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نمذجة تفاعلات السوائل والجسيمات في قناة مفتوحة ومتدفقة في مركزات جاذبة

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الخلاصة

فهم حركة الجسيمات من خلال تدفق السوائل وترسبها وآلية توزيعها في قناة مفتوحة لها أهمية هندسية كبيرة. والبحث هو محاولة لاختبار مدى امكانية مبادىء ميكانيكا الموائع بأن توفر معلومات عن تركيز الجسيمات وشكل سرعة السائل، والدراسة اجريت على قناة مستطيلة مائلة ميلان بسيط مع سائل بمعدلات تدفق مختلفة وجسيمات مصنوعة من حبات زجاجية ومخاليط إلمينيت بتركيزات مختلفة. وأوليت عناية كبيرة لابتكار تدفق الفصال في نهاية القناة والتي تسمح بالقياس الدقيق لشكل تدفق السائل وجمع عينات من الجسيمات، والسماح لتقييم تركيز الجسيمات المتنوعة لغرض التحقق من صحة النموذج. وتظهر توقعات النتائج من فصل الجسيمات كما هو مبين في البحث لتكون وظيفة قوية للسرعات وتسوية الجزيئات المستخدمة والتوقعات الناتجة من فصل الجسيمات وهي جيدة بشكل ملحوظ وهو ما يؤكد في نهاية طريقة النمذجة المستخدمة في البحث.

Modeling fluid-particle interactions in flowing film type gravity concentrators

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ABSTRACT

Understanding of particle motion through water and the particle sedimentation as well as distribution mechanism in an open channel flow is of considerable engineering importance. In an effort to test the extent to which fundamental fluid mechanics can provide a prediction of particle concentration profile and the fluid velocity profile, a study on a simple inclined rectangular channel has been carried out using glass bead and ilmenite mixtures at various particle concentrations and flow rates. Great care was taken to devise a flow splitter at the end of the channel that permitted accurate measurement of the fluid flow profile and collection of slurry samples, allowing evaluation of the particle species concentration profiles for model validation purpose. The resulting predictions of particle separation are shown to be a strong function of the settling velocities of the particles used and the resulting predictions of particle separation are remarkably good, which ultimately validates the modelling methodology.

Keywords: Flowing film concentrator; fluid-particle interaction; gravity concentration; modelling; particle sedimentation.

INTRODUCTION

A typical mechanical property of a fluid is that, when it flows down an inclined plane the velocity of the fluid adjacent to the plane becomes zero and it becomes maximum at or very near the top layer of the flow stream. This means that there exists a velocity gradient along the depth of the flowing fluid. When particles are added to the flowing fluid they try to settle at the bottom of the bed. The factors that are important in determining the relative movement of a particle in the fluid include the specific gravity, size and particle shape, not only in absolute terms but also relative to all other particles in the system. Shear and turbulent eddies, however, keep the particles suspended in the flow, moving relative to each other. These important properties of the fluids and the particles settling behaviour in a fluid have been exploited to separate minerals according to their differences in settling velocities, which in general is termed as flowing film concentration technique.

Sluices are the simplest forms of the flowing film concentrators and have been used in the mineral industry since at least the 15th century (Agricola, 1556). Sluicing is an operation, where separation by gravity is attained by the settling of particles and the transportation of the non-cohesive ("loose") bed thus formed in a slurry flowing through a trough, which is essentially inclined and flat-bottomed (Sivamohan & Forssberg, 1985). Various mineral particles are classified horizontally in thin layers across the wide bottom, causing inefficient separation between adjacent beds. To overcome this problem, pinched sluices are developed where the pulp is pinched between the constricting walls, thereby increasing the flow depth at the discharge end, which helps in better separation of stratified mineral beds by splitters in general.

It is reported that the efficiency of separation of the pinched sluices improves with increased feed concentration, increased particle size, and reduced flow rates (Abdinegoro & Partridge, 1979). The main disadvantage of a pinched sluice is that the sidewalls cause turbulence, resulting in inefficient operation, and large middling streams must be recirculated, if high recoveries are to be obtained (Graves, 1973). To avoid the detrimental effects of the sidewalls in a pinched sluice, cone concentrators are developed, which comprises a number of pinched sluices, without walls, arranged in a circle. The most commonly used cone concentrator is a Reichert cone. The advantages of a Reichert cone over a pinched sluice are that there is no turbulence due to the effects of side walls, the capacity is higher and the circulating load is lower (Graves, 1973).

The basic principle of flowing film concentration technique has been used to design various other separators like spiral concentrators, tables, vanners, Falcon concentrator and Mozley multi gravity separator, which find wide applications in mineral industry to meet specific requirements. However, tailor-made designs are still not available, as the basic mechanism of particle separation in a flowing film is still not properly understood.

Apparently, the particle sedimentation and distribution mechanism in a flowing slurry is very simple, but unfortunately the development of a universal theory which explains the behaviour of a suspension through a open channel is still not achieved (Sivamohan & Forssberg, 1985). The difficulty arises because the presence of the particles influences the fluid flow pattern and when particles are fully transported by the flow, the flowing slurry may not have the same viscosity as water and does not have the same density (Lyman 1994). However, there are some isolated defined sorting mechanisms which can be related to the behaviour of individual devices (Mayer, 1964; Subasinghe, 1983). The other development on the theoretical side is the simulation models based on regression analyses. Here, the equations can be purely empirical or be based on hydrodynamic relations with the empirical constants, as found in the work of (Subasinghe, 1983).

Lyman (1994) has rightly pointed out that most of the traditional textbooks contain analyses of the forces acting on single particles alone in the flowing fluid and employ expressions for fluid drag forces that consider the motion of single particles in isolation. Further, they describe the flow field in the neighbourhood of the particle as the field for flow of water alone; the influence of the presence of the particle on the flow field is generally neglected. The equations that result from such a treatment of the problem can only provide the most approximate guide to the phenomenon the model seeks to describe.

Various theories, like Bagnolds theory, Mayer's potential energy theory and Rouse's diffusion theory are also available in the literature (Majumder, 2002) to describe the particles suspension behaviour in flowing slurry. However, it has not yet been shown', how these theories can be applied to predict accurately the particle sedimentation behaviour in flowing slurry. An attempt has therefore been made in this article, to demonstrate how the basic principles of fluid mechanics can be applied to develop a generic model towards better understanding of the particle sorting mechanism in a flowing film concentrator.

MODELLING STRATEGY

So far, probably, Lyman (1994) and Majumder (2010) have given the best descriptions of the physics of operation of the water-based separators through the elementary sedimentation–back–mixing model as illustrated in Figure 1. This has its origins with the work of Schubert (1979), which is preceded by the work of Hunt (1969). An inclined rectangular channel may be considered as the simplest separator with some fluid velocity profile and some steady-state distribution of particle concentration with depth in the fluid. Then, by considering a splitter to be located at some flow height, the partitioning of the solids between underflow and overflow can be calculated by appropriate integration of the velocity and concentration profiles.

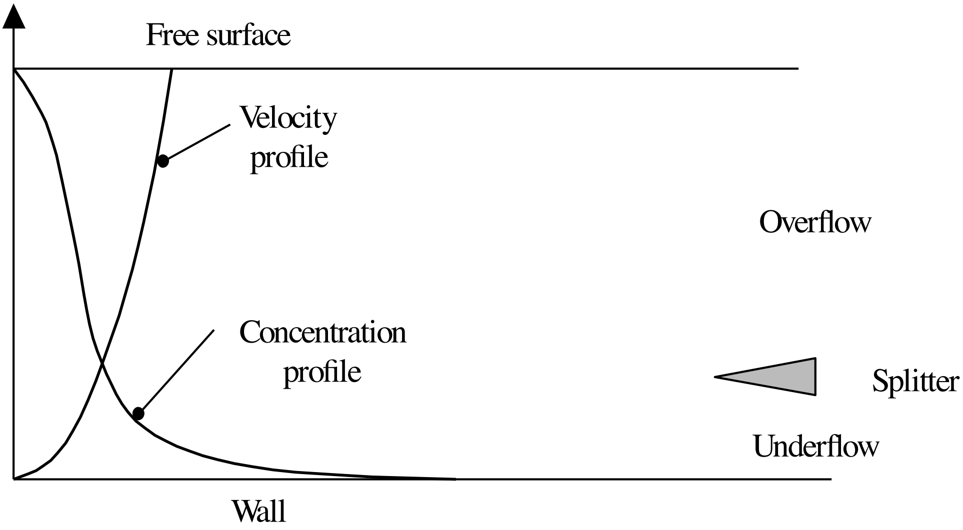


Fig. 1. Elementary separator model

Therefore, strategically the modelling procedure has been divided into two parts; the first is to predict the horizontal slurry flow velocity profile and the second is to predict the solid particle concentration over the depth of the flow at different flow conditions. The predictions have then to be validated with some measured data sets. This can easily be done, if the slurry flow is split at different fractional flow depths so that the mass flow of solids over different flow heights is measured accurately over a fixed period of time. The same can be predicted by integrating the product of the predicted horizontal slurry velocity profile and the predicted mass fractions of solids from the channel bed to the top surface.

Prediction of slurry velocity profile

If it is assumed that the fluid flow down the channel is two dimensional and fully developed, then the basic Reynolds Averaged Navier – Stokes equations can be written as:

$$u_x(x) = f(y) \tag{1}$$

$$\frac{d}{dy}\left[(\mu + \mu_t)\frac{du_x}{dy}\right] + \rho g \sin\theta = 0 \tag{2}$$

$$\frac{dp}{dy} + \rho g \cos\theta = 0 \tag{3}$$

The validity of this assumption for the designed experimental set up has been described in detail elsewhere (Majumder, 2009). The average fluid velocity normal to the channel floor is everywhere zero ($u_y = 0$). For the eddy viscosity, the simple

Prandtl mixing length model is chosen, which has the form

$$\mu_t = \rho k^2 y^2 \left| \frac{du_x}{dy} \right| \tag{4}$$

After integrating Equation (2) once and substituting the above expression for the eddy viscosity, a non-linear ordinary differential equation providing the velocity profile in the channel can be derived as

$$\rho k^2 y^2 \left(\frac{du_x}{dy} \right)^2 + \mu \frac{du_x}{dy} + \rho g y \sin\theta + C_1(x) = 0 \tag{5}$$

Similarly Equation (3) can be integrated to arrive at a hydrostatic relationship for the pressure in the form

$$p + \rho g y \cos\theta + C_2(x) = 0 \tag{6}$$

The two arbitrary functions of x in Equations (5) and (6) can be evaluated from the boundary conditions that the pressure is atmospheric at the upper surface of the water at $y = h_f$ and it may be assumed that there is no shear at the upper fluid surface, which means $du_x / dy = 0$ at this point. This gives

$$\rho k^2 y^2 \left(\frac{du_x}{dy} \right)^2 + \mu \left(\frac{du_x}{dy} \right) - \rho g (h_f - y) \sin\theta = 0 \tag{7}$$

$$p = \rho g (h_f - y) \cos\theta \tag{8}$$

If y^+ and u^+ are introduced as dimensionless distance and dimensionless velocity defined respectively as

$$y^+ = \frac{u^* y}{\nu} \tag{9}$$

$$u^+ = \frac{u_x}{u^*} \tag{10}$$

then, equation (5) can be rewritten as

$$\left(\frac{du^+}{dy^+} \right)^2 + \frac{1}{(ky^+)^2} \left(\frac{du^+}{dy^+} \right) - \frac{1}{(ky^+)^2} \left(1 - \frac{y^+}{h_f^+} \right) = 0 \tag{11}$$

Finally the velocity profile can explicitly be written in dimensionless form from equation (11) as

$$\frac{du^+}{dy^+} = \frac{1}{2\left(ky^+\right)^2} \left[\sqrt{1 + 4\left(ky^+\right)^2 \left(1 - \frac{y^+}{h_f^+} \right)} - 1 \right] \tag{12}$$

Prediction of solids concentration profile

In gravity concentrators, mixtures of different particle classes generally have very high concentrations and they are given very short time to settle. Due to the differences in the fall velocities of individual particle classes, a particle concentration gradient along the depth of the flow is developed, which ultimately helps in sorting various particle classes. However, the complexity arises in determining the correct solids concentration profile at a given flow condition as the turbulence in the flow is responsible for momentum transfer and also produces a mixing effect in the direction perpendicular to the main flow. Various theories are proposed and different models are developed to quantify the influences of turbulence on the particle motion. However, Hunt's (1969) diffusion approach in explaining the particle stratification behaviour analytically in a turbulent shear flow seems to be the most appropriate in this case.

For N particle species in the slurry, Hunt's (1969) concentration profile (volume fraction) is given as

$$c_i(y) = \frac{c_i(\alpha)\exp\left[u_i\{\varphi(\alpha)-\varphi(y)\}\right]}{1-\sum_{j=1}^{N} c_j(\alpha)\left[1-\exp\{u_j(\varphi(\alpha)-\varphi(y))\}\right]} \tag{13}$$

and

$$\varphi(y) = \int \frac{dy}{\varepsilon(y)} \tag{14}$$

where $i = 1,2,\ldots,N$ and α denotes a reference height where concentrations are known.

Usually, the diffusion of fluid momentum, $\varepsilon(y)$, is described (Van Rijn, 1984) by a parabolic distribution over the flow depth, h_f, in the following manner

$$\varepsilon(y) = \frac{y}{h_f}(1-\frac{y}{h_f})ku^* h_f \tag{15}$$

Substituting Equation (15) in Equation (14) results

$$\varphi(y) = \int \frac{dy}{\left[\frac{y}{h_f}(1-\frac{y}{h_f})ku^* h_f\right]} \tag{16}$$

Putting the value of $k = 0.41$ (Van Driest, 1956) in Equation (16) then results

$$0.41u^*\varphi(y) = \ln\left(\frac{\frac{y}{h_f}}{(1-\frac{y}{h_f})}\right) + \text{constant} \tag{17}$$

Now, if the particle concentration at the middle of the flow depth i.e. at $\alpha = \frac{h_f}{2}$ is known, then it can be written conveniently,

$$\{\varphi(\alpha) - \varphi(y)\} = \frac{1}{0.41u^*} \ln\left(\frac{1-\frac{y}{h_f}}{\frac{y}{h_f}}\right) \tag{18}$$

Considering only one particle species in the slurry, having settling velocity u, it may be written from Equations (13) to (18)

$$c(y) = \frac{c(\alpha)\left(\frac{1-\frac{y}{h_f}}{\frac{y}{h_f}}\right)^{\frac{2.439u}{u*}}}{1-c(\alpha)\left[1-\left(\frac{1-\frac{y}{h_f}}{\frac{y}{h_f}}\right)^{\frac{2.439u}{u*}}\right]} \tag{19}$$

Similarly, Equation (19) can be expanded accordingly for the concentration profile of the N^{th} particle class in an assemblage of N particle classes as is given in Equation 20.

$$c_N(y) = \frac{c_N(\alpha)\left(\frac{1-\frac{y}{h_f}}{\frac{y}{h_f}}\right)^{\frac{2.439u_N}{u*}}}{1-\left[\begin{array}{l} c_N(\alpha)\left\{1-\left(\frac{1-\frac{y}{h_f}}{\frac{y}{h_f}}\right)^{\frac{2.439u_N}{u*}}\right\}+c_{(N-1)}(\alpha)\left\{1-\left(\frac{1-\frac{y}{h_f}}{\frac{y}{h_f}}\right)^{\frac{2.439u_{(N-1)}}{u*}}\right\}+ \\ ....+c_1(\alpha)\left\{1-\left(\frac{1-\frac{y}{h_f}}{\frac{y}{h_f}}\right)^{\frac{2.439u_1}{u*}}\right\} \end{array}\right]} \tag{20}$$

To validate the afore-mentioned models, experimental data were generated in a carefully designed test rig. The description of the test rig and the data acquisition techniques are briefly discussed hereunder.

EXPERIMENTAL

Design of the test rig

The basic philosophy behind the design of the test rig was to split the flow of slurry down the channel at different flow depths, without hindering the principal nature of the flow, so that the mass flow of slurry at various fractional flow depths could be measured accurately.

The test rig is shown schematically in Figure 2. The channel was made of 6mm thick glass sheet to facilitate direct visualisation of the flow. It was 2400mm in length, 370mm in width and the side walls were made of 6 mm thick and 60 mm high perspex sheets. Proper arrangements were made to tilt the channel at a desired slope and the feed distributor could be mounted anywhere in the channel to vary the channel length, if necessary.

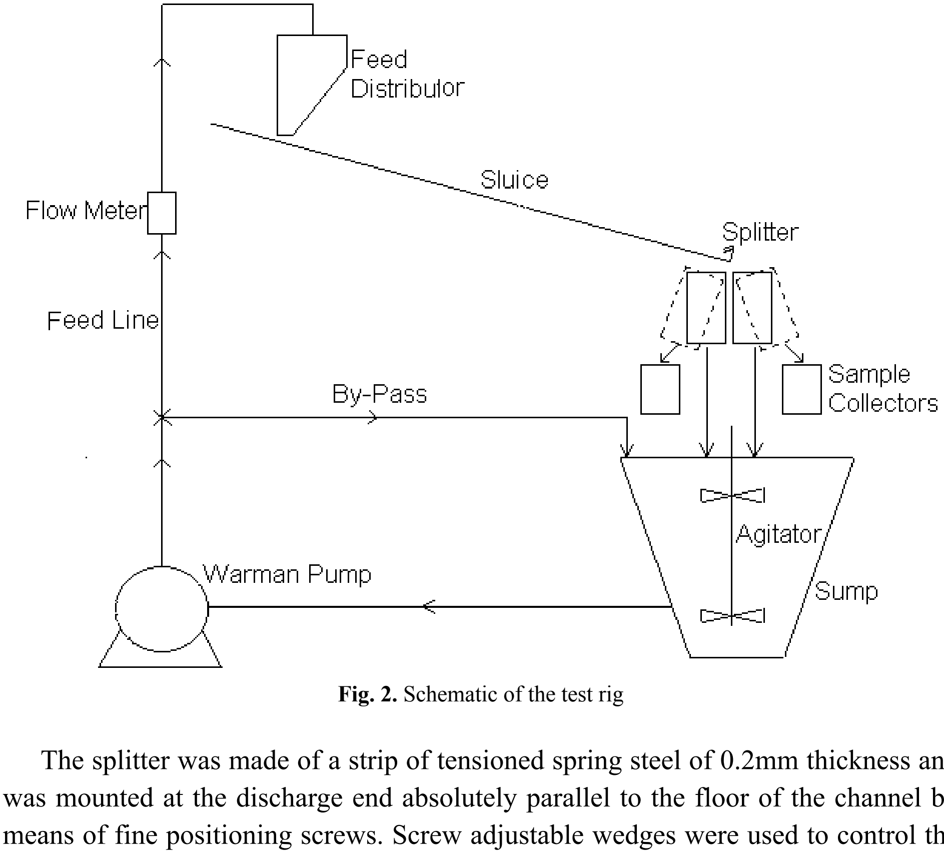


Fig. 2. Schematic of the test rig

The splitter was made of a strip of tensioned spring steel of 0.2mm thickness and was mounted at the discharge end absolutely parallel to the floor of the channel by means of fine positioning screws. Screw adjustable wedges were used to control the vertical movements of the splitter blade with the required precision. The splitter height was accurately measured with a precision of 0.01mm by means of two dial gauges fitted at the two horizontal ends of the splitter blade assembly. The splitter blade was made so thin to present a minimal cross-sectional area to the flow, minimising any component of form drag. The detailed description of the test rig may be found elsewhere (Majumder *et al.*, 2006).

Materials and experimental conditions

Closely sized (nominally –180 + 125 microns) spherical glass beads (GB) and mixtures of glass beads and ilmenite (ILM) particles (nominally –180 + 125 microns) were used to carry out experiments with solid particles of two different densities. For all the experiments, the channel inclination was kept constant at 17.5 degrees and the effective length of the channel was also kept constant at 1350 millimetre. Densities of the solid particles were measured with a helium gas displacement pycnometer. The average particle densities of glass beads and ilmenites were measured to be 2494 kg/m^3 and 4595 kg/m^3 respectively.

Experiments were carried out at different flow rates with different proportions of glass beads and ilmenite particles and at different feed solids concentrations. The experimental conditions are summarised in Table 1 below.

Table 1. Experimental conditions with mixtures

Sample	Feed Solids Concentration	Ratio	Flow Rates
	(weight %)	(GB/ILM)	(l/s)
Mixture 1	22.96	1.08	2.32
	44.22	1.08	2.32
Mixture 2	22.96	0.82	1.99
	19.39	0.86	1.46
Mixture 3	44.22	6.2	0.69
	38.29	6.97	1.05
	37.45	6.59	1.40
Mixture 4	39.80	1.32	0.90

Data acquisition procedure

Slurry was pumped and fed at one end of the channel through the feed distributor for uniform distribution over the width of the channel. The pump was connected to a mechanical variable speed drive and a by-pass valve was installed in the feed line to control the flow rate. A flow meter was also installed in the feed line to reproduce the experimental conditions, but the actual flow rates were measured accurately by collecting slurries through Vezin samplers over a fixed interval of time.

When the slurry flow over the channel became steady, the flow was split at different flow heights by positioning the knife-edge splitter at the desired flow heights. Slurry weights in the underflow and the overflow were noted. After drying, the glass beads and ilmenite in underflow and overflow were separated magnetically and the respective weights were noted to calculate the fractional recovery of glass beads and ilmenite in the underflow. The actual operating flow rate was then back calculated from the measured mass of total slurry collected as underflow and overflow and water as well as glass beads & ilmenite densities. The actual feed solids concentration was also back calculated from the measured weights of slurry and glass beads as well as ilmenite collected as underflow and overflow. Thus the solids splits over different flow heights at a particular flow rate were determined experimentally. Once a set of experimental data was generated at an almost identical solids concentration and at different flow rates, another set of experimental data was then generated with new feed solids concentration, following the similar procedures.

MODEL VALIDATION

The experimental data thus generated at different solids concentrations were then compared with the predicted data, by solving Equations 12 and 19 for the slurry velocity and concentration profiles respectively, at the same experimental conditions. Brauer & Thiele's (1973) hindered settling model was used throughout while predicting the data at a given operating condition. Figures 3 & 4 show the comparative slurry velocity profiles and concentration profiles respectively, whereas Figures 5 & 6 show the comparison between the model predicted values and the experimentally observed values at total solids concentrations of 22.96 and 44.22 weight per cent at equal glass beads and ilmenite ratio and at a total slurry flow rate of 2.32 litres/second. Plots at various other conditions mentioned in Table 1 are not shown for the brevity of this article.

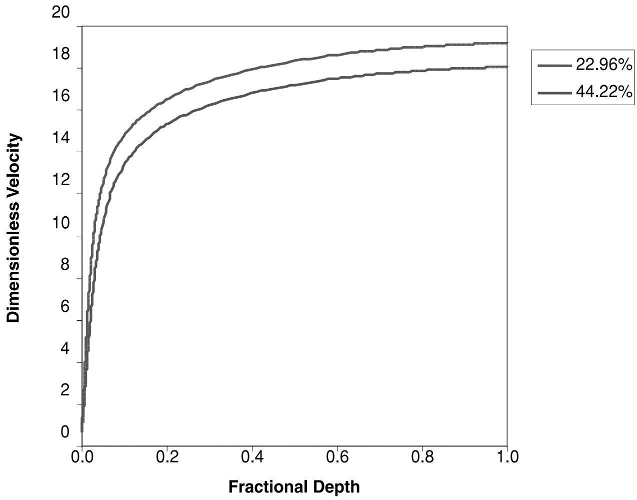
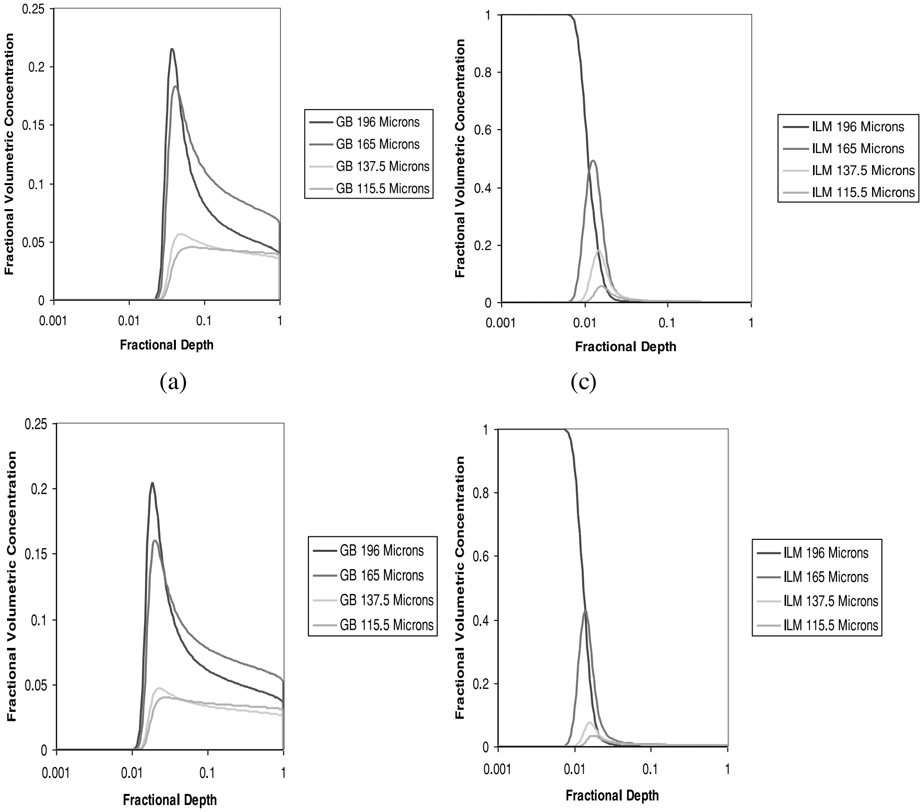


Fig. 3. Predicted slurry velocity profiles



(a) (c)

(b) (d)

Fig. 4. Concentration profiles of glass beads (a & b) and ilmenite (c & d) particles at 22.96 and 44.22% solids concentrations respectively

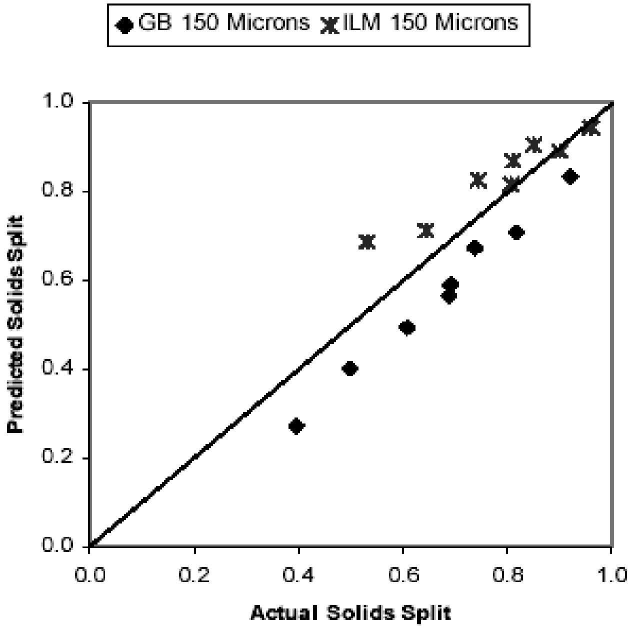


Fig. 5. Comparative plot at 22.96% solids concentration

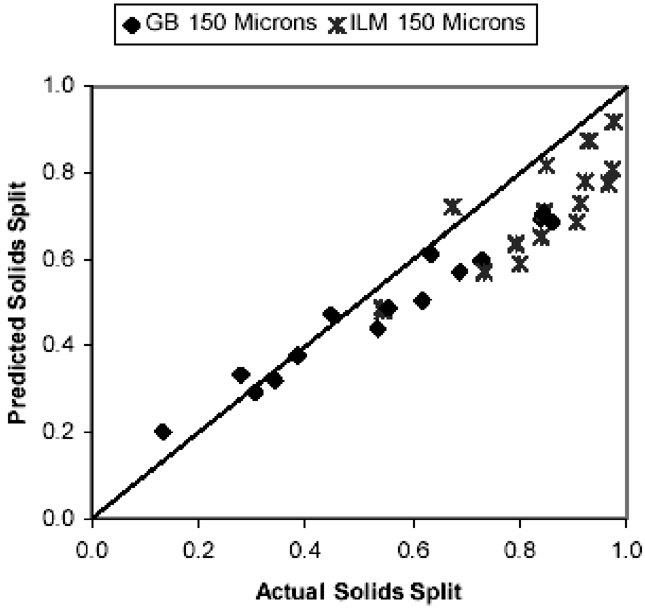


Fig. 6. Comparative plot at 44.22% solids concentration

Figures 3 & 4 show that the predicted slurry velocity profiles and the concentration profiles of glass beads as well as ilmenite particles are realistic enough to the general trend observed in any flowing film type gravity concentrator. As mentioned before, the product of the horizontal slurry velocity profiles and the concentration profiles of various glass beads and ilmenite size fractions were first integrated over a desired fractional flow height to predict the volumetric flows per unit width of the channel of those size fractions over that flow height. The integration was performed numerically using the trapezoidal rule. From the product of this integral, densities of respective particles and the channel width, the mass flow of each size fraction over that splitter height was then calculated. From the feed size distribution, average solids concentration and the slurry flow rate data, the overall mass flow of each fraction of glass beads and ilmentie particles were also calculated. The ratio of these two then gave the values of the predicted fractional mass recoveries of each size fraction over that flow height. Following this methodology the fractional recoveries (solids splits) of four different size fractions of glass beads and ilmenite particles at the identical experimental conditions were predicted and compared with the experimental data, which are presented in Figures 5 & 6. Similar procedures were also adapted to validate the experimental data generated at various other conditions mentioned in Table 1.

Figures 5 & 6 show that the predicted data is reasonably close (relative errors are 11 and 18% respectively) to the actual experimental data, considering the uncertainties associated with some constant values used and the hindering settling velocity model chosen. Brauer & Thiele (1973) hindered settling model may not be the accurate model to predict the settling velocities of individual particle classes in a poly-disperse suspension. As the present model is very sensitive to the settling velocities of individual particle classes, use of an inappropriate hindered settling model will introduce error, even in the predicted data. However, no attempt has been made towards obtaining the best fitted values of these parameters, as the aim of this research was to demonstrate that the established theories of fluid mechanics, if applied judiciously can predict the particle sorting mechanism in a complicated flow field with reasonable accuracy.

SUMMARY

This entire exercise has shown for the first time that a simple consideration of the interaction of gravity (settling forces) and turbulence (causing mixing or diffusion) can lead to a generic separator model for the flowing film type gravity concentrators very similar to those observed in practice. Opportunities are many to extend the basic model structure in various other engineering disciplines like slurry transportation, suspended load transport in particular and solid-fluid interactions in open channel flows in general.

NOMENCLATURE (SI Units are used throughout)

g	Gravitational acceleration
h_f	Flow depth
h_f^+	Dimensionless depth
k	Von Karman's constant
l_{mix}	Prandtl's mixing length
p	Fluid pressure
p^+	Integrating factor
u_x	Flow velocity in the horizontal (x) direction
u_y	Flow velocity in the direction (y) normal to the channel bed
u^*	Shear or friction velocity
u^+	Dimensionless fluid velocity in the horizontal (x) direction
y	Position normal to the channel bed
y^+	Dimensionless distance normal to the channel bed
ρ	Density of the fluid
ρ_w	Density of water
ρ^+	Dimensionless fluid density
z	Momentum variable ($z = \rho u_x$)
z^+	Dimensionless momentum variable
θ	Inclination of the channel
μ	Molecular viscosity of the fluid
ν	Kinematic viscosity of the fluid defined as $\nu = \mu / \rho$
μ_t	Turbulent eddy viscosity
ε	Eddy diffusivity
c_i	Local volumetric fraction of the ith particle type in slurry
c_j	Local volumetric fraction of the jth particle type in slurry
q	Volumetric flow of fluid per unit width of the sluice
u	Local settling velocity
u_i	Local settling velocity for the ith particle type
u_j	Local settling velocity for the jth particle type
u_x	Mean flow velocity in the horizontal (x) direction

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