}]}\bar{X},$$

where, $\bar{Y}_{[\mathbb{R}]} = \frac{\sum_{l=1}^{r}\sum_{j=1}^{k}\sum_{i=1}^{n}Y_{[]jl}}{rkn}$, $\bar{X} = \frac{\sum_{j=1}^{n}X_{(i)}}{n}$, and the conditional mean vector (μ) and variance covariance matrix (Σ) of $\hat{\alpha}_{[\mathbb{R}]}$ and $\hat{\beta}_{[\mathbb{R}]}$ are given as follows:

$$\mu_{[\mathbb{R}]} = \left(E\left[\hat{\alpha}_{[\mathbb{R}]}|X\right], E\left[\hat{\beta}_{[\mathbb{R}]}|X\right]\right)^T = (\alpha\ ,\beta)^T,$$

$$\Sigma_{[\mathbb{R}]} = \begin{pmatrix} \sigma^2_{e[\mathbb{R}]}\left[\frac{1}{rkn} + \frac{\bar{X}^2}{S_{xx}}\ \right] & -\frac{\sigma^2_{e[\mathbb{R}]}\bar{X}}{S_{xx}} \\ -\frac{\sigma^2_{e[\mathbb{R}]}\bar{X}}{S_{xx}} & \frac{\sigma^2_{e[\mathbb{R}]}}{S_{xx}} \end{pmatrix}.$$

The mean square error is an unbiased estimator of $\sigma^2_{e[\mathbb{R}]}$ which is expressed as follows:

$$MSE_{[\mathbb{R}]} = \frac{\sum_{j=1}^{k} MSE_{[\mathbb{R}]j}}{k},$$

where, $MSE_{[\mathbb{R}]j} = \frac{\sum_{l=1}^{r}\sum_{i=1}^{n} e_{[i]jl}^2}{r(n-2)}$ and $e_{[i]jl} = y_{[i]jl} - \hat{y}_{[i]jl}$ which is the i^{th} residual in j^{th} subgroup of l^{th} cycle and $\hat{y}_{[i]jl}$ is the i^{th} fitted regression line in j^{th} subgroup of l^{th} cycle. Moreover, the vector of the estimators is denoted by $Z_{[\mathbb{R}]}$ and defined as $Z_{[\mathbb{R}]} = \left(\hat{\alpha}_{[\mathbb{R}]}, \beta_{[\mathbb{R}]}\right)^T$ while the sample mean vector of estimators expressed as $\bar{Z}_{[\mathbb{R}]}$, sample variance-covariance matrix ($S_{1[\mathbb{R}]}$) and $\hat{\Sigma}_{[\mathbb{R}]}$ matrix by replacing $\sigma_{e[\mathbb{R}]}^2$ with its estimate $MSE_{[\mathbb{R}]}$ are defined as follows:

$$\bar{Z}_{[\mathbb{R}]} = \left(\frac{\sum_{l=1}^{r}\sum_{j=1}^{k}\hat{\alpha}_{[\mathbb{R}]jl}}{rk}, \frac{\sum_{l=1}^{r}\sum_{j=1}^{k}\hat{\alpha}_{[\mathbb{R}]jl}}{rk}\right)^T = \left(\bar{\alpha}_{[\mathbb{R}]}, \bar{\beta}_{[\mathbb{R}]}\right)^T, \tag{3}$$

$$S_{1[\mathbb{R}]} = \begin{pmatrix} \frac{\sum_{l=1}^{r}\sum_{j=1}^{k}\left(\hat{\alpha}_{[\mathbb{R}]jl} - \bar{\alpha}_{[\mathbb{R}]}\right)^2}{r(k-1)} & \frac{\sum_{l=1}^{r}\sum_{j=1}^{k}\left(\hat{\alpha}_{[\mathbb{R}]jl} - \bar{\alpha}_{[\mathbb{R}]}\right)\left(\beta_{[\mathbb{R}]jl} - \bar{\beta}_{[\mathbb{R}]}\right)}{r(k-1)} \\ \frac{\sum_{l=1}^{r}\sum_{j=1}^{k}\left(\hat{\alpha}_{[\mathbb{R}]jl} - \bar{\alpha}_{[\mathbb{R}]}\right)\left(\beta_{[\mathbb{R}]jl} - \bar{\beta}_{[\mathbb{R}]}\right)}{r(k-1)} & \frac{\sum_{l=1}^{r}\sum_{j=1}^{k}\left(\beta_{[\mathbb{R}]jl} - \bar{\beta}_{[\mathbb{R}]}\right)^2}{r(k-1)} \end{pmatrix}, \tag{4}$$

$$\hat{\Sigma}_{[\mathbb{R}]} = \begin{pmatrix} MSE_{[\mathbb{R}]}\left[\frac{1}{rkn} + \frac{\bar{X}^2}{S_{xx}}\right] & -\frac{MSE_{[\mathbb{R}]}\bar{X}}{S_{xx}} \\ -\frac{MSE_{[\mathbb{R}]}\bar{X}}{S_{xx}} & \frac{MSE_{[\mathbb{R}]}}{S_{xx}} \end{pmatrix}. \tag{5}$$

In simultaneous monitoring, it is necessary to make the intercept and slope independently while error variance is already independent of the slope and intercept. Therefore, Kim et al. (2003) preferred the coded model (given in equation (6)) instead of the model (2) which is providing zero covariance between intercept and slope. The coded model is obtained by implementing the transformation on $X_{(i)}$ values, (i.e. $X_{(i)}^* = X_{(i)} - \bar{X}$). The transformed model with $\alpha^* = \alpha + \beta\bar{X} + (\partial\sigma)\bar{X}$ and $\beta^* = (\beta + \partial\sigma)X_{(i)}^*$ is represented as;

$$Y_{[i]jl} = \alpha^* + \beta^* X_{(i)}^* + \varepsilon_{[i]jl} \tag{6}$$

where the shift in the slope of the model (2) is obtained in terms of σ units (i.e. $\partial\sigma$). The least square estimates of intercept and slope under $\mathbb{R}$ schemes using model (6) are denoted by $\hat{\alpha}_{[\mathbb{R}]}^*$ and $\hat{\beta}_{[\mathbb{R}]}^*$ respectively. Further, Phase I methods under RSS schemes for the monitoring of SLP parameters are discuss below.

Method A: Stover and Brill (1998) proposed a method based on Hotelling's T^2 chart which is estimated through the vector of intercept and slope. The plotting statistic depends on the $\bar{Z}_{[\mathbb{R}]}$ and $S_{1[\mathbb{R}]}$ given in equation (3) and (4), is defined as:

$$T_{1[\mathbb{R}]}^2 = \left(Z_{[\mathbb{R}]} - \bar{Z}_{[\mathbb{R}]}\right)^T S_{1[\mathbb{R}]}^{-1}\left(Z_{[\mathbb{R}]} - \bar{Z}_{[\mathbb{R}]}\right),$$

However, the control limit of this method depends on the charting constant $(L_{1[\mathbb{R}]})$ which is defined as:

$$UCL_A = (k-1)^2 L_{1[\mathbb{R}]}/k,$$

Method B: Kang and Albin (2000a) initiated two different Phase II methods for the monitoring of SLP parameters. The first approach consists of Hotelling's T^2 charting structure while second based on $EWMA/R$ approach. They also recommended that one may obtain these Phase II structures as Phase I structures by replacing the estimates of unknown parameters. In this study, we are intended to use the first approach under ranked set samplings. The plotting statistic $T^2_{2[\mathbb{R}]}$ (based on $\bar{Z}_{[\mathbb{R}]}$ and $\hat{\Sigma}_{[\mathbb{R}]}$ given in equation (3) and (5)) is defined as:

$$T^2_{2[\mathbb{R}]} = k\left(Z_{[\mathbb{R}]} - \bar{Z}_{[\mathbb{R}]}\right)^T \hat{\Sigma}^{-1}_{[\mathbb{R}]} \left(Z_{[\mathbb{R}]} - \bar{Z}_{[\mathbb{R}]}\right)/(k-1),$$

whereas, UCL_B is the probability control limit used as a threshold for this method.

Method C: Kim et al. (2003) proposed a linear profile method based on the coded model (6), used for the detection of shift(s) in linear profile parameters such as (error variance, intercept and slope). The Shewhart control chart for each linear profile parameter under ranked set samplings are defined as follow:

for $\hat{\alpha}^*_{[\mathbb{R}]}$;
$$LCL_{CI} = \overline{\alpha^*}_{[\mathbb{R}]} - L_{3[\mathbb{R}]}\sqrt{\frac{(k-1)MSE_{[\mathbb{R}]}}{rkn}}, \quad UCL_{CI} = \overline{\alpha^*}_{[\mathbb{R}]} + L_{3[\mathbb{R}]}\sqrt{\frac{(k-1)MSE_{[\mathbb{R}]}}{rkn}},$$

for $\hat{\beta}^*_{[\mathbb{R}]}$;
$$LCL_{CS} = \overline{\beta^*}_{[\mathbb{R}]} - L_{4[\mathbb{R}]}\sqrt{\frac{(k-1)MSE_{[\mathbb{R}]}}{rkS_{xx}}}, \quad UCL_{CS} = \overline{\beta^*}_{[\mathbb{R}]} + L_{4[\mathbb{R}]}\sqrt{\frac{(k-1)MSE_{[\mathbb{R}]}}{rkS_{xx}}},$$

where $L_{3[\mathbb{R}]}$ and $L_{4[\mathbb{R}]}$ are the control charting constant for the intercept and slope respectively. Further, for the monitoring of error variance $\sigma^2_{e[\mathbb{R}]}$, the plotting statistic $(F_{[\mathbb{R}]jl})$ is used which is defined as follows:

$$F_{[\mathbb{R}]jl} = \frac{MSE_{[\mathbb{R}]j}}{MSE_{[\mathbb{R}](-j)}},$$

and the control limits (LCL_{CF} and UCL_{CF}) are used as a threshold for the monitoring of error variance.

Method D: Mahmoud and Woodall (2004) proposed a Phase I structure which comprises typical charting setups for the monitoring of slope and intercept (cf. Kim et al. (2003)) while global F-test is used for the monitoring of error variance. In this approach, global F-test (based on dummy variable) is used, and the structure of dummy variable technique under $\mathbb{R}$ comprises r cycles, where k samples of bivariate observation are pooled to create $r(k-1)$ indicator variables:

$$Z_{[\mathbb{R}]} = \begin{cases} 1, & \text{In } l^{\text{th}} \text{ cycle, if observation } i \text{ is from sample } j \\ 0, & \text{otherwise} \end{cases};$$

$$i = 1,2,3,\ldots,kn\,; j = 1,2,3,\ldots,k-1;\;\; l = 1,2,3,\ldots,r,$$

whereas, k^{th} sample is termed reference sample and multiple regression model on the pooled data set can be obtained by using the following equation:

$$Y_{[i]l} = \alpha_l + \beta_l X_{[i]l} + \beta_{01l} Z_{[i]1l} + \beta_{02l} Z_{[i]2l} +, \ldots\ldots, +\beta_{0(k-1)l} Z_{[i](k-1)l} + \beta_{11l} Z_{[i]1l} X_{[i]l} +$$
$$\beta_{12l} Z_{[i]2l} X_{[i]l} +, \ldots\ldots, +\beta_{1(k-1)l} Z_{[i](k-1)l} X_{[i]l} + \varepsilon_{[i]l}\,;$$
$$i = 1,2,3,\ldots,kn\,; l = 1,2,3,\ldots,r; \tag{7}$$

So, to check the equality of k regression lines, the null hypothesis is developed on the following model

$$Y_{[i]jl} = \alpha + \beta X_{[i]} + \varepsilon_{[i]l}\,;\; i = 1,2,3,..,kn;\; l = 1,2,3,..,r \tag{8}$$

To monitors the process variation by global F-test, we use $F_{[\mathbb{R}]g}$ statistic which is calculated by

$$F_{[\mathbb{R}]g} = \frac{SSE_{[\mathbb{R}]}(reduced) - SSE_{[\mathbb{R}]}\,(full)}{2r(k-1) * MSE_{[\mathbb{R}]}\,(full)},$$

where $SSE_{[\mathbb{R}]}\,(reduced)$ is the sum of squares (SS) of residuals from the reduced model given in equation (7), $SSE_{[\mathbb{R}]}(full)$ is the SS of full model reported in equation (8) and $MSE_{[\mathbb{R}]}(full)$ is the MSE of the full model (cf. equation (8)). The control limits (UCL_{DF}) is used as a threshold for the detection of change in the error variance while the statistics and the limits for the detection of change in intercept and slope are defined as follow:

$$\text{for } \hat{\alpha}^*_{[\mathbb{R}]}; \quad \begin{aligned} LCL_{DI} &= \overline{\alpha^*}_{[\mathbb{R}]} - L_{3[\mathbb{R}]}\sqrt{\frac{(k-1)MSE_{[\mathbb{R}]}}{rkn}} \\ UCL_{DI} &= \overline{\alpha^*}_{[\mathbb{R}]} + L_{3[\mathbb{R}]}\sqrt{\frac{(k-1)MSE_{[\mathbb{R}]}}{rkn}} \end{aligned},$$

$$\text{for } \hat{\beta}^*_{[\mathbb{R}]}; \quad \begin{aligned} LCL_{DS} &= \overline{\beta^*}_{[\mathbb{R}]} - L_{4[\mathbb{R}]}\sqrt{\frac{(k-1)MSE_{[\mathbb{R}]}}{rkS_{xx}}} \\ UCL_{DS} &= \overline{\beta^*}_{[\mathbb{R}]} + L_{4[\mathbb{R}]}\sqrt{\frac{(k-1)MSE_{[\mathbb{R}]}}{rkS_{xx}}} \end{aligned},$$

Further, the performance of the aforementioned methods used for the monitoring of SLP parameters under RSS, MRSS and ERSS discusses in the next section.

3. PERFORMANCE EVALUATIONS AND COMPARISONS

In this section, we provide a brief discussion on the in-control (IC) parameters of the modified Phase I linear profile methods. Moreover, we will discuss the performance evaluation and comparative study between existing and modified Phase I linear profile methods.

3.1. Designing of in-control parameters

For the original IC simple linear model given in equation (2), we assumed $\alpha = 0$ and $\beta = 1$ by following Mahmoud and Woodall (2004) (i.e. $Y_{[i]jl} = X_{(i)} + \varepsilon_{[i]l}$). Where the fixed values of the explanatory variable are taken as $X_{(i)} = 0, 0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6,$ and 1.8 and the error term is obtained as $\varepsilon_{[i]jl} \sim BN(s, t;\ \mu_s = 0, \sigma_s = 1, \mu_t = 0, \sigma_t = 1, \rho = 1)$. Moreover, the transformed model (6) is obtained by substituting the $\alpha^* = \alpha + \beta\bar{X} + (\partial\sigma)\bar{X}$ and $\beta^* = (\beta + \partial\sigma)X^*_{(i)}$. whereas, the fixed transformed values of the explanatory variable are $X^*_{(i)} = -0.9, -0.7, -0.5, -0.3, -0.1,\ 0.1,\ 0.3,\ 0.5,\ 0.7$ and 0.9 with average equals to zero.

In Phase I study, samples are collected in the form of rational subgroups (k). Subgroups are introduced in such a way that in the presence of instable values, the chances of variation with in subgroups will be minimized while chances of variation between subgroups will be maximized. Further, subgroups are categorized as m_0 (stable subgroups) and m_1 (inconsistent subgroups) (i.e. $k = m_0 + m_1$). In this study, we have considered 20 subgroups ($k = 20$) each of sample size ($n = 10$) and to examine the performance of modified Phase I SLP methods, we introduced 10% inconsistent subgroups (i.e. $m_1 = 2$).

The performance of modified Phase I SLP methods is evaluated in terms of overall probability to signal (PTS), which is defined as the detection ability of a chart in terms of probability when the process is actually out-of-control

(OOC). It is to be noted that under the null hypothesis, the overall probability to signal (PTS) is termed as the overall false-alarm probability (FAP) which is further denoted by θ. The control limit(s) are chosen for each method in such a way that individual FAP may be set to achieve a specified value of θ. For method A and B, individual FAP (θ_1) is used to set the upper control limits (i.e., UCL_A and UCL_B), where θ_1 is obtained by $\theta_1 = 1 - \sqrt[k]{1-\theta}$. The control limits of method C are obtained by the individual FAP (θ_2), which is calculated as $\theta_2 = 1 - \sqrt[3]{1-\theta_1}$. Moreover, in method D, the global F-test performed at a level of significance $\theta_3 = 1 - \sqrt{1-\theta}$ and the limits of slope and intercept are obtained by using $\theta_4 = 1 - \sqrt[k]{1-\theta_3}$.

3.2. Algorithm to obtain control limits

As mentioned above, the control limits of method A and B are represented by UCL_A and UCL_B while in method C, LCL_{CI} and UCL_{CI} are the threshold for the intercept parameter, LCL_{CS} and UCL_{CS} are used for the slope parameter and LCL_{CF} and UCL_{CF} are the limits for the F-statistic. However, in method D, LCL_{DI} and UCL_{CI} are the threshold for the intercept parameter, LCL_{CS} and UCL_{CS} are used for the slope parameter and LCL_{CF} and UCL_{CF} are the limits for the F-statistic. The procedure to find control limits of the stated methods is illustrated in the following steps:

i. Generate a subgroup of a fixed size n using bi-variate normal distribution i.e., $\varepsilon_{[i]1l} \sim BN(s,t;\ \mu_s = 0, \sigma_s = 1, \mu_t = 0, \sigma_t = 1, \rho)$ for a specific choice of ρ and ranked set strategy (discussed in section 2.1).

ii. Plug the generated values into equation (6) to obtain response variable values $Y_{[i]1l}$ by fixing the IC parameters $\alpha^* = 0$ and $\beta^* = 1$.

iii. Regress the obtained $Y_{[i]1l}$ values against $X^*_{(i)} = -0.9, -0.7, -0.5, -0.3, -0.1,\ 0.1,\ 0.3,\ 0.5,\ 0.7,$ and 0.9 to obtain the least square estimates of $\hat{\alpha}^*_{[\mathbb{R}]}$, $\hat{\beta}^*_{[\mathbb{R}]}$ and $MSE_{[\mathbb{R}]}$.

iv. For method A and B, repeat steps i-iii k times and obtained k estimates of Hotelling's T^2 statistics. However, for method C and D, repeat step i-iii k times and from the k estimates obtained the mean and standard errors of intercept and slope. Further, calculate the standardized version of intercept, slope and global F-statistic.

v. Repeat steps i-iv, a large number of times in order to get a complete empirical behavior about the distribution of the statistics.

vi. On the fixed value of FAP, obtained the limits by taking the $(FAP/2)^{th}$ and $(1-(FAP/2))^{th}$ quantiles of the plotting statistics.

vii. For method A and B, set individual FAP ($\theta_1 = 0.000135$) to obtain desired overall FAP ($\theta = 0.0027$) while in method C, use $\theta_2 = 0.000045$ to obatain the limits. In method D fix $\theta_3 = 0.001351$ and $\theta_4 = 0.000068$ to obtain the control limits at fixed overall FAP $\theta = 0.0027$. The control limit(s) of the method A, B, C and D are given in Table 1 are computed by an extensive Monte Carlo simulation study with 10^6 iteration.

Table 1. Control limits of Phase I linear profile methods under consideration

Method	Limits	$\mathbb{R}$		
		RSS	**MRSS**	**ERSS**
A	UCL_A	11.92967	11.76558	12.02049
B	UCL_B	23.70728	18.29335	6.64172
C	LCL_{CI}	-1.42915	-1.34988	-1.58348
	UCL_{CI}	1.42915	1.34988	1.58348
	LCL_{CS}	-3.67964	-1.97462	-4.62615
	UCL_{CS}	3.67964	1.97462	4.62615
	LCL_{CF}	0.14528	0.10828	0.80014
	UCL_{CF}	0.30400	0.21445	1.36564
D	LCL_{DI}	-1.43404	-1.34712	-1.58433
	UCL_{DI}	1.43404	1.34712	1.58433
	LCL_{DS}	-3.68369	-1.97056	-4.62205
	UCL_{DS}	3.68369	1.97056	4.62205
	LCL_{DF}	2.01881	2.02858	2.05190

Table 1 presents the in-control performance of different profile methodologies under RSS strategies at fixed nominal false-alarm probabilities such as $\theta = 0.001, 0.003, 0.005, 0.010,\ 0.015, 0.020, 0.025, 0.030, 0.040$ and 0.050. The findings given in Table 2 are obtained by simulated study with 10^6 iterations, which provides the evident that the linear profile methodologies under different ranked set schemes have no difference between the nominal and simulated false-alarm probabilities.

Table 2. Overall False-alarm probabilities under ranked set strategies ($\mathbb{R}$)

θ	Method A			Method B			Method C			Method D		
	RSS	**MRSS**	**ERSS**	**RSS**	**MRSS**	**ERSS**	**RSS**	**MRSS**	**ERSS**	**RSS**	**MRSS**	**ERSS**
0.0010	0.0012	0.0011	0.0011	0.0010	0.0012	0.0009	0.0013	0.0009	0.0011	0.0013	0.0010	0.0014
0.0030	0.0035	0.0029	0.0029	0.0030	0.0030	0.0031	0.0028	0.0025	0.0030	0.0033	0.0031	0.0027
0.0050	0.0057	0.0048	0.0048	0.0051	0.0057	0.0048	0.0054	0.0051	0.0051	0.0054	0.0051	0.0053
0.0100	0.0106	0.0090	0.0095	0.0110	0.0102	0.0097	0.0106	0.0094	0.0099	0.0111	0.0106	0.0102
0.0150	0.0153	0.0143	0.0144	0.0156	0.0150	0.0162	0.0162	0.0136	0.0146	0.0156	0.0151	0.0151
0.0200	0.0211	0.0197	0.0180	0.0211	0.0199	0.0207	0.0202	0.0196	0.0198	0.0209	0.0200	0.0201
0.0250	0.0248	0.0252	0.0244	0.0252	0.0248	0.0253	0.0256	0.0253	0.0255	0.0258	0.0260	0.0254
0.0300	0.0302	0.0297	0.0275	0.0312	0.0294	0.0301	0.0310	0.0293	0.0299	0.0321	0.0307	0.0312
0.0400	0.0400	0.0396	0.0384	0.0410	0.0397	0.0406	0.0422	0.0394	0.0396	0.0428	0.0398	0.0421
0.0500	0.0491	0.0496	0.0488	0.0527	0.0505	0.0520	0.0520	0.0490	0.0396	0.0522	0.0462	0.0505

3.3. Performance evaluations

In order to measure the performance of Phase I methods under several sampling techniques, we have considered a different amount of shifts in the SLP parameters. The description of the shifts in SLP parameters are reported as follows:

i. Shifts in the intercept parameter $(\alpha^* \ to\ \alpha^* + \lambda(\sigma_{e[\mathbb{R}]}/\sqrt{n}))$,

ii. Shifts in the slope parameter $(\beta \ to\ \beta + \partial(\sigma_{e[\mathbb{R}]}/\sqrt{S_{xx}}))$,

iii. Shifts in the slope parameter $(\beta^* \ to\ \beta^* + \delta(\sigma_{e[\mathbb{R}]}/\sqrt{S_{xx}}))$,

iv. Shifts in the error variance $(\sigma^2_{e[\mathbb{R}]} \ to\ \gamma^2\sigma^2_{e[\mathbb{R}]})$,

Where the size of shifts for intercept, slope and slope under transformed model (6) is quantified as; λ, ∂ and $\delta = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5,$ and 5.0 while the size of shifts in the error variance are enumerated as; $\gamma^2 = 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8$ and 3.0. It is to be noted that the process is said to be IC when the shift in intercept, slope and slope under the transformed model is zero while the variability of the error term is one otherwise, the process is declared as OOC.

3.4. Comparative analysis

In this section, we provide the comparative analysis of Phase I methods under several sampling techniques (i.e., SRS, RSS, MRSS and ERSS). The probability to signal of each method under several samplings are provided in Tables 3-6, and selected cases are plotted in Figures 1-2.

Shifts in intercept parameter: The performance of modified Phase I methods under several amounts of shifts in intercept are given in Table 3.

- In general, method A under sampling schemes performs very poor in the presence of shifts in the intercept. The reason behind these findings is that when the mean is not consistent, then the covariance matrix is imperfectly assessed by the pooled sample variance-covariance matrix (cf. Mahmoud and Woodall (2004)).

- In method B, increase in the intercept parameter $(\lambda = 2.5)$ may cause 0.0466 units to increase in the PTS for the SRS scheme while 0.0125, 0.0501 and 0.2970 unit increase is reported in the PTS of the schemes RSS, MRSS and ERSS respectively.

- In the case of method C, increase in the intercept parameter $(\lambda = 3.5)$ causes 0.1541, 0.9907, 0.9925 and 0.9973 unit increase in the PTS for the SRS, RSS, MRSS and ERSS respectively.

- Further, increase in the intercept parameter $(\lambda = 4.5)$ causes 0.9973, 0.9972, 0.9972 and 0.9973 unit increase in the PTS of the method D with respect to SRS, RSS, MRSS and ERSS scheme.

Table 3. Probability to signal under intercept shifts from $\boldsymbol{\alpha}^*$ **to** $\boldsymbol{\alpha}^* + \boldsymbol{\lambda}\left(\boldsymbol{\sigma}_{\boldsymbol{e}[\mathbb{R}]}/\sqrt{\boldsymbol{n}}\right)$.

λ	Method A				Method B				Method C				Method D			
	SRS	**RSS**	**MRSS**	**ERSS**	**SRS**	**RSS**	**MRSS**	**ERSS**	**SRS**	**RSS**	**MRSS**	**ERSS**	**SRS**	**RSS**	**MRSS**	**ERSS**
0.50	0.0026	0.0029	0.0021	0.0025	0.0036	0.0030	0.0039	0.0038	0.0013	0.0094	0.0089	0.0283	0.0098	0.0093	0.0104	0.0267
1.00	0.0026	0.0025	0.0024	0.0016	0.0062	0.0037	0.0063	0.0106	0.0033	0.0436	0.0416	0.2423	0.0426	0.0393	0.0441	0.2387
1.50	0.0021	0.0021	0.0018	0.0014	0.0115	0.0052	0.0133	0.0361	0.0077	0.1516	0.1537	0.7872	0.1514	0.1425	0.1625	0.7821
2.00	0.0014	0.0019	0.0012	0.0010	0.0251	0.0084	0.0260	0.1144	0.0181	0.4062	0.4116	0.9940	0.4059	0.3845	0.4284	0.9943
2.50	0.0012	0.0016	0.0010	0.0011	0.0493	0.0152	0.0528	0.2997	0.0396	0.7362	0.7438	1.0000	0.7382	0.7139	0.7585	0.9999
3.00	0.0011	0.0014	0.0011	0.0010	0.0966	0.0291	0.1014	0.6111	0.0826	0.9401	0.9457	1.0000	0.9411	0.9313	0.9507	1.0000
3.50	0.0011	0.0013	0.0001	0.0012	0.1719	0.0556	0.1843	0.8909	0.1568	0.9934	0.9952	1.0000	0.9942	0.9932	0.9957	1.0000
4.00	0.0011	0.0013	0.0011	0.0011	0.2968	0.1000	0.3158	0.9897	0.2813	0.9996	0.9997	1.0000	0.9998	0.9995	0.9999	1.0000
4.50	0.0012	0.0013	0.0010	0.0010	0.4604	0.1786	0.4866	0.9998	0.4506	0.9999	1.0000	1.0000	1.0000	0.9999	0.9999	1.0000
5.00	0.0012	0.0010	0.0009	0.0011	0.6491	0.2968	0.6803	1.0000	0.6464	0.9891	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

- On the other hand, in SRS, the findings depict that increase in intercept $(\lambda = 1)$ may causes 0.0035, 0.0006 and 0.0399 unit increase in the PTS of the method B, C and D respectively. However, in RSS, increase in intercept parameter $(\lambda = 2)$ causes 0.0057, 0.4035 and 0.3818 unit increase in the PTS with respect to method B, C and D.
- In the case of MRSS, increase in the intercept $(\lambda = 3)$ may causes 0.0987, 0.9430 and 0.9480 unit increase in the PTS for the method B, C and D respectively. Moreover, in ERSS, increase in the intercept parameter $(\lambda = 4)$ causes 0.9870, 0.9973 and 0.9973 unit increase in the PTS with respect to method B, C and D.
- Overall, it is to be noted that in SRS, method B and D have better performance while in RSS strategies, method C and D have superior performance as compared to other methods under the shifts in intercept parameter. Moreover, from sampling strategies ERSS offering attractive performance as compared to other schemes (cf. Figures 1-2).

Shifts in the slope (original model) parameter: Table 4 presents the performance of the modified Phase I methods under several amounts of shift in the slope parameter.

- In the presence of shifts in the slope of the original model (2), method A has lower performance while in method B an increase in slope $(\partial = 2)$ causes 0.2202, 0.0838, 0.2357 and 0.9520 units increase in the PTS for SRS, RSS, MRSS and ERSS respectively.
- In case of method C, increase in slope parameter $(\partial = 3)$ may cause 0.0802 unit increase in the PTS for SRS while 0.6186, 0.9366 and 9973 unit increase is reported with respect to RSS, MRSS and ERSS schemes. Further, the increase in slope $(\partial = 4)$ causes 0.9970, 0.9581, 0.9970 and 0.9973 unit increase in the PTS of method D with respect to SRS, RSS, MRSS and ERSS schemes.
- In SRS, results reveal that an increase in slope $(\partial = 1.5)$ may causes 0.0685, 0.0047 and 0.1494 unit increase in the PTS of method B, C and D respectively. However, in the case of RSS, an increase in slope parameter $(\partial = 2.5)$ causes 0.2374, 0.3584 and 0.3485 unit increase in the PTS with respect to method B, C and D.
- In MRSS, increase in slope $(\partial = 3.5)$ may causes 0.9850, 0.9916 and 0.9918 unit increase in the PTS for the method B, C and D respectively. Further, in ERSS, an increase in the slope parameter $(\partial = 4.5)$ causes 0.9973 unit increase in the PTS of Modified Phase I methods (i.e. B, C and D).

- Some particular cases of shifts in the slope of the original model are also portrayed in Figures 1 and 2. To sum up, it is noted that modified Phase I methods under ERSS offers an attractive performance as compared to other schemes while in SRS, method B and D provides superior performance and in all RSS strategies methods, C and D have excellent performance as compared to other methods.

Table 4. Probability to signal under slope (original model) shifts from $\boldsymbol{\beta}$ *to* $\boldsymbol{\beta} + \boldsymbol{\partial}(\boldsymbol{\sigma}_{\boldsymbol{e}[\mathbb{R}]}/\sqrt{\boldsymbol{S}_{\boldsymbol{xx}}})$.

∂	Method A				Method B				Method C				Method D			
	SRS	**RSS**	**MRSS**	**ERSS**	**SRS**	**RSS**	**MRSS**	**ERSS**	**SRS**	**RSS**	**MRSS**	**ERSS**	**SRS**	**RSS**	**MRSS**	**ERSS**
0.50	0.0026	0.0028	0.0026	0.0025	0.0053	0.0039	0.0055	0.0095	0.0012	0.0062	0.0092	0.0186	0.0094	0.0033	0.0089	0.0188
1.00	0.0015	0.0032	0.0024	0.0028	0.0201	0.0096	0.0215	0.0836	0.0032	0.0215	0.0409	0.1494	0.0416	0.0067	0.0427	0.1547
1.50	0.0013	0.0030	0.0027	0.0028	0.0712	0.0292	0.0768	0.4674	0.0074	0.0633	0.1464	0.6005	0.1521	0.0613	0.1538	0.6079
2.00	0.0012	0.0026	0.0023	0.0031	0.2229	0.0865	0.2384	0.9547	0.0180	0.1656	0.3986	0.9645	0.4045	0.1609	0.4068	0.9665
2.50	0.0011	0.0030	0.0025	0.0025	0.5181	0.2401	0.5431	0.9999	0.0395	0.3611	0.7301	0.9997	0.7326	0.3512	0.7402	0.9998
3.00	0.0011	0.0031	0.0023	0.0030	0.8402	0.5265	0.8622	1.0000	0.0829	0.6213	0.9393	1.0000	0.9388	0.6174	0.9446	1.0000
3.50	0.0010	0.0029	0.0021	0.0031	0.9838	0.8391	0.9877	1.0000	0.1581	0.8497	0.9943	1.0000	0.9940	0.8428	0.9945	1.0000
4.00	0.0009	0.0028	0.0025	0.0027	0.9997	0.9806	0.9998	1.0000	0.2813	0.9624	0.9997	1.0000	0.9997	0.9608	0.9997	1.0000
4.50	0.0009	0.0033	0.0026	0.0028	1.0000	0.9995	1.0000	1.0000	0.4498	0.9949	1.0000	1.0000	1.0000	0.9941	1.0000	1.0000
5.00	0.0011	0.0030	0.0026	0.0024	1.0000	1.0000	1.0000	1.0000	0.6473	0.9994	1.0000	1.0000	1.0000	0.9994	1.0000	1.0000

Shifts in slope (transformed model) parameter: The performance of modified Phase I methods under several amounts of shifts in the slope of the model (6) are reported in Table 5.

- In this case, again method A has imperfect performance while in method B, increase in the slope ($\delta = 2.5$) may cause 0.0459 units to increase in the PTS for SRS while 0.0365, 0.0653 and 0.3544 unit increase is reported with respect to RSS, MRSS and ERSS schemes.
- In the case of method C, an increase in the slope parameter ($\delta = 3.5$) causes 0.1565, 0.8458, 0.9910 and 0.9973 unit increase in the PTS of SRS, RSS, MRSS and ERSS schemes respectively. Further, an increase in the slope parameter ($\delta = 4.5$) causes 0.9973, 0.9915, 0.9973 and 0.9973 unit increase in the PTS of the method D with respect to SRS, RSS, MRSS and ERSS schemes.
- On the other hand, in SRS, the findings depict that increase in slope ($\delta = 1$) may causes 0.0035, 0.0000 and 0.0386 unit increase in the PTS of method B, C and D respectively. In the case of RSS, an increase in slope parameter ($\delta = 2$) causes 0.0208, 0.1615 and 0.1602 unit increase in the PTS with respect to method B, C and D.
- Whereas, in MRSS, an increase in the slope ($\delta = 3$) may causes 0.1252, 0.9376 and 0.9418 unit increase in the PTS for the method B, C and D respectively. Moreover, in ERSS, increase in slope parameter ($\delta = 4$) causes 0.9874, 0.9973 and 0.9973 unit increase in the PTS with respect to method B, C and D.
- To conclude, it is observed that in the SRS scheme, method B and D have better performance while in RSS strategies, method C and D have superior performance as compared to other methods. Moreover, modified Phase I methods under ERSS have attractive performance as compared to other schemes (cf. Figures 1-2).

Table 5. Probability to signal under slope (transformed model) shifts from $\boldsymbol{\beta}^* \; \boldsymbol{to} \; \boldsymbol{\beta}^* + \boldsymbol{\delta}\left(\boldsymbol{\sigma}_{\boldsymbol{e}[\mathbb{R}]}/\sqrt{\boldsymbol{S_{xx}}}\right)$.

δ^*	Method A				Method B				Method C				Method D			
	SRS	**RSS**	**MRSS**	**ERSS**	**SRS**	**RSS**	**MRSS**	**ERSS**	**SRS**	**RSS**	**MRSS**	**ERSS**	**SRS**	**RSS**	**MRSS**	**ERSS**
0.50	0.0030	0.0024	0.0028	0.0023	0.0036	0.0098	0.0047	0.0054	0.0010	0.0073	0.0089	0.0188	0.0093	0.0072	0.0100	0.0206
1.00	0.0023	0.0021	0.0026	0.0017	0.0062	0.0117	0.0085	0.0175	0.0027	0.0227	0.0398	0.1510	0.0413	0.0216	0.0423	0.1557
1.50	0.0022	0.0016	0.0020	0.0010	0.0117	0.0158	0.0171	0.0528	0.0075	0.0647	0.1472	0.5970	0.1497	0.0602	0.1509	0.6093
2.00	0.0016	0.0012	0.0017	0.0009	0.0243	0.0235	0.0341	0.1495	0.0188	0.1642	0.3948	0.9638	0.4021	0.1629	0.4077	0.9679
2.50	0.0011	0.0009	0.0013	0.0008	0.0486	0.0392	0.0680	0.3571	0.0400	0.3623	0.7283	0.9998	0.7306	0.3536	0.7409	0.9998
3.00	0.0010	0.0009	0.0013	0.0009	0.0943	0.0664	0.1279	0.6600	0.0812	0.6265	0.9403	1.0000	0.9388	0.6171	0.9445	1.0000
3.50	0.0010	0.0008	0.0013	0.0009	0.1729	0.1085	0.2197	0.9049	0.1592	0.8485	0.9937	1.0000	0.9942	0.8448	0.9949	1.0000
4.00	0.0012	0.0006	0.0013	0.0009	0.2981	0.1777	0.3543	0.9901	0.2800	0.9638	0.9997	1.0000	0.9997	0.9601	0.9998	1.0000
4.50	0.0010	0.0008	0.0012	0.0007	0.4637	0.2787	0.5239	0.9998	0.4521	0.9949	1.0000	1.0000	1.0000	0.9942	1.0000	1.0000
5.00	0.0010	0.0007	0.0012	0.0008	0.6509	0.4094	0.7033	1.0000	0.6456	0.9995	1.0000	1.0000	1.0000	0.9994	1.0000	1.0000

Shifts in the variance of error term: Table 6 presents the performance of modified Phase I methods under several amounts of shift in the variability of error variance.

- In method A, an increase in the error variance ($\gamma^2 = 1.4$) may cause 0.0126 unit increase in the PTS for SRS scheme while 0.0218, 0.0116, 0.0122 unit increase in the PTS is reported with respect to RSS, MRSS and ERSS schemes.
- Further, an increase in variability of disturbance term ($\gamma^2 = 1.8$) causes 0.0067, 0.0081, 0.0077 and 0.0055 unit increase in the PTS for method B under SRS, RSS, MRSS and ERSS schemes respectively.
- In the case of method C, an increase in the error variance ($\gamma^2 = 2.2$) may cause 0.1526 unit increase in the PTS for SRS while 0.9972, 0.9973 and 0.9973 unit increase is reported with respect to RSS, MRSS and ERSS schemes.
- Moreover, an increase in variability of disturbance term ($\gamma^2 = 2.4$) causes 0.2226, 0.0281, 0.0082 and 0.0007 unit increase in the PTS of the method D with respect to SRS, RSS, MRSS and ERSS schemes.
- In SRS, results depict that increase in the error variance ($\gamma^2 = 1.6$) may causes 0.0300, 0.0034, 0.0403 and 0.0258 unit increase in the PTS of method A, B, C and D respectively. However, in the case of RSS, an increase in variability of disturbance term ($\gamma^2 = 2$) causes 0.1245, 0.0122, 0.9972 and 0.0142 unit increase in the PTS with respect to method A, B, C and D.
- Under the MRSS scheme, an increase in the error variance ($\gamma^2 = 2.4$) may causes 0.1289, 0.0195, 0.9973 and 0.0061 unit increase in the PTS for the method A, B, C and D respectively. Further, in ERSS, an increase in variability of disturbance term ($\gamma^2 = 2.8$) causes 0.9973, 0.0206, 0.9973 and 0.0010 unit increase in the PTS of Modified Phase I methods (i.e. A, B, C and D).
- Some selected cases of shifts in the disturbance term are also portrayed in Figures 1 and 2. To sum up, it is noted that in the SRS scheme, method C and D offers superior performance and in all RSS strategies, methods A and C have superior performance as compared to other methods. Moreover, modified Phase I methods (A and C) under ERSS offers an attractive performance as compared to other schemes while in method B, RSS performs better and in method D, RSS schemes have relatively lower performance.

Table 6. Probability to signal under error variance shifts from $\sigma^2_{e[\mathbb{R}]}$ ***to*** $\gamma^2\sigma^2_{e[\mathbb{R}]}$.

γ^2	Method A				Method B				Method C				Method D			
	SRS	RSS	MRSS	ERSS	SRS	RSS	MRSS	ERSS	SRS	RSS	MRSS	ERSS	SRS	RSS	MRSS	ERSS
1.20	0.0052	0.0074	0.0044	0.0051	0.0026	0.0027	0.0030	0.0026	0.0068	0.1001	0.1289	0.2655	0.0055	0.0039	0.0029	0.0027
1.40	0.0153	0.0245	0.0143	0.0149	0.0044	0.0045	0.0045	0.0036	0.0197	0.5980	0.7108	0.9265	0.0135	0.0055	0.0030	0.0028
1.60	0.0327	0.0533	0.0328	0.0324	0.0061	0.0069	0.0067	0.0055	0.0430	0.9450	0.9758	0.9994	0.0285	0.0083	0.0037	0.0030
1.80	0.0581	0.0889	0.0581	0.0548	0.0094	0.0108	0.0104	0.0082	0.0733	0.9969	0.9992	1.0000	0.0499	0.0127	0.0044	0.0030
2.00	0.0820	0.1272	0.0839	0.0815	0.0131	0.0149	0.0133	0.0102	0.1097	0.9999	1.0000	1.0000	0.0746	0.0169	0.0054	0.0031
2.20	0.1085	0.1642	0.1090	0.4933	0.0171	0.0191	0.0180	0.0139	0.1553	0.9999	1.0000	1.0000	0.1063	0.0222	0.0073	0.0031
2.40	0.1346	0.1968	0.1316	0.9999	0.0214	0.0233	0.0222	0.0166	0.2026	1.0000	1.0000	1.0000	0.1429	0.0272	0.0088	0.0032
2.60	0.1588	0.2286	0.1575	1.0000	0.0263	0.0285	0.0259	0.0210	0.2811	1.0000	1.0000	1.0000	0.2253	0.0308	0.0109	0.0034
2.80	0.1803	0.2570	0.1746	1.0000	0.0314	0.0337	0.0312	0.0233	0.4519	1.0000	1.0000	1.0000	0.4364	0.0358	0.0130	0.0037
3.00	0.2008	0.2815	0.1969	1.0000	0.0359	0.0392	0.0365	0.0272	0.6924	1.0000	1.0000	1.0000	0.7101	0.0405	0.0157	0.0141

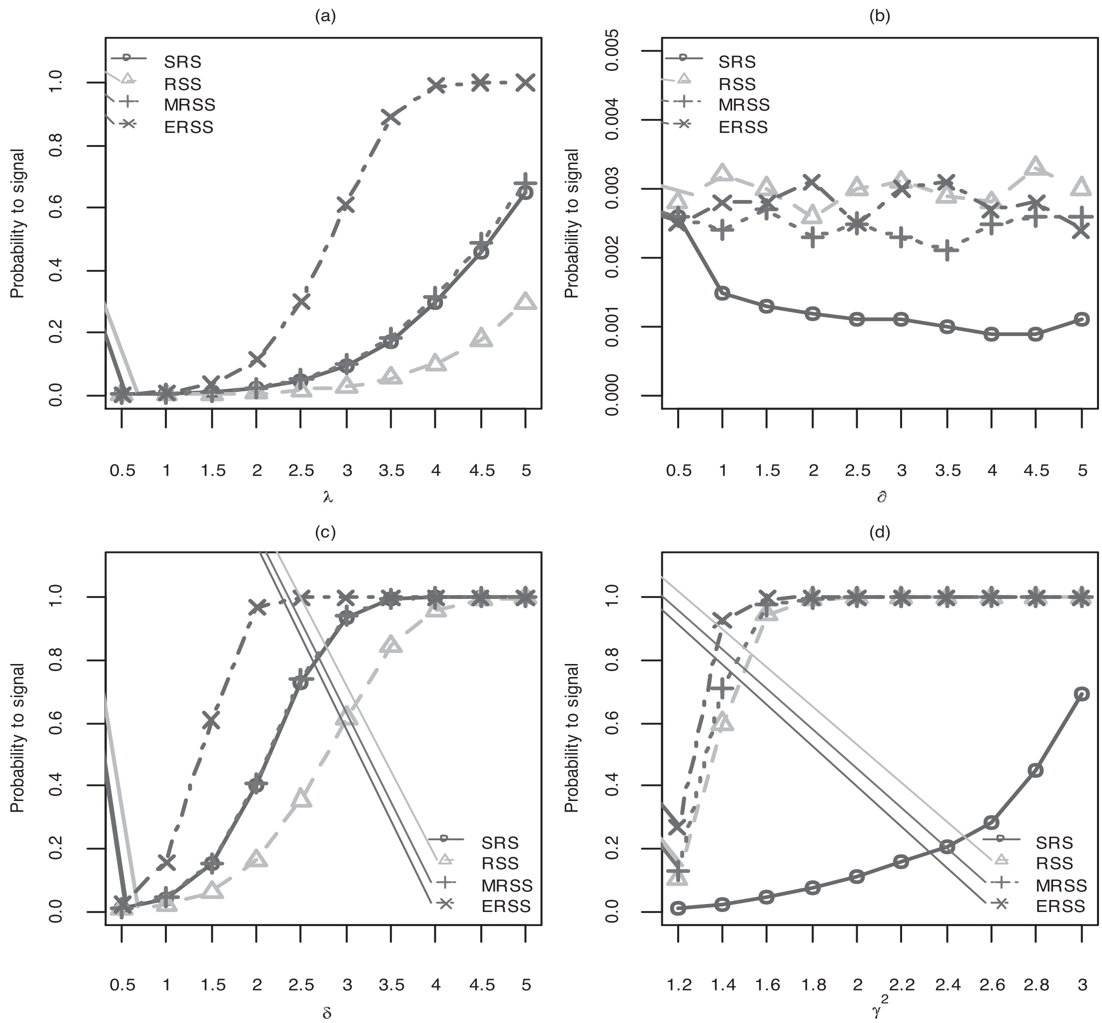

Figure 1. Comparison of sampling strategies in (a) method B under the shifts in intercept parameter; (b) method A under the variations in the slope of the original model; (c) method D under the changes in the slope of the transformed model; (c) method C under the shifts in error variance.

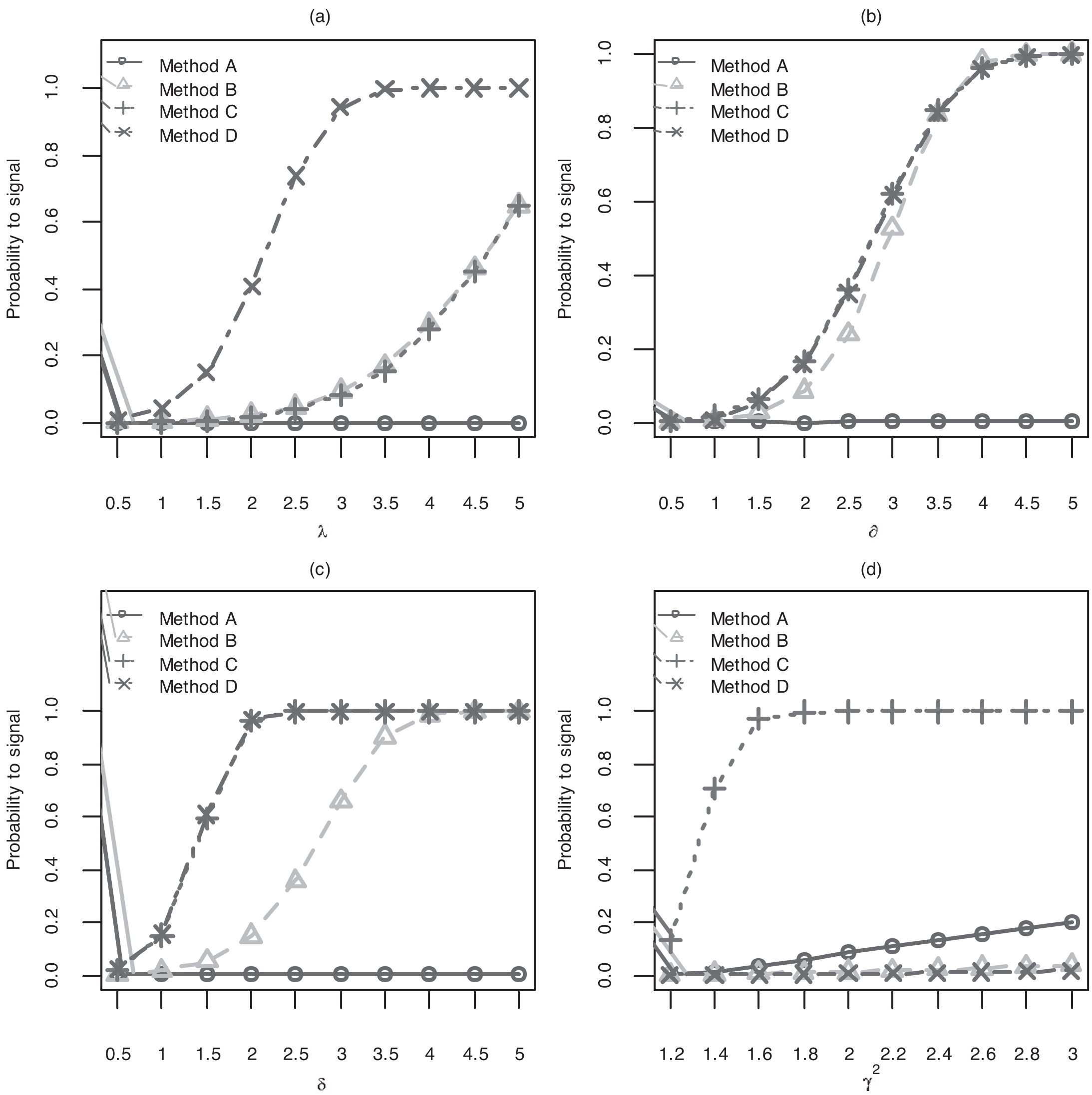

Figure 2. Comparison of Phase I methods under (a) SRS for the shifts in intercept parameter; (b) RSS for the changes in the slope of original model; (c) ERSS for the variations in the slope of the transformed model; (c) MRSS for the shifts in error variance.

4. A REAL APPLICATION: ELECTRICAL ENGINEERING

In the following section, an illustrative example is presented to elaborate the performance of linear profile methodologies with the application of ranked set sampling and its modified schemes.

4.1. Description of a grid-connected photovoltaic system

Nowadays, solar energy is used as an efficient source of electricity for households as well as for small industrial projects. The illustration of grid-connected photovoltaic (PV) system is portrayed in Figure 3 which consist of solar PV panels that are connected with DC/AC inverter through a switch. The solar panels absorb the sun rays and generate the DC voltage which is further converted into AC voltage by using different inverters. There exist several inverters such as VSI (voltage source inverter; buck inverter), CSI (current source inverter; boost inverter) and ZSI

(Z-source inverter; buck-boost inverter). The buck inverter provides voltage to a smaller extent and boosts inverter provides better voltage in contrast to input DC voltage. Recently, an emerging inverter ZSI is used to improve the overall system efficiency and eliminate the problems of CSI and VSI. For more details see Mukhtar (2015, Riaz et al. (2017).

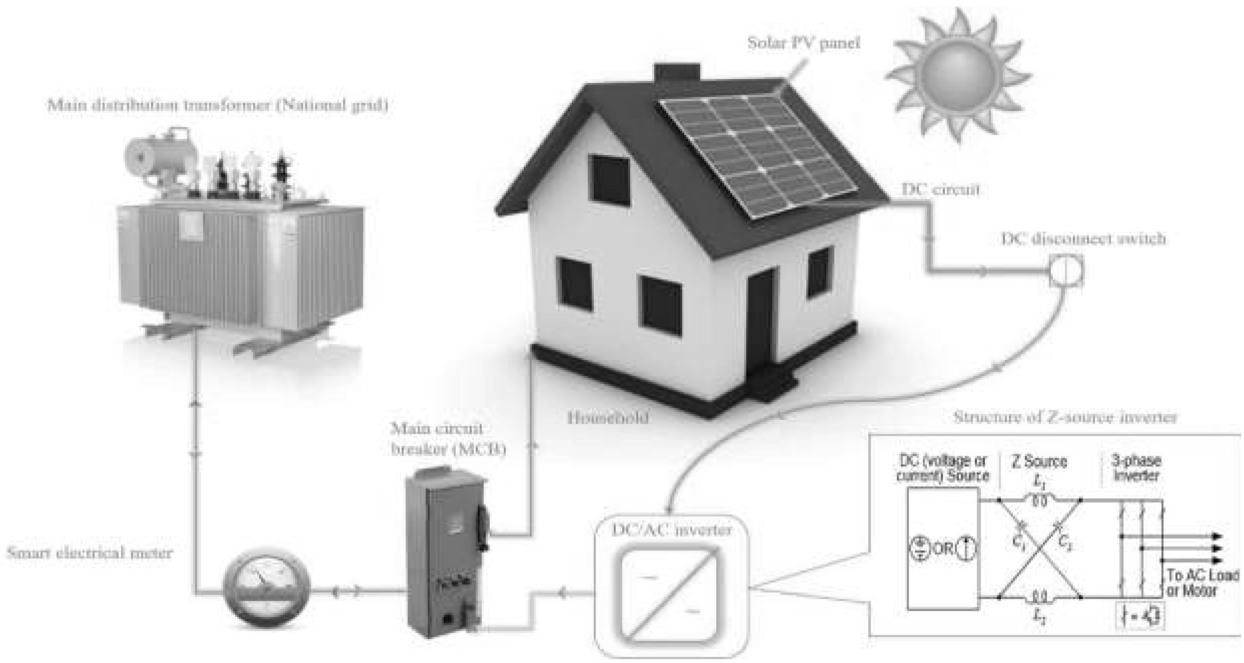

Figure 3. Structure of grid-connected PV panel system

In PV mechanism, capacitance (C) has an inverse relation with voltage (V) on fixed charge (Q) ($+q$ and $-q$) such that $C \propto Q/V$. The DC current is rehabilitated into AC current through ZSI which is further controlled by the main circuit breaker. The circuit breaker plays a bridge between the solar panel voltage and grid voltage. In the day time, when solar panels are in working position, the circuit breaker provides the voltage (generated by solar panels) to households; otherwise, the grid voltage is delivered for the household usage. Moreover, a bi-directional meter is used to exchange the rate of electricity between solar supply and power grid supply.

4.2. Execution of Phase I methods

Generally, electrical engineers want to detect any severe variation in the voltage of the PV system. As mentioned above that capacitance (C) has an inverse relationship with voltage (V) at a fixed charge (Q). we have used V values against 7 levels of C ($C = 50\mu F - 350\mu F$) reported in Mukhtar (2015). In stated example, we considered the explanatory variable (V) and explanatory variable (C). It is noted that only method C and D are efficient for the detection of shifts in SLP parameters (see section 3.3). So, for the brevity, we have discussed method C and D with execution of RSS, MRSS and ERSS schemes in the real data set. Further, the following steps are made to execute the SLP methods for given example;

Step 1: For a single linear profile, we have considered two replications of V against each C. Further, we applied RSS, MRSS and ERSS on the values of V against C and get 1344 RSS observations of V against each level of C.

Step 2: For IC regression models, we run 1344 profiles each of sample size ($n = 14$) against C and get the following models

$$\text{for RSS,}\quad \hat{V} = 399.006229 - 0.007326\ C\ \ (\hat{\sigma} = 84.56122)$$
$$\text{for MRSS,}\quad \hat{V} = 399.586079 - 0.009238\ C\ (\hat{\sigma} = 84.52078)$$
$$\text{for ERSS,}\quad \hat{V} = 399.006329 - 0.007336\ C\ (\hat{\sigma} = 84.56322)$$

Step 3: We have fixed overall FAP $(\theta = 0.05)$ with $k = 50$ to obtain the control limits of the linear profile methods under ranked set schemes. For method C, individual FAP $(\theta_1 = 0.000342)$ is used to obtain desired overall FAP $(\theta = 0.05)$ while in method D, $\theta_3 = 0.025321$ and $\theta_4 = 0.000531$ are used to obtain $\theta = 0.05$. The control limits of method C and D are given in Table 7 which are computed by an extensive Monte Carlo simulation study with 1e[5] iteration.

Step 4: Once, we get 1344 IC profiles and control limits, we used only 50 IC profiles (pink shaded) in Figures (4-9) for each method under RSS, MRSS and ERSS. Further, following perturbation mechanisms are used to produce shifted data

i. For detection of shifts in intercept, we used

$C = -650, -650, -600, -600, -550, -550,$

$-500, -500, -450, -450, -400, -400, -350$

and -350 against 1344 sets of V and only 50 profiles with index 51 to 100 for each method under RSS, MRSS and ERSS are portrayed in Figures (4-9).

ii. For detection of shifts in slope parameter, we used $C = -25, -25, -50, -50, -75, -75, -100, -100, -125, -125, -150, -150, -175$ and -175 against 1344 sets of V and only 50 profiles with index 101 to 150 for each method under RSS, MRSS and ERSS are plotted in Figures (4-9).

iii. For the detection of shifts in error variance, we multiply 1344 sets of V with $\sqrt{2.5}$ only 50 profiles with index 151 to 200 for each method under RSS, MRSS and ERSS are represented in Figures (4-9).

For methods C and D under RSS, MRSS and ERSS scheme, the number of OOC profiles with their index are reported in Table 7. In the presence of shifts in intercept and slope, the findings reveal that method D under ERSS scheme has better detection ability relative to method C. Further, for the shifts in error variance, method D has inferior performance in the detection of OOC profiles.

Table 7: The summary of a case study (number of OOC points (OOC index)).

ℝ	Method	Shifted	Control limits		Parameters		
			LCL	UCL	Intercept	Slope	Error variance
RSS	C	Intercept	787.4137	805.3415	7 (65,66,68,78,86,87,92)	0	0
		Slope	-0.14253	0.110963	0	15 (103,104,111,114-116,123,124 ,126,128,131,134,135,140,145)	0
		Error variance	0.009528	4.129614	50 (151-200)	4 (153,155,165,184)	6 (152,164,169, 183,191,199)
	D	Intercept	368.9844	427.8868	20 (51,52,57,58,60,63,67,69, 72,75,76,78,81,83,85,90,9 3,94,96,98)	0	0
		Slope	-0.11214	0.107097	0	26 (101,105,106,108-110,112,116,118, 120-122,125-130,132,134,138,144-147,149)	0
		Error variance	1.570277	33383.48	50 (151-200)	16 (152,153,155,164,168,179-181, 184,186,188,189,192,195,197,199)	0
MRSS	C	Intercept	787.6332	804.4683	8 (51,66,69,83,85,87,88,91)	0	0
		Slope	-0.13014	0.107112	0	21 (107,110,111,113,114,117,118,12 0,126, 129,130,132,133,139,141-145,147,149)	0
		Error variance	0.008838	4.105156	50 (151-200)	9 (165,169,176,177,184,185,187,19 0,190)	10 (152,155,161,162, 171,185,192 ,194,195,198)
	D	Intercept	368.9126	427.8711	22 (53,54,57,64,68,72,73,75, 78,80-83,87,88,91-95,98,99)	0	0
		Slope	-0.11223	0.105214	0	22 (106,107,111,112,113,115,116,117, 122-124,126-130,134,135,140,145,148,150)	0
		Error variance	1.294069	38556.17	50 (151-200)	22 (153,154,159,160,162,163,168-173, 175,177-179,189,194,196-198,200)	0
ERSS	C	**Intercept**	786.4668	805.208	**9** (57,61,64,75-77,79,89,93)	**0**	**0**
		Slope	-0.14205	0.121621	**0**	14 (101,108,110-112,125,131, 134,139,142,144,145,147,148)	**0**
		Error variance	0.005641	3.988217	**50** (151-200)	**9** (153,160,161,164,167, 170,189,192,196)	**12** (164,167,169,173, 178,187-192,196)
	D	**Intercept**	368.9844	427.8868	**26** (54,56-59,61,62,64-69,72,75,76,78,82-84,88,90,94,97,98,100)	**0**	**0**
		Slope	-0.11214	0.107097	**0**	**32** (101,102,104,105,110,112,114,118, 119,122,123,125-129,131-143,145,146,148)	**0**
		Error variance	1.570277	33383.48	**50** (151-200)	**16** (151,153-155,157,164,166,172, 173,177,179,184,188,191,197,199)	**0**

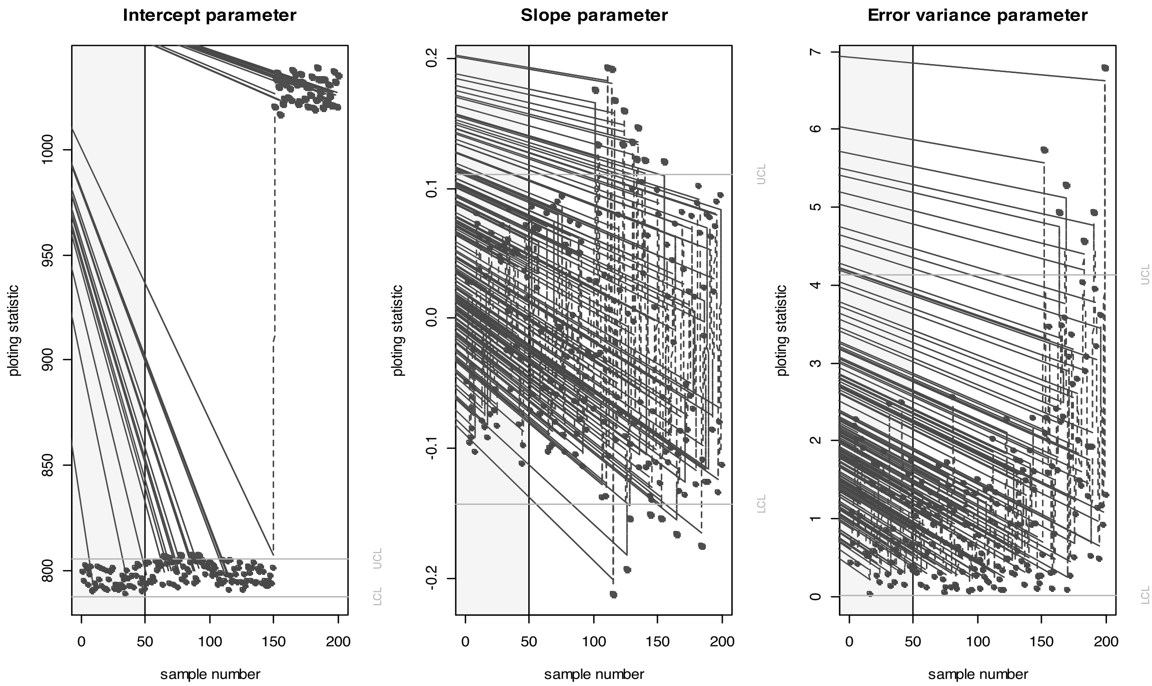

Figure 4: The Diagnosis ability of method C in the presence of shifted profile parameters under the RSS scheme

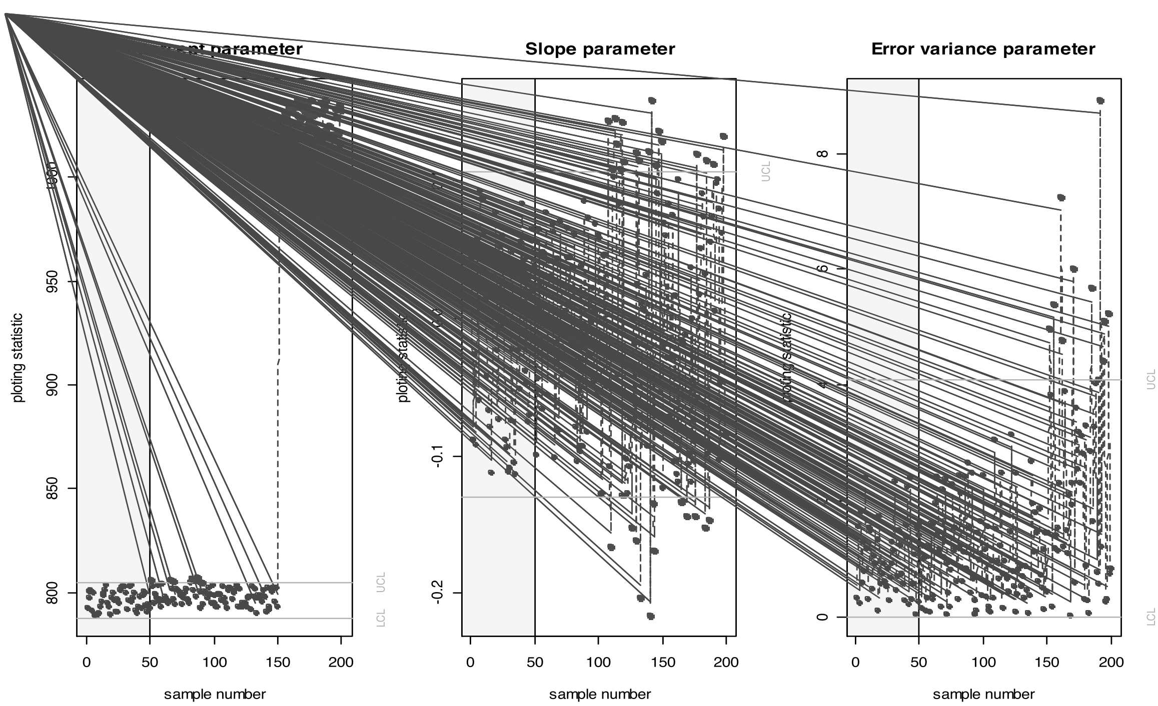

Figure 5: The Diagnosis ability of method C in the presence of shifted profile parameters under MRSS scheme

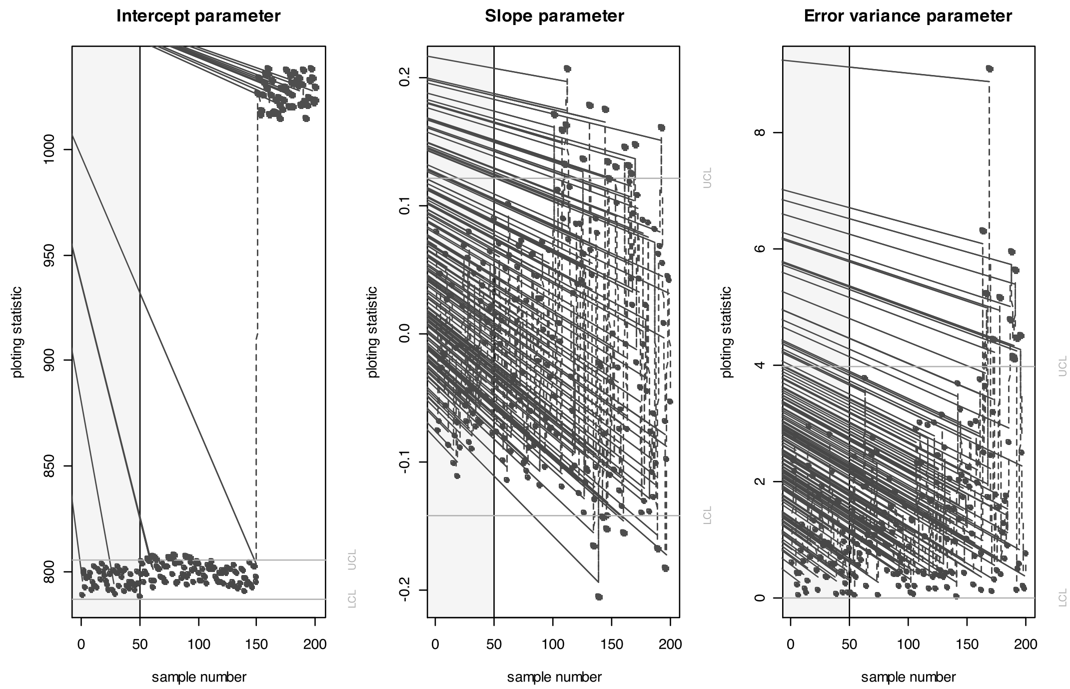

Figure 6: The Diagnosis ability of method C in the presence of shifted profile parameters under ERSS scheme

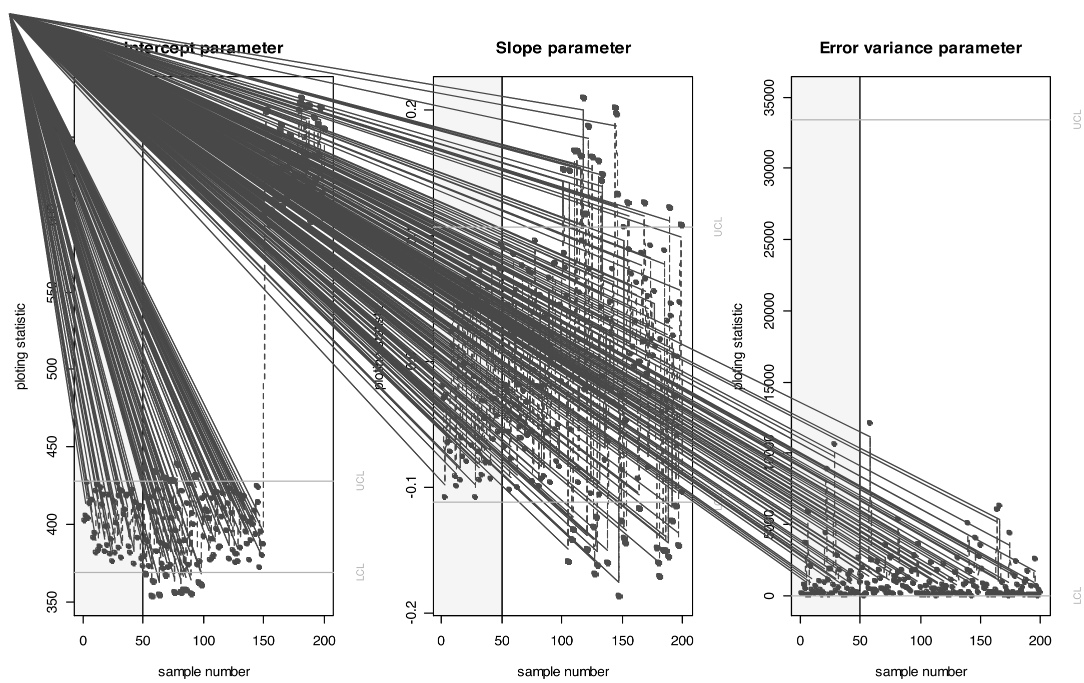

Figure 7: The Diagnosis ability of method D in the presence of shifted profile parameters under the RSS scheme

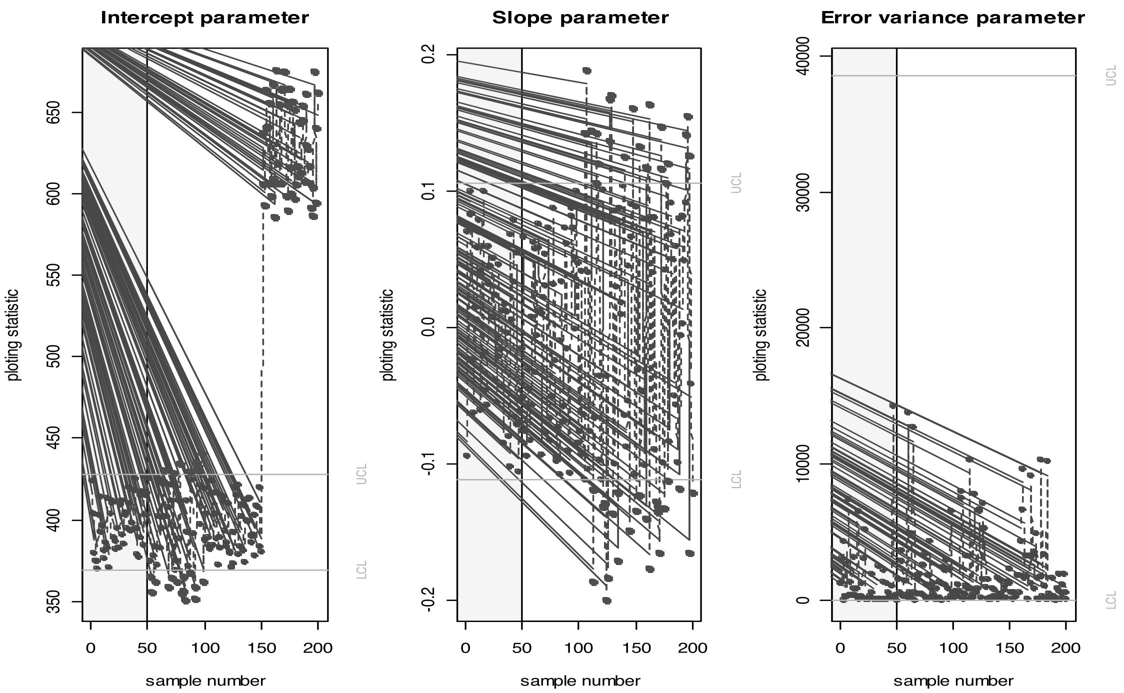

Figure 8: The Diagnosis ability of method D in the presence of shifted profile parameters under MRSS scheme

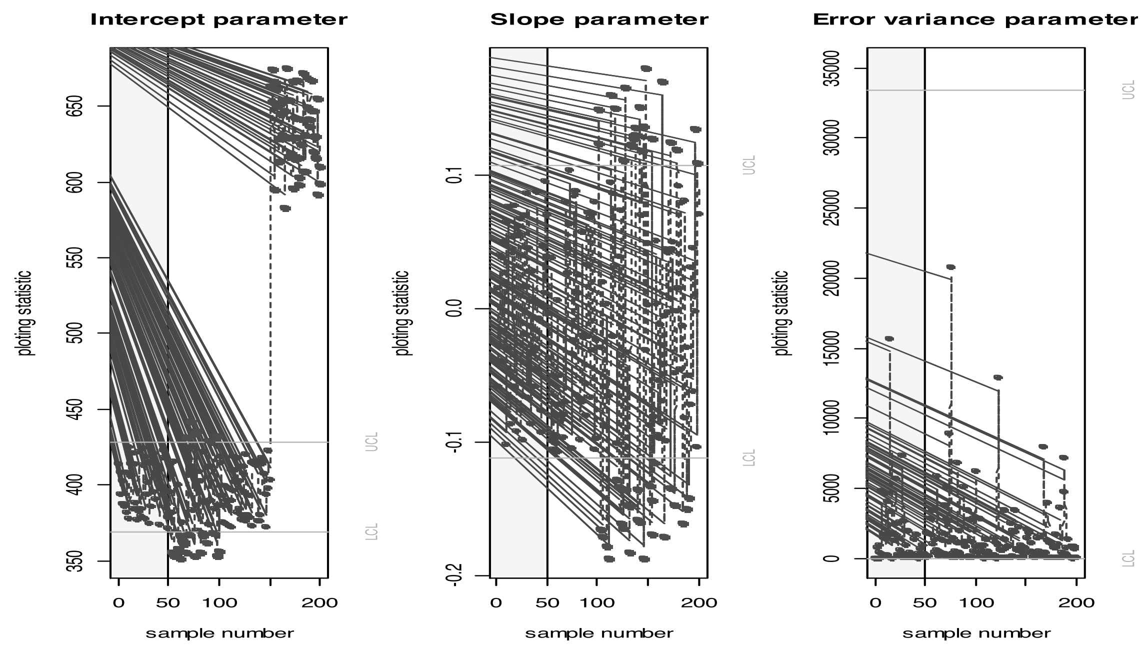

Figure 9: The Diagnosis ability of method D in the presence of shifted profile parameters under ERSS scheme

In precise, the implementation of ranked set schemes in phase I methods enhanced their performance for the detection of OOC linear profile parameters. In the ranked set environment, the methods C and D appeared as efficient methods to detect the variations in the voltage which is linearly associated with capacitance.

5. SUMMARY AND CONCLUDING REMARKS

Ranked set sampling is an efficient selection mechanism that covers a variety of sampling schemes such as RSS and its modified forms namely MRSS and ERSS. Nowadays, profile monitoring is an emerging field that is used to monitor the study variable(s), linearly associated with other explanatory variable(s) of the process. We have proposed different SLP methodologies under RSS schemes (RSS, MRSS and ERSS) and derived their design structures under normally distributed setups. We have observed that the proposed methods offer superior detection abilities for SLP parameters for shifts in intercept, slope (original model) and slope of the transformed model. We have noticed that under RSS schemes, method C and D have better performance as compared to other methods. Moreover, modified Phase I methods under ERSS have relatively attractive performance as compared to the other schemes. For shifts in error variance, the analysis shows that methods A and C are dominant over the other methods under RSS strategies, while modified Phase I methods (A and C) under ERSS offer relatively more appealing performance. Furthermore, under RSS method B performs quite well, while method D falls at the end in the performance order. The real application in electrical engineering supports the practical aspects of our study. The current research focuses on linear profiles; however, the scope of ongoing research work may be extended to cover the monitoring of multiple linear or non-linear profiles.

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