Satisfying product demand with a quality-assured hybrid EMQ-based replenishment system

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ABSTRACT

With the intention of minimizing operating cost and/or leveling limited production resources, managers of manufacturing firms usually fully assess diverse fabrication alternatives for meeting demand in their decision-making processes. This study presents a quality-assured hybrid economic manufacturing quantity- (EMQ-) based replenishment system considering an outsourcing option, rework, and scrap. The objectives are to determine optimal lot size for the proposed system and learn in-depth characteristics of key system parameters. A precise model is carefully built to portray the proposed system. Through mathematical analysis and optimization, a closed-form optimal lot size is obtained. Using a numerical example, we further confirm the applicability of the research result and demonstrate that crucial joint effects of various system variables on the optimal lot size and on total system cost are obtainable.

Keywords: Hybrid replenishment system; economic manufacturing quantity; outsource; rework; scrap.

INTRODUCTION

Different fabrication alternatives are usually evaluated by managers of manufacturing firms when seeking to minimize operating cost and/or to level limited production resources. This study explores a quality-assured hybrid economic manufacturing quantity- (EMQ-) based replenishment system with an outsourcing option, rework, and scrap. The classic EMQ model was first introduced by Taft (1918) with simple assumptions including the perfection of the in-house manufacturing process. However, in real manufacturing environments, various inevitable factors (either controllable or uncontrollable features) may cause production equipment failures (Bielecki and Kumar, 1988; Moinzadeh and Aggarwal, 1997; Giri and Dohi, 2005; Nourelfath, 2011), or process deterioration with a certain amount of defective items being produced (Yum and McDowell, 1987; Hariga and Ben-Daya, 1998; Chelbi and Rezg, 2006; Jaber et al., 2013; Wu et al., 2014; Cao et al., 2015; Al-Refaie et al., 2016; Chiu et al., 2016a,b; Romero-Jabalquinto et al., 2016; Nazir et al., 2017).

Some defective products may be repairable, so overall fabrication inventory relevant costs can be reduced accordingly. Zargar (1995) examined two reworking policies and their effects on the cycle length. The difference between his proposed policies was on the timing of repairing defective items. Queuing theory and simulation were employed to investigate effects of the policies on cycle length. Inderfurth et al. (2007) considered a single-machine fabrication system with two separate stages. Stage 1 is the regular fabrication and stage 2 repairs reworkable defective products. The rate of deterioration of defective items was also assumed. Polynomial time algorithms were proposed to derive the optimal lot size that minimizes overall fabrication relevant cost. Studies relating to fabrication systems with different aspects of rework and quality assurance issues can also be found in (Flapper and Teunter, 2004; Taleizadeh et al., 2010; Abilash and Sivapragash, 2016; Jawla and Singh, 2016; Zhang et al., 2016; Chiu et al., 2017; Khanna et al., 2017).

Capacity constraint is another key issue for production managers. Alternatives for smoothing of fabrication schedules or adoption of outsourcing policy are constantly evaluated by managers to streamline the operations and/or

to reduce cycle length and system cost. Amaral et al. (2006) offered a few policies to original equipment manufacturers on determining types of business activities to outsource and on how to effectively monitor and control the outsourcing procedure based on risk minimization. Paul and Yasar (2009) studied the effects of input composition and productivity of subcontracting on Turkish textile firms. Various performance indicators from separately employing domestic and foreign outsourcers were investigated and compared. Factors of labor skills and productivity were also taken into consideration to draw a few conclusions from their findings. Proff (2011) explored the conflicts of the competitive strategy used by automobile firms (particularly, those pursuing a differentiation strategy) to outsource specific parts to their suppliers. Specifically, the conflicts from aiming at short-term or medium-term cost-saving give up competencies. Based on core competency theory and transaction cost theory, he proposed possible tactical actions and performed content analysis to demonstrate these actions. Studies relating to different features of manufacturing systems with outsourcing choices may also be found elsewhere (Forni, 2010; Narayanan et al., 2011; Balachandran et al., 2013; Hahn et al., 2016; Kenyon et al., 2016; Rakyta et al., 2016; Oblak et al., 2017). With the purpose of offering decisionmakers crucial information regarding alternative fabrication plans for meeting demand, this study uses mathematical modeling and analysis techniques to explore a hybrid EMQ-based replenishment system with rework and scrap. The objectives are to determine the optimal lot size for the proposed system and to learn the in-depth characteristics of key system parameters. Because little attention has been paid to this specific research area, we intend to fill the gap.

METHOD

The proposed hybrid EMQ-based supply system

A quality-assured hybrid EMQ-based replenishment system is used to meet a specific product demand λ per year, wherein a partial outsourcing option is incorporated to level workloads of the production equipment. In each replenishment cycle, we assume that a fraction π (where $0 < \pi < 1$) of lot size Q is outsourced. All outsourced products are promised to have perfect quality, and the scheduled time of receipt is at the end of the in-house rework process (see Figure 1). Consequently, a specific fixed setup cost K_{π} and unit purchase cost C_{π} are associated with this outsourcing policy.



Fig. 1. Status of perfect quality items in the proposed hybrid EMQ-based system (in green) compared to that in an EMQ-based system with no outsourcing option (in black).

The other $(1 - \pi)$ portion of the lot is fabricated by an in-house EMQ-based system at an annual production rate of *P*, and the production process may randomly produce *x* portion of defective items at a rate of *d*. Defective products

are further inspected and categorized as scrap (a θ portion) and reworkable (the other $(1 - \theta)$ portion) items. A rework process starts in each replenishment cycle when regular fabrication ends, at a rate of P_1 units per year. During rework, a θ_1 portion of reworked items fails and becomes scrap. It is also assumed that no shortages are allowed in the proposed system, so $P - d - \lambda$ must be greater than zero. The status of defective products in the proposed hybrid EMQ-based system is depicted in Figure 2. The additional parameters used in this study are shown in Appendix A.



Fig. 2. Status of defective products in the proposed hybrid EMQ-based system.

Based on the assumptions of the proposed system and from Figures 1 and 2, we can directly observe the following formulas:

$$t_{1\pi} = \frac{(1-\pi)Q}{P} = \frac{H_1}{P-d-\lambda} \tag{1}$$

$$t_{2\pi} = \frac{x \left[(1-\pi)Q \right] (1-\theta)}{P_1} \tag{2}$$

$$t_{3\pi} = \frac{H}{\lambda} = \frac{H_2 + \pi Q}{\lambda} \tag{3}$$

$$T_{\pi} = t_{1\pi} + t_{2\pi} + t_{3\pi} = \frac{Q(1 - x\phi(1 - \pi))}{\lambda}$$
(4)

$$H_1 = (P - d - \lambda)t_{1\pi} \tag{5}$$

$$H_2 = H_1 + (P_1 - d_1 - \lambda) t_{2\pi}$$
(6)

$$H = H_2 + \pi Q = \lambda \cdot t_{3\pi} \tag{7}$$

The level of defective products at the end of fabrication uptime $t_{1\pi}$ is as follows:

$$dt_{1\pi} = xPt_{1\pi} = x\left[\left(1-\pi\right)Q\right] \tag{8}$$

As stated earlier, among the defective items, a θ portion is scrap and the other $(1 - \theta)$ portion is reworkable (see Figure 2). During rework, a θ_1 portion of the reworked items fails and becomes scrap. Therefore, the total number of scrap items in each cycle is

$$\theta \left[xQ(1-\pi) \right] + \theta_1 \left\{ (1-\theta) \left[xQ(1-\pi) \right] \right\} = \left[\theta + \theta_1 (1-\theta) \right] \left[x(1-\pi)Q \right] = \varphi x \left[(1-\pi)Q \right]$$
(9)

Total relevant costs per cycle, TC(Q), for the proposed hybrid EMQ-based system include the variable outsourcing and setup costs, the in-house variable fabrication and setup costs, variable reworking cost, disposal cost for scraps, and inventory holding costs for reworked items, perfect quality products, and defective items in the fabrication cycle. Thus, TC(Q) is

$$TC(Q) = C_{\pi}\pi Q + K_{\pi} + C(1-\pi)Q + K + C_{R}(1-\theta)x[(1-\pi)Q] + C_{S}\varphi x[(1-\pi)Q] + h_{1}\frac{P_{t_{2\pi}}}{2}(t_{2\pi}) + h\left[\frac{H_{1} + dt_{1\pi}}{2}(t_{1\pi}) + \frac{H_{1} + H_{2}}{2}(t_{2\pi}) + \frac{H_{2}}{2}(t_{2\pi})\right]$$
(10)

Substituting K_{π} with $[(1 + \beta_1)K]$ and C_{π} with $[(1 + \beta_2)C]$ in Eq. (10), we obtain the following:

$$TC(Q) = \left[(1+\beta_2)C \right] \pi Q + (1+\beta_1)K + C \left[Q(1-\pi) \right] + K + C_R (1-\theta)x \left[Q(1-\pi) \right] + C_S \varphi x \left[(1-\pi)Q \right] + h_1 \frac{P_1 t_{2\pi}}{2} (t_{2\pi}) + h \left[\frac{H_1 + dt_{1\pi}}{2} (t_{1\pi}) + \frac{H_1 + H_2}{2} (t_{2\pi}) + \frac{H}{2} (t_{3\pi}) \right]$$
(11)

Using the expected values of x to cope with the randomness of the defect rate in the fabrication process, substituting all relevant variables from equations (1) to (9) in Eq. (11), and with extra derivations, the expected system costs E[TCU(Q)] can be found as follows:

$$E\left[TCU(Q)\right] = \frac{E\left[TC(Q)\right]}{E[T_{\pi}]} = \frac{\lambda\left[K + (1+\beta_{1})K\right]}{Q(1-(1-\pi)\varphi E[x])} + \frac{\lambda C\left[\pi(1+\beta_{2}) + (1-\pi)\right]}{(1-\varphi E[x](1-\pi))} + \frac{\lambda(1-\pi)E[x]\left[C_{R}(1-\theta) + C_{S}\varphi\right]}{(1-\varphi E[x](1-\pi))} + \frac{\lambda Q\left[h_{1}(1-\theta) - h\right]}{2(1-\varphi E[x](1-\pi))}\left[\frac{E[x]^{2}(1-\pi)^{2}(1-\theta)}{P_{1}}\right] + \frac{hQ}{2(1-(1-\pi)\varphi E[x])} \left\{ \frac{\left[1-\varphi E[x](1-\pi)\right]^{2} - \frac{(1-\pi)\lambda}{P}\left[(1+\pi) - 2\varphi E[x](1-\pi)\right]}{P_{1}} + \frac{E[x](1-\pi)(1-\theta)\lambda}{P_{1}}(\varphi(1-\pi)E[x] - 2\pi) \right\}$$

$$(12)$$

Let

$$E_{0} = \frac{1}{\left(1 - (1 - \pi)\varphi E[x]\right)}; \quad E_{1} = \frac{E[x]}{\left(1 - (1 - \pi)\varphi E[x]\right)}$$
(13)

Thus,

$$E[TCU(Q)] = \frac{\lambda[(1+\beta_{1})K+K]E_{0}}{Q} + \lambda C[\pi(1+\beta_{2})+(1-\pi)]E_{0} + \lambda(1-\pi)[C_{R}(1-\theta)+C_{S}\varphi]E_{1} + \frac{\lambda Q[h_{1}(1-\theta)-h]E_{0}}{2}\left[\frac{E[x]^{2}(1-\pi)^{2}(1-\theta)}{P_{1}}\right] + \frac{hQE_{0}}{2}\left\{\frac{E_{0}^{-2}-\frac{(1-\pi)\lambda}{P}[(1+\pi)-2\varphi E[x](1-\pi)]}{P_{1}}\right\} + \frac{E[x](1-\pi)(1-\theta)\lambda}{P_{1}}(\varphi E[x](1-\pi)-2\pi)\right\}$$
(14)

RESULT AND DISCUSSION

The optimal replenishment lot size

To determine the optimal replenishment lot size, one can apply the first and second derivatives of E[TCU(Q)] with respect to Q and obtain the following:

$$\frac{dE[TCU(Q)]}{dQ} = -\frac{\lambda[(1+\beta_{1})K+K]E_{0}}{Q^{2}} + \frac{\lambda[h_{1}(1-\theta)-h]E_{0}}{2} \left[\frac{E[x]^{2}(1-\pi)^{2}(1-\theta)}{P_{1}}\right] + \frac{hE_{0}}{2} \left\{ \frac{E_{0}^{-2} - \frac{(1-\pi)\lambda}{P}[(1+\pi) - 2\varphi E[x](1-\pi)]}{P_{1}} + \frac{E[x](1-\pi)(1-\theta)\lambda}{P_{1}}(\varphi E[x](1-\pi) - 2\pi) \right\}$$

$$\frac{d^{2}E[TCU(Q)]}{dQ^{2}} = \frac{2\lambda[(1+\beta_{1})K+K]E_{0}}{Q^{3}}$$
(16)

Because λ , $(1 + \beta_1)$, K, E_0 , and Q are all positive, the Eq. (16) result is positive. We confirm that E[TCU(Q)] is a strictly convex function for all Q different from zero. Then, by setting the first derivative of E[TCU(Q)] equal to zero, and with further derivations, one can solve for the optimal replenishment lot size Q^* as follows:

$$\frac{dE[TCU(Q)]}{dQ} = -\frac{\lambda[(1+\beta_{1})K+K]E_{0}}{Q^{2}} + \frac{\lambda[h_{1}(1-\theta)-h]E_{0}}{2} \left[\frac{E[x]^{2}(1-\pi)^{2}(1-\theta)}{P_{1}}\right] + \frac{hE_{0}}{2} \left\{ \frac{E_{0}^{-2} - \frac{(1-\pi)\lambda}{P}[(1+\pi) - 2\varphi E[x](1-\pi)]}{P_{1}} + \frac{E[x](1-\pi)(1-\theta)\lambda}{P_{1}}(\varphi E[x](1-\pi) - 2\pi) \right\} = 0$$
(17)

and

$$Q^{*} = \sqrt{\frac{2K\lambda(\beta_{1}+2)}{\lambda[h_{1}(1-\theta)-h]\left[\frac{E[x]^{2}(1-\pi)^{2}(1-\theta)}{P_{1}}\right] + h\left\{\frac{E_{0}^{-2} - \frac{(1-\pi)\lambda}{P}[(1+\pi)-2\varphi E[x](1-\pi)]}{\frac{E[x](1-\pi)(1-\theta)\lambda}{P_{1}}(\varphi E[x](1-\pi)-2\pi)\right\}}}$$
(18)

Numerical example with discussion

A numerical example with sensitivity analysis is provided in this section to demonstrate the applicability of the research result. Suppose a hybrid EMQ-based system can fabricate a specific product at an annual rate of P = 20,000 units to satisfy an annual demand rate $\lambda = 4,000$ units. The in-house fabrication setup cost K = \$5,000, unit cost C = \$100, and unit holding cost h = \$30. To cope with limited fabrication capacity, a portion $\pi = 0.4$ of each replenishment lot is outsourced, with $K_{\pi} = \$1,500$ (calculated by assuming $\beta_1 = -0.7$) and $C_{\pi} = \$120$ (computed by assuming $\beta_2 = 0.2$), respectively. The random defect rate *x* is assumed to obey a uniform distribution over the range [0, 0.2]. Among defective products, a portion $\theta = 0.1$ is scrap and the other $(1 - \theta)$ portion can be reworked at a rate of $P_1 = 5000$ units/ year, in each cycle. The unit cost of rework $C_R = \$60$ and unit holding cost for a reworked item $h_1 = \$40$. It is also assumed that a $\theta_1 = 0.1$ portion of reworked items fails and is discarded at disposal cost $C_s = \$20$ per item.

The optimal replenishing lot size $Q^* = 1492$ can be obtained first from the computation of Eq. (18). Then, applying Q^* in Eq. (14), the expected total relevant costs per unit time E[TCU(Q)] = \$486,265 can be found. The performance of E[TCU(Q)] relating to different values of replenishment lot size Q is illustrated in Figure 3.



Fig. 3. Performance of E[TCU(Q)] relating to different batch size Q.

Table 1 exhibits analytical results of the effects of differences in π on Q^* , fabrication uptime, rework time, optimal replenishment cycle time T^* , machine utilization, and decrease in percentage of utilization. It is noted that as π increases, Q^* declines slightly; and uptime t_1 , reworking time t_2 , and machine utilization $(t_1 + t_2)/T^*$ all decrease accordingly.

When $\pi = 0$ (i.e., no outsourcing option is adopted), the machine utilization is 20.79%; and when π is set to 0.4 (as in our example), machine utilization drops to 12.28% (also see Figure 4); i.e., it decreases by 40.9%.

related decrease percentage.											
π	Q^*	Fabrication	Reworking	working $(t_1 + t_2)$ Time t_2	<i>T</i> *	Machine Utilization	Utilization				
		Uptime t_1	Time t_2			$(t_1 + t_2) / T^*$	Decrease %				
0	1314	0.0657	0.0236	0.0893	0.3159	20.79%	-				
0.05	1503	0.0714	0.0257	0.0971	0.3622	19.71%	-5.2%				
0.10	1507	0.0678	0.0244	0.0922	0.3638	18.64%	-10.4%				
0.15	1509	0.0641	0.0231	0.0872	0.3650	17.57%	-15.5%				
0.20	1509	0.0604	0.0217	0.0821	0.3657	16.50%	-20.6%				
0.25	1507	0.0565	0.0203	0.0768	0.3661	15.44%	-25.7%				
0.30	1504	0.0526	0.0189	0.0715	0.3660	14.38%	-30.8%				
0.35	1499	0.0487	0.0175	0.0662	0.3654	13.33%	-35.9%				
0.40	1492	0.0448	0.0161	0.0609	0.3645	12.28%	-40.9%				
0.45	1484	0.0408	0.0147	0.0555	0.3632	11.23%	-46.0%				
0.50	1474	0.0369	0.0133	0.0502	0.3615	10.19%	-51.0%				
0.55	1463	0.0329	0.0118	0.0447	0.3595	9.16%	-56.0%				
0.60	1450	0.0290	0.0104	0.0394	0.3571	8.12%	-60.9%				
0.65	1437	0.0251	0.0091	0.0342	0.3544	7.09%	-65.9%				
0.70	1422	0.0213	0.0077	0.029	0.3514	6.07%	-70.8%				
0.75	1406	0.0176	0.0063	0.0239	0.3482	5.05%	-75.7%				
0.80	1389	0.0139	0.0050	0.0189	0.3447	4.03%	-80.6%				
0.85	1372	0.0103	0.0037	0.014	0.3411	3.02%	-85.5%				
0.90	1354	0.0068	0.0024	0.0092	0.3372	2.01%	-90.3%				
0.95	1336	0.0033	0.0012	0.0045	0.3332	1.00%	-95.2%				
1	577	0	0.0000	0	0.1443	0%	-100.0%				

Table 1: Effects of variations of π on system uptime, rework time, T^* , and machine utilization and its related decrease percentage.

Figure 5 depicts the analytical results of joint effects of variations in Q and outsourcing proportion π on E[TCU(Q)]. It can be seen that as π increases, the optimal lot size Q decreases slightly, but E[TCU(Q)] increases significantly.



Fig. 4. Effect of variations of π on in-house machine utilization.



Fig. 5. Joint effects of variations of *Q* and π on E[TCU(Q)].

The joint effects of variations of the unit outsourcing cost factor β_2 and random defect rate x on E[TCU(Q)] are analyzed and illustrated in Figure 6. It shows that E[TCU(Q)] rises drastically as β_2 increases; and as x moves up, E[TCU(Q)] also increases significantly.



Fig. 6. Joint effects of variations of β_2 and *x* on E[TCU(Q)].

Figure 7 illustrates the analytical results of joint effects of variations of overall scrap rates φ and random defect rates x on E[TCU(Q)]. It can be seen that, according to this numerical example, as φ increases, the expected system cost per unit time E[TCU(Q)] increases.



Fig. 7. Joint effects of variations of φ and x on E[TCU(Q)].

The proposed study enables us to analyze the effect of the overall scrap rate φ on E[TCU(Q)] for supporting managerial "make-or-buy" decision-making (see Figure 8). For example, in the case of $\theta = 0.1$ (i.e., the scrap rate is 10% among the defective items produced in uptime), the analytical result reveals that if the overall scrap rate φ (refer to Eq. (9)) exceeds 0.874, then it is better to "buy" (i.e., to outsource the entire lot). Similarly, in the case of $\theta = 0.25$, the critical values for choosing the buy decision are when φ exceeds 0.939.



Fig. 8. Analysis of the effect of overall scrap rate φ on E[TCU(Q)] for supporting make-or-buy decision-making.

The impacts of changes in π on Q^* and relevant system cost components including outsourcing cost, in-house fabrication cost, and E[TCU(Q)] are explored and displayed in Table 2.

π	Q^*	Outsourcing Cost	In-house Fabrication Cost	$E[TCU(Q^*)]$	Cost Increase %
0	1314	\$0	\$462,357	\$462,357	0.0%
0.05	1503	\$28,507	\$440,520	\$469,027	1.4%
0.10	1507	\$52,887	\$418,495	\$471,382	2.0%
0.15	1509	\$77,224	\$396,548	\$473,773	2.5%
0.20	1509	\$101,520	\$374,680	\$476,200	3.0%
0.25	1507	\$125,773	\$352,889	\$478,663	3.5%
0.30	1504	\$149,985	\$331,177	\$481,161	4.1%
0.35	1499	\$174,154	\$309,542	\$483,696	4.6%
0.40	1492	\$198,282	\$287,983	\$486,265	5.2%
0.45	1484	\$222,367	\$266,501	\$488,868	5.7%
0.50	1474	\$246,411	\$245,094	\$491,505	6.3%
0.55	1463	\$270,413	\$223,762	\$494,175	6.9%
0.60	1450	\$294,374	\$202,502	\$496,876	7.5%
0.65	1437	\$318,293	\$181,316	\$499,609	8.1%
0.70	1422	\$342,170	\$160,200	\$502,370	8.7%
0.75	1406	\$366,006	\$139,155	\$505,161	9.3%
0.80	1389	\$389,799	\$118,179	\$507,978	9.9%
0.85	1372	\$413,551	\$97,270	\$510,822	10.5%
0.90	1354	\$437,262	\$76,429	\$513,690	11.1%
0.95	1336	\$460,930	\$55,652	\$516,582	11.7%
1	577	\$500,785	\$0	\$500,785	8.3%

Table 2. Impacts of changes in π on Q^* and system cost components.

It can be seen that when π is 0.40, $Q^* = 1492$ and $E[TCU(Q^*)] = $486,265$. If $\pi = 0$, the proposed hybrid EMQbased system turns into a pure "make" system (i.e., all products are made in-house) and $Q^* = 1314$ and $E[TCU(Q^*)] = $462,357$. When $\pi = 1$, the proposed hybrid EMQ-based system becomes the same as a pure "buy" system (i.e., all products are outsourced) and based on the economic order quantity model we have $Q^* = 577$ and $E[TCU(Q^*)] = $500,785$. It is also noted that, at $\pi = 0.40$, the cost increases 5.2% as compared to $\pi = 0$ but the machine utilization declines 40.9% (see Table 1).

Finally, the proposed hybrid EMQ-based system further enables us to determine a critical outsourcing proportion at approximately $\pi = 0.671$ (see Table 2). This means that, based on the given values of system parameters in this numerical example, once π goes over 0.671, it is better to switch to a pure "buy" system because $E[TCU(Q^*)]$ will be greater than \$500,785. In other words, for any given (known) values of system variables, the proposed hybrid EMQ-based system can provide a critical proportion π for supporting managerial "make-or-buy" decision-making.

CONCLUSIONS

With the purpose of offering decision-makers crucial information regarding an alternative fabrication plan for meeting product demand, this study uses mathematical modeling and analysis techniques to investigate a hybrid EMQ-based replenishment system with rework and scrap. Specifically, an outsourcing option and issues of quality assurance are incorporated into a classic EMQ-based system. This in-depth exploration enables production managers to obtain the optimal lot size for the system and determine the joint effects of various system parameters, as the outsourcing percentage of a batch and its relevant costs, random defect and scrap rates of the manufacturing process, and quality assurance related costs in fabrication, among others, on the optimal lot size and on the expected total system cost, as well as information applicable to make-or-buy decision-making.

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Appendix A

Additional parameters used in the proposed hybrid EMQ-based supply system are as follows:

 T_{π} = replenishment cycle time,

- $t_{1\pi}$ = production uptime,
- $t_{2\pi}$ = rework time,

 $t_{3\pi}$ = delivery time,

- Q =lot size the decision variable,
- H_1 = maximum perfect quality inventory level when in-house fabrication uptime ends,
- H_2 = on-hand level of perfect quality items when rework process finishes,
- H = on-hand level of perfect quality items when outsourcing items are received,

K = in-house setup cost,

- C = unit in-house fabrication cost,
- h = unit holding cost,
- $C_{\rm R}$ = unit reworking cost,
- h_1 = holding cost per reworked item per year,

 $C_{\rm S}$ = unit disposal cost,

- φ = overall scrap rate of defective items, where $\varphi = [\theta + (1 \theta)\theta_1]$,
- K_{π} = fixed outsourcing setup (order) cost in a cycle,
- C_{π} = unit outsourcing cost,
- β_1 = the connecting variable between K_{π} and K, where $K_{\pi} = (1 + \beta_1)K$ and $-1 < \beta_1 < 0$,

 β_2 = the connecting variable between C_{π} and C, where $C_{\pi} = (1 + \beta_2)C$ and $\beta_2 > 0$,

TC(Q) = total relevant costs per cycle,

E[TCU(Q)] = the expected total relevant costs per unit time.

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Submitted: 03/06/2017 *Revised:* 03/02/2019 *Accepted:* 24/03/2019 تلبية المتطلبات على المنتجات من خلال نظام مضمون يعتمد على جودة التصنيع الاقتصادي المختلط

الخلاصة

عادةً ما يقوم مديرو شركات التصنيع بإجراء تقييم كامل لبدائل التصنيع المختلفة لتلبية الطلبات من خلال عمليات صنع القرار الخاصة بهم وذلك بقصد تقليل تكلفة التشغيل و / أو رفع مستوى موارد الإنتاج المحدودة. تقدم هذه الدراسة نظاماً مضموناً يعتمد على جودة التصنيع الاقتصادي المختلط (EMQ)، مع الأخذ في الاعتبار خيار الاستعانة بمصادر خارجية وإعادة تشغيل الخردة. تتمثل الأهداف في تحديد حجم الدفعات الأمثل للنظام المُقترح ومعرفة الخصائص المتعمقة لمعلمات النظام الأساسية. تم تصميم نموذج دقيق لوصف النظام المقترح. ومن خلال التحليل الرياضي وتحقيق الأمثلية، تم الحصول على الحجم الأمثل للدفعات مغلق الشكل. وباستخدام مثال رقمي، قمنا بالتأكيد على قابلية تطبيق نتائج البحث وإظهار أنه يمكن الحصول على تأثيرات مشتركة حاسمة لمتغيرات النظام المختلفة على حجم الدفعات الأمثل ومن التوابية تطبيق نتائج المعرفة الإمثلية، الم