Journal of Engg. Research Vol. 6 No. (3) September 2018 pp 170-188

Robust trajectory tracking control of robotic manipulators based on model-free PID-SMC approach

Tolgay Kara* and Ali Hussien Mary**

**University of Gaziantep Department of Electrical and Electronics Engineering, Gaziantep, Turkey*

***University of Baghdad Al-Khwarizmi College of Engineering, Baghdad, Iraq*

**Corresponding Author: kara@gantep.edu.tr*

ABSTRACT

In this paper, a robust controller based on proportional-derivative control and sliding mode control for trajectory tracking problem of a nonlinear robotic manipulator is presented. Actuator dynamics is taken into account in tracking control simulations for verification of good precision trajectory tracking. A low pass filter is employed for the elimination of chattering, high frequency components, and noises. The proposed control scheme combines the simplicity feature of the proportional-integral-derivative (PID) controller and the robustness feature of the sliding mode control (SMC). There is no need to know the dynamic model of controlled systems, unlike most robust controllers, and only the upper bound of the dynamic system is required in the proposed method. Lyapunov's stability method is used to prove robustness of the proposed controller for the robot manipulator subjected to system uncertainties and external disturbance. The performance of the proposed controller is simulated by MATLAB-Simulink environment and is compared with other control schemes to verify its efficiency with various control methods commonly preferred in robotic manipulators. Robustness tests of the proposed controller against uncertainties in robot and actuator dynamics and external disturbance are illustrated.

Keywords: proportional-integral-derivative control; robotic manipulator; robust control; sliding mode control.

INTRODUCTION

Robotic manipulators are widely used in different aspects of life, such as manufacturing, medical fields, industrial applications, nuclear plants, and space applications (Sharma et al., 2015; Ghosh et al., 2015). Accurate positioning is an important desired feature of robotic manipulators, and controlling these complicated systems with their high nonlinearity and coupling effect is a difficult task. Nevertheless, there has been a significant and fast progress in this field of control in the recent years, and different control strategies have been suggested to solve this problem (Sharma et al., 2015). A vast majority of these control schemes are model based, where the dynamic model of the controlled system must be known. On many occasions it is not easy to obtain a precise dynamic model of the robotic manipulator due to the presence of structured and unstructured uncertainties and external disturbances. Therefore, many control schemes have been presented to deal with unknown dynamic models of robotic manipulators. Adaptive control is an efficient scheme used in this case, although it may pose assumptions such as linear parameterization, or parameters of the robotic manipulator being constant or slowly varying (Ho et al., 2007). Sliding mode control

(SMC) is one of the efficient robust control schemes with the property of invariance to model uncertainties and external disturbances. To ensure stability, SMC requires a priori knowledge about the system's dynamic model and bound of uncertainty. From a practical point of view, it is difficult to obtain system dynamics for a robotic manipulator. Moreover, SMC may produce large control chattering, which is undesirable in practical applications for it could damage the mechanical components of a system (Fallaha et al., 2011; Islam & Liu, 2011). Different strategies for solving the chattering problem have been presented, mostly based on saturation function (Chern & Wu, 1993), fuzzy sliding mode (Roopaei & Jahromi, 2009), and the usage of a boundary layer around the switching surface (Slotine & Li, 1991). Additionally, several methods in the literature use a higher order sliding mode controller for chattering reduction (Levant, 2007; Bartolini et al., 2000; Laghrouche, et al., 2007; Plestan et al., 2008). Most of these proposed methods are insufficient in eliminating high frequencies in noisy environments or the noises caused by measurement devices. Different intelligent techniques presented for estimating the dynamic model of robotic manipulator are available. The neural network is a method widely used for modelling the manipulator dynamics with its ability to approximate the nonlinear functions (Liu et al., 2014; Abdollahi et al., 2006; Ye, 2014; Hoseini et al., 2008; Dehghan et al., 2015). Most of these model based methods require complicated calculations (Liu et al., 2014). Despite the progress in control engineering and computational intelligence, proportional-integral-derivative (PID) controller is still the first choice in industrial control applications. This is not only due to its important features such as simplicity in structure, easy implementation, and low cost, but also due to its effectiveness in most industrial plants and ability in dealing with nonlinearities of system dynamics. Furthermore, it is very effective in achieving positioning goals, which makes it a commonly preferred control strategy in robotic manipulator applications (Craig, 2005). Many modified methods for PID controller are proposed to improve its performance. Slotine and Li (1987) suggest using proportional-derivative (PD) control with computed feed forward, while Tarn, Bejczy, Marth, and Ramadorai (1993), as well as De and Banens (1994), employ PD control in computed torque control (CTC). In addition to the aforementioned approaches, different fractional PID controllers are also suggested for trajectory tracking control of robotic manipulators (Richa et al., 2014; Huang et al., 2012; Bingul & Karahan, 2012).

Simplicity of PID and robustness of SMC are two motivating factors to propose a hybrid controller that can overcome the problem of necessity to determine the dynamics of the controlled system and weakness of PID in controlling nonlinear systems such as a robotic manipulator. On the other hand, the chattering problem is tackled by considering the dynamics of the actuator, which, in practice, is where chattering affects the system, and it is ignored in many papers published in this field to simplify the analysis (Liu & Zhang, 2013). As a solution to the chattering problem, a low pass filter (LPF) is used to cancel the chattering that occurs due to external disturbance and uncertainties and eliminate the noisy signals that come from measurement devices and, specifically, from derivative signals. This filtering method is advantageous as control accuracy can be maintained by using LPF, while the boundary layer that cancels only high frequencies in control signal reduces the control accuracy (Tseng and Chen, 2010). Lyapunov's second method is used to select the controller parameters that ensure stability of the proposed method with bounded reference trajectory. Three aspects of the proposed method are addressed in this paper. The first one is suggesting a robust control scheme that can be obtained without a priori knowledge about the dynamic model of the controlled system. The second aspect is the actuator dynamics being taken into account for better accuracy and practicality of simulation results. The third one refers to eliminating the chattering by the low pass filter, which also cancels the high frequency noises caused by differentiation of measurement signals in real systems. The proposed control method is applied to a three-link rigid robotic manipulator through simulations and compared with standard SMC and CTC. Simulation results with comparative analyses illustrate the effectiveness of the proposed method and its superiority to other methods under different cases of model uncertainty, load disturbance, payload variations, and external noise. It is worth mentioning that most common methods for reducing chattering are based on boundary layer, which decreases the accuracy performance since these methods enforce the states to move towards a boundary layer instead of the sliding surface. In this paper, using the sign function keeps the principle of standard SMC that enforces states to move towards the sliding surface, and the chattering problem is solved as mentioned above.

This paper is organized as follows. Section 2 presents preliminaries and robotic manipulator's dynamic model with its main characteristics. Section 3 presents the proposed method and stability analysis, which is followed by the simulation tests and obtained results in Section 4. Section 5 presents conclusions of the study.

MANIPULATOR DYNAMICS

Dynamics of an *n* link rigid robotic manipulator are expressed as follows:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) = \tau \tag{1}$$

where $q \in R^n$ is the joint angular position vector and *n* is the number of joints, $\tau \in R^n$ is the torque vector, $M(q) \in R^{n\times n}$ is the inertia matrix as a function of q, q, $C(q,\dot{q}) \in R^{n\times n}$ is the centrifugal matrix, $G(q) \in R^n$ is the gravity vector, and $F(\dot{q}) \in R^n$ is the frictional force vector. The dynamic model of the armature controlled motor used in robotic manipulator systems can be expressed as follows:

$$\tau_m = J_m\ddot{q}_m + D_m\dot{q}_m + \tau_l \tag{2}$$

$$\tau_m = k_\tau u_f \tag{3}$$

where $\tau_m \in R^n$ is the electromagnetic torque vector, $q_m \in R^n$ is the vector of motor angular position, $\tau_l \in R^n$ refers to the load torque at the motor shaft, $J_m \in R^{n\times n}$ is the moment of inertia diagonal matrix, $D_m \in R^{n\times n}$ denotes the diagonal matrix of torsional damping coefficients, $u_f \in R^n$ is the armature input voltage vector, and $k_\tau \in R^{n\times n}$ refers to the torque constants diagonal matrix. The relation between the position of the motor shaft and the position of the joint can be expressed as follows:

$$N = \frac{q_m}{q} = \frac{\tau}{\tau_l} \tag{4}$$

where $N \in R^{n\times n}$ refers to the gear ratios diagonal matrix with *n* joints. As a result, the dynamic model of the robotic manipulator including dynamic of the actuator is

$$M_H(q)\ddot{q} + C_H(q,\dot{q})\dot{q} + F_H(\dot{q}) + G_H(q) = u_f \tag{5}$$

where $M_H = K_\tau^{-1}(N^{-1}M + NJ_m)$, $C_H = K_\tau^{-1}N^{-1}\,C$, $F_H = K_\tau^{-1}(N^{-1}F + ND_m)$, and $G_H = K_\tau^{-1}N^{-1}\,G$.

PROPOSED METHOD

The main objectives of proposed approach can be summarized as follows: i) Designing a robust controller without the need to determine the dynamic model of the robot manipulator unlike conventional SMC; ii) eliminating the chattering caused by discontinues term in SMC; iii) rejecting high frequency noises caused by differentiation of signals; and iv) ensuring stability of the proposed control scheme, taking into account all dynamic terms of real robotic manipulators including actuator and low pass filter dynamics. The block diagram of the proposed control scheme is shown in Figure 1.

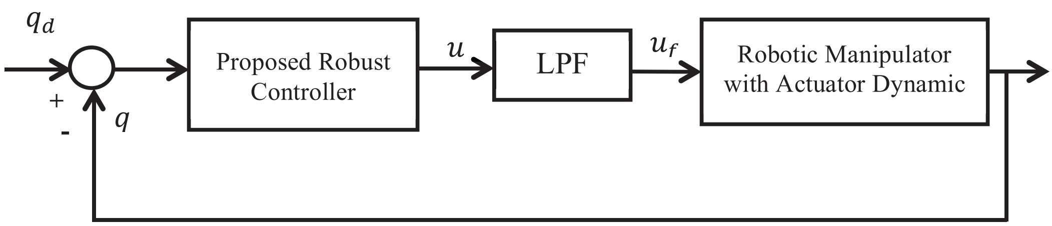

Fig 1. Block diagram of proposed control scheme for three link robotic manipulator.

The control design for the robotic manipulator relies upon a number of properties that the manipulator is known or assumed to possess. These properties are related to the upper and lower bounds of uncertainties for some positive β_i, $i = 1,\cdots,6,$ as given below.

Property 1: M_H is a positive definite matrix, and it is bounded as shown below:

$$\beta_0 < \lambda_m(M(q)) \le \|M(q\,)\| \le \lambda_M(M(q)) < \beta_1 \tag{6}$$

$\lambda_m(M(q))$ and $\lambda_M(M(q)$ are the minimum and maximum eigenvalues of $M(q)$, respectively.

Property 2: The matrix $\dot{M}(q) - 2C(q,\dot{q}\,)$ is skew–symmetric matrix, that is,

$$X^T\left[\dot{M}(q) - 2C(q,\dot{q}\,)\right]X = 0,\ X \in R^n \tag{7}$$

Property 3: $$\|C_H(q,\dot{q}\,)\dot{q}\| \le \beta_2\|\dot{q}\| \tag{8}$$

Property 4: $$\|F_H(\dot{q})\| \leq \beta_3\|\dot{q}\| + \beta_4 \tag{9}$$

Property 5: $$\|G_H(q)\| \leq \beta_5 \tag{10}$$

Property 6: $$\|\tau_d\| \leq \beta_6 \tag{11}$$

It is depicted in Figure 1 that the real control input is u_f, which represents the control signal u after filtering. The transfer function of the low pass filter is

$$G(s) = \frac{\gamma_i}{s+\gamma_i} \tag{12}$$

where γ_i is a positive scalar. According to the transfer function of the low pass filter, the relation between u_f and u is

$$\dot{u}_f + \gamma\, u_f = \gamma\, u \tag{13}$$

where $\gamma = diag\ (\gamma_1, \gamma_2, \dots, \gamma_n)$. $\dot{u}_f$ can be obtained by taking the derivative of u_f in (5):

$$\dot{u}_f = \dot{M}_H(q)\ddot{q} + M_H(q)\dddot{q} + \dot{C}_H(q,\dot{q})\dot{q} + C_H(q,\dot{q})\ddot{q} + \dot{F}_H(\dot{q}) + \dot{G}_H(q) \tag{14}$$

Then, substituting (14) and (5) into (13) instead of $\dot{u}_f$ and u_f, respectively, yields

$$\left[\dot{M}_H(q)\ddot{q} + M_H(q)\dddot{q} + \dot{C}_H(q,\dot{q})\dot{q} + C_H(q,\dot{q})\ddot{q} + \dot{F}_H(\dot{q}) + \dot{G}_H(q)\right]$$
$$+\gamma\ [M_H(q)\ddot{q} + C_H(q,\dot{q})\dot{q} + F_H(\dot{q}) + G_H(q)] = \gamma u \tag{15}$$

Then, after simple rearrangement, (15) becomes

$$M_H(q)\dddot{q} + C_H(q,\dot{q})\ddot{q} + \left[\dot{M}_H(q) + \gamma\, M_H(q)\right]\ddot{q} + \left[\dot{C}_H(q,\dot{q}) + \gamma C_H(q,\dot{q})\right]\dot{q}$$
$$+\dot{F}_H(\dot{q}) + \gamma\, F_H(\dot{q}) + \dot{G}_H(q) + \gamma\, G_H(q) = \gamma u \tag{16}$$

$$M_H\dddot{q} + C_H(q,\dot{q})\ddot{q} + H_1\ddot{q} + H_2\dot{q} + H_3 = \gamma u \tag{17}$$

where

$$H_1 = \dot{M}_H(q) + \gamma\, M_H(q) \tag{18}$$

$$H_2 = \dot{C}_H(q,\dot{q}) + \gamma C_H(q,\dot{q}) \tag{19}$$

$$H_3 = \dot{F}_H(\dot{q}) + \gamma\, F_H(\dot{q}) + \dot{G}_H(q) + \gamma\, G_H(q) \tag{20}$$

Remark 1: The control performance is affected by the dynamics of the low pass filter. In practical implementations, an appropriate value for γ can be selected by a trial and error method, choosing a small value and gradually increasing it until the required noise rejection performance is achieved.

Since the robotic manipulator is linearly parameterized (Vega *et al.,* 2003), its dynamics can be represented by the product of a known regressor matrix $Y(t, q, \dot{q}, \ddot{q}, q_d, \dot{q}_d, \ddot{q}_d) \in R^{n\times p}$ with a vector $a \in R^p$ in terms of a nominal reference $\ddot{q}_r$, where p refers to the number of unknown parameters. Matrix Y is based only on desired and actual trajectories, while vector a contains the unknown manipulator parameters.

$$M\,\ddot{q}_r + C_H\,\ddot{q}_r + H_1\ddot{q} + H_2\dot{q} + H_3 = Ya \tag{21}$$

$$\ddot{q}_r = \ddot{q}_d + A_1\dot{e} + A_2 e \tag{22}$$

where $A_1 \in R^{n\times n}$ and $A_2 \in R^{n\times n}$ are positive definite diagonal matrices.

Remark 2: Based on the result obtained by Vega et al. (2003), for a bounded desired trajectory and robotic manipulator properties (6-11), there exists a state-dependent vector $\rho(t) \in R^n$ such that $Ya < \rho(t)$, where $\rho(t)$ refers to the upper bound of the manipulator dynamics.

Proposed Control Law

Let the position tracking error $e(t) \in R^n$ be defined as

$$e(t) = q_d(t) - q(t) \tag{23}$$

where $q_d(t) \in R^n$ is the desired position signal. According to the SMC principles in selecting the sliding surface that is expressed below for an n^{th} order system,

$$S(t) = \left(\frac{d}{dt} + \sigma\right)^{n-1} e(t), \sigma > 0 \tag{24}$$

for the robotic manipulator with low pass filter, the sliding surface is

$$S(t) = A_1 e(t) + A_2\dot{e}(t) + \ddot{e}(t) \tag{25}$$

The proposed control law consists of pseudo-equivalent control term and robust control term,

$$u(t) = u_{peq}(t) + u_{ro}(t) \tag{26}$$

$$u_{peq}(t) = k_{P1}\, e(t) + k_{d1}\dot{e}(t) + k_{d2}\ddot{e}(t) \tag{27}$$

$$u_{ro}(t) = k_I \int_0^t s(\delta)d\delta + k\, sgn(s) \tag{28}$$

where k_{P1}, k_{d1}, k_{d2}, k_I, and k are constant diagonal matrices. It should be mentioned here that the pseudo-equivalent control term in (26) as described in (27) is not of the same nature as the equivalent control term in standard SMC, as discussed in Remark 4.

Remark 3: In real applications, most measurement devices cause a noise signal, and most papers focusing on reducing chattering ignore the high frequency components that come from measurement devices. This motivated us to use LPF to cancel all high frequencies.

Remark 4: The dynamic model of the manipulator is not included in the proposed control law in (26), which is based only on the tracking errors and their derivatives. This means that the proposed control scheme is model-free, unlike standard SMC, in which a nominal model is needed to determine the equivalent control.

Remark 5: The proposed control law combines the features of PID controller and SMC. The PD controller is used to stabilize the robotic manipulator system, while the SMC compensates for the uncertainties and external disturbance while improving the trajectory control performance as well. With this structure, the simplicity of the proposed control makes it easy to be implemented in real applications.

Theorem: For nonlinear robotic manipulator systems including the actuator dynamics in (5) with the low pass filter in (12) and sliding surface in (25), if the proposed control law presented by (26) is used, then the closed loop system is guaranteed to be asymptotically stable in tracking desired trajectories $q_d(t)$ provided that the controller parameters are selected based on the following conditions:

$$k_I > 0 \tag{29}$$

$$k_{d2} > 0 \tag{30}$$

$$k_p = k_{d2} A_1 \tag{31}$$

$$k_{d1} = k_{d2}A_2 \quad (32)$$

$$\gamma k > \|\rho(t)\| \quad (33)$$

These conditions are obtained when Lyapunov's theory is used to guarantee the stability of the proposed control scheme, which is discussed in detail in the following proof.

Proof: Lyapunov's theory is used to ensure the stability of the controlled system. Lyapunov's function candidate is

$$V(t) = \frac{1}{2}S^T M_H S + \frac{1}{2}(\int_0^t Sdt)^T k_I \int_0^t Sdt \quad (34)$$

Differentiation of (34) yields

$$\dot{V}(t) = S^T \left[\frac{1}{2}\dot{M}_H S + k_I \int_0^t Sdt\right] \quad (35)$$

$$\dot{V}(t) = S^T \left[M_H \dot{S} + C_H S + k_I \int_0^t sdt\right] \quad (36)$$

$$\dot{S} = \dddot{e} + A_2 \ddot{e} + A_1 \dot{e} \quad (37)$$

$$\dot{V}(t) = S^T \left[M_H(\dddot{q}_r - \dddot{q}) + C(\ddot{q}_r - \ddot{q}) + k_I \int_0^t Sdt\right] \quad (38)$$

$$\dot{V}(t) = S^T \left[M_H \dddot{q}_r + C_H \ddot{q}_r - M_H \dddot{q} - C_H \ddot{q} + k_I \int_0^t Sdt\right] \quad (39)$$

Combining (17) with (36) the following equation is obtained:

$$\dot{V}(t) = S^T \left[M_H \dddot{q}_r + C_H \ddot{q}_r + H_1 \ddot{q} + H_2 \dot{q} + H_3 - \gamma u + \gamma\, k_I \int_0^t sdt\right] \quad (40)$$

Substituting (14) in (40),

$$\dot{V}(t) = S^T \left[Ya - \gamma[k_P\, e(t) + k_{d1}\dot{e}(t) + k_{d2}\, \ddot{e}(t) + k_I \int_0^t sdt + k\, sgn(S)] + \gamma\, k_I \int_0^t Sdt\right] \quad (41)$$

$$\dot{V}(t) = S^T\left[Y\, a - \gamma k_{d2}\left[k_{d2}^{-1}\, k_p\, e(t) + k_{d2}^{-1} k_{d1}\dot{e}(t) + \ddot{e}(t)\right] - \gamma k\, sgn(S)\right] \quad (42)$$

If we select

$$k_p = k_{d2}A_1 \text{ and } k_{d1} = k_{d2}A_2 \quad (43)$$

$$\dot{V}(t) = S^T[Y\, a - \gamma k_{d2}[A_1\, e(t) + A_2\dot{e}(t) + \ddot{e}(t)] - \gamma k\, sgn(S)] \quad (44)$$

$$\dot{V}(t) = S^T[Y\, a - \gamma k_{d2}\, S - \gamma k\, sgn(S)] \quad (45)$$

$$\dot{V}(t) = S^T[Y\, a - \gamma k_{d2}S - \gamma k\, sgn(S)] \quad (46)$$

$$\dot{V}(t) = S^T[-\gamma k_{d2}S - \gamma k\, sgn(S) + Y\, a] \quad (47)$$

$$\dot{V} \leq -\gamma k_{d2}\|S\|^2 - \|S\|(\gamma k - \|\rho(t)\|) \quad (48)$$

If γk is selected to be large enough such that $\gamma k > \|\rho(t)\|$, then

$$\dot{V} \leq -S^T k_{d2}S \quad (49)$$

$$\dot{V}(t) \leq -S^T k_{d2}S \leq 0 \quad (50)$$

$$\dot{V} \leq 0 \quad (51)$$

According to Lyapunov's stability theory, the robotic manipulator system in (1) controlled by the proposed controller in (19-25) is globally asymptotically stable, and also the tracking error and its derivative converge to zeros. □

SIMULATION TESTS

The design procedure for the proposed controller consists of determining a sliding surface $S(t)$ using (25), finding pseudo-equivalent control term in (27) based on conditions in (29-31), and using (28) to calculate the robust control term based on the condition in (32).

This section presents the simulation results of the proposed control scheme applied on a three-link SCARA robot manipulator with the following dynamic model (Schilling, 2003; Wai *et al.,* 2003):

$$\begin{bmatrix}\tau_1\\\tau_2\\\tau_3\end{bmatrix}=\begin{bmatrix}D_{11} & D_{12} & 0\\ D_{12} & D_{22} & 0\\ 0 & 0 & D_{33}\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\\ddot{q}_2\\\ddot{q}_3\end{bmatrix}+\begin{bmatrix}C_{11} & C_{12} & 0\\ C_{21} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\dot{q}_1\\\dot{q}_2\\\dot{q}_3\end{bmatrix}$$

$$+\begin{bmatrix}v_1\dot{q}_1\\v_2\dot{q}_2\\v_3\dot{q}_3\end{bmatrix}+\begin{bmatrix}p_1 sgn(\dot{q}_1)\\p_2 sgn(\dot{q}_2)\\p_3 sgn(\dot{q}_3)\end{bmatrix}+\begin{bmatrix}G_1\\G_2\\G_3\end{bmatrix}+\begin{bmatrix}D_1\\D_2\\D_3\end{bmatrix} \quad (52)$$

$$D_{11}=\frac{1}{3}l_1^2m_1+\left[l_1^2+l_1l_2\cos(q_2)+\frac{1}{3}l_2^2\right]m_2+(l_1^2+2l_1l_2+l_2^2)m_3 \quad (53)$$

$$D_{12}=-\left(\frac{1}{2}l_1l_2\cos(q_2)+\frac{1}{3}l_2^2\right)m_2-(l_1l_2+l_2^2)m_3 \quad (54)$$

$$D_{22}=\frac{1}{3}l_2^2m_2+l_2^2m_3 \quad (55)$$

$$C_{11}=-l_1l_2\sin(q_2)\,\dot{q}_2m_2-2l_1l_2\sin(q_2)\,\dot{q}_2m_3 \quad (56)$$

$$C_{12}=-\frac{1}{2}l_1l_2\sin(q_2)\,\dot{q}_2m_2-l_1l_2\sin(q_2)\,\dot{q}_2m_3 \quad (57)$$

$$C_{21}=-\frac{1}{2}l_1l_2\sin(q_2)\,\dot{q}_1m_2-l_1l_2\sin(q_2)\,\dot{q}_1m_3 \quad (58)$$

where q_1, q_2 and q_3 are angular positions, τ_1, τ_2 and τ_3 are torques, l_1, l_2 and l_3 are lengths, m_1, m_2 and m_3 are masses, v_1, v_2 and v_3 are coefficients of viscous friction, and p_1, p_2 and p_3 are coefficients of dynamic friction of Link 1, Link 2, and Link 3, respectively. The parameters of the robot manipulator are selected as follows (Wai *et al.,* 2004): $m_1=1\,kg$, $m_2=0.8\,kg$, $m_3=0.5\,kg$, $P_1=P_2=P_3=12$, $V_1=V_2=V_3=0.2$.

The parameters of the actuator dynamics used in this simulation are $J_m=diag(0.67\times10^{-4},\ 0.42\times10^{-4},\ 0.67\times10^{-4})$, $D_m=diag(0.21, 0.15, 0.21)$, and $N=diag(9,8,1)$, $k_\tau=diag\left(\frac{19}{40},\frac{19}{80},\frac{19}{80}\right)$. The control objective is to control the joint angles of the three-link SCARA robot manipulator to track desired signals of $q_d(t)=[q_{d1}\ \ q_{d2}\ \ q_{d3}]^T$, where

$$q_{d1}=0.3+0.1\sin(t)+0.3\sin(2t)+0.2\sin(3t) \quad (59)$$

$$q_{d2}=0.4+0.1\cos(t)+0.3\cos(3t)+0.2\cos(4t) \quad (60)$$

$$q_{d3}=0.1+0.1\sin(t)+0.2\cos(2t)+0.3\sin(3t) \quad (61)$$

Table 1. Controller parameters.

Method	Control law	Parameter	Value
Proposed	$u(t) = u_{peq}(t) + u_{robust}(t)$ $u_{peq}(t) = k_P\, e(t) + k_{d1}\dot{e}(t) + k_{d2}\ddot{e}(t)$ $u_{robust}(t) = ksgn(S) + k_I \int_0^t S(\delta)d\delta$ $s(t) = A_1 e(t) + A_2\dot{e}(t) + \ddot{e}(t)$	k_P	diag (50,50,50)
		k_{d1}	diag (50,50,50)
		k_{d2}	diag (10,10,10)
		k	diag (50,50,50)
		k_I	diag (100,100,100)
		A_1	diag (5,5,5)
		A_2	diag (5,5,5)
IOSMC	$u = \widehat{M}(q)\ddot{q}_r + \widehat{N}(q)\dot{q}_r + \widehat{G}(q) + \widehat{H}(q) + k_1 sat(s, \emptyset) + s$ $\dot{q}_r(q) = \dot{q}_d - c(q - q_d)$ $s(t) = ce(t) + \dot{e}(t)$	k_1	diag (25,25,25)
		$\emptyset$	0.05
		c	diag (10,10,10)
SMC	$u = M_0(q)\ddot{q}_r + N_0(q)\dot{q}_r + G_0(q) + H_0(q) + k_2 sat(s, \emptyset) + s$ $\dot{q}_r(q) = \dot{q}_d - c(q - q_d)$ $s(t) = ce(t) + \dot{e}(t)$	k_2	diag (30,30,30)
		$\emptyset$	0.05
		c	diag (10,10,10)
CTC	$\tau = M(q)\left[\ddot{q}_d + k_v\dot{e} + k_p e\right] + C(q, \dot{q})\dot{q} + G(q)$	k_v	diag (20,20,20)
		k_p	diag (100,100,100)

In order to demonstrate the effectiveness of the proposed scheme, it is compared with conventional CTC, conventional SMC, and SMC based on input-output stability (IOSMC) (Slotine & Li, 1987). Table 1 lists the control laws and values of the controller parameters of the proposed, IOSMC, SMC, and CTC methods. The integral of the absolute value of the error *(IAE)* and integral of the square of the control input *(ISV)* are used to evaluate the effectiveness of the control schemes:

$$IAE = \int_0^{tf} |e(t)|dt \tag{62}$$

$$ISV = \int_0^{tf} u^2(t)dt \tag{63}$$

Effectiveness and robustness of the proposed method is revealed by simulation tests conducted in a comparative fashion with SMC, IOSMC, and CTC in the case of varying masses and viscous frictions of links. Additionally, various disturbance signals are enforced at different time instances of simulation tests in the following manner:

1) At the beginning of simulation, a disturbance signal of (*t*) is enforced.
2) At second 1.5, an additional disturbance function of (3*t*) is imposed.
3) Two seconds after initiation, another addition to the disturbance signal given by the sinusoid of 4 sin 5(*t*) is embedded to yield an overall disturbance of 3 sin (*t*)+ 5 sin 5(3*t*)+ 4 sin(5*t*) .
4) At the 3rd second, the mass of each link is perturbed by a magnitude of 0.5 kg, and the dynamic and static frictions of each link are changed to 1.3 and 0.4, respectively.

In simulation tests, the initial values of all system variables, which are angular displacements of the three links, and their derivatives are assumed to be zero. The results of link trajectories, tracking errors, and control signals are displayed in a graphical form versus time, using a logarithmic time scale for better visualization of transient responses. This is preferred for the fact that significant deviations from the set values are observed specifically during early transient durations rather than steady state responses in simulation tests. From the simulation results shown in Figures 2 and 3, one can deduct that the CTC method is highly affected by the uncertainties and external disturbance, while the other methods (the proposed, SMC, and IOSMC) are more robust to these variations. However, these figures indicate clearly that the proposed method is better than CTC and SMC and slightly better than IOSMC, especially in terms of fast response when the tracking error signal of the proposed method for all three links converges to zero very fast with respect to the other robust methods SMC and IOSMC. Moreover, the proposed method is model-free, where there are no dynamic parameters in the control law, while the other methods are model-based. The control input voltage values for Link 1, Link 2, and Link 3 are shown in Figure 3. The figure indicates that the control efforts paid by all controllers are almost equal, except a short duration of time at the beginning. Tables 2 and 3 list the *IAE* and *ISV* values for the proposed control scheme as well as other methods. These indices are clear indications of superiority of the proposed control scheme in reducing the tracking error while reducing the control effort. This improved performance in all cases of parameter variations is observable with respect to SMC and IOSMC and is significantly better when compared to CTC. As a final remark, it should be noted that all simulation results indicate high robustness of the proposed scheme against model uncertainties with better accuracy than CTC and SMC and slightly better than IOSMC methods.

Table 2. *IAE* values of three links using four methods with standard simulation.

	Proposed	**IOSMC**	**SMC**	**CTC**
Link 1	0.0030	0.0032	0.0037	0.0060
Link 2	0.0023	0.0059	0.0068	0.0077
Link 3	0.0040	0.0041	0.0052	0.0058

Table 3. *ISV* values of three links using four methods with standard simulation.

	Proposed	**IOSMC**	**SMC**	**CTC**
Link 1	2.7351	2.7230	2.7170	2.7120
Link 2	2.2880	2.2900	2.2870	2.2530
Link 3	12.133	11.866	11.878	11.810

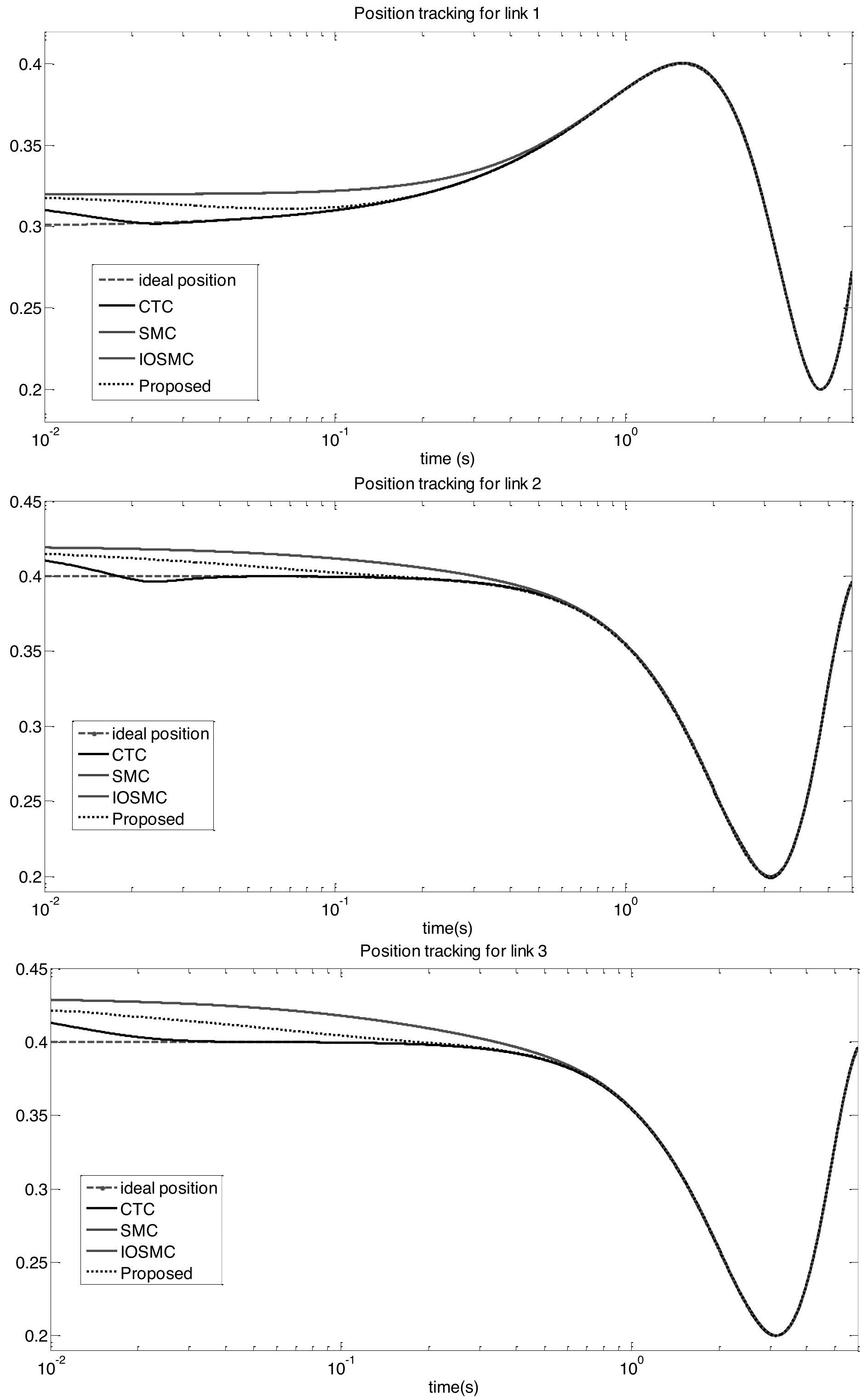

Fig. 2. Position tracking for Link 1, Link 2, and Link 3 in radians.

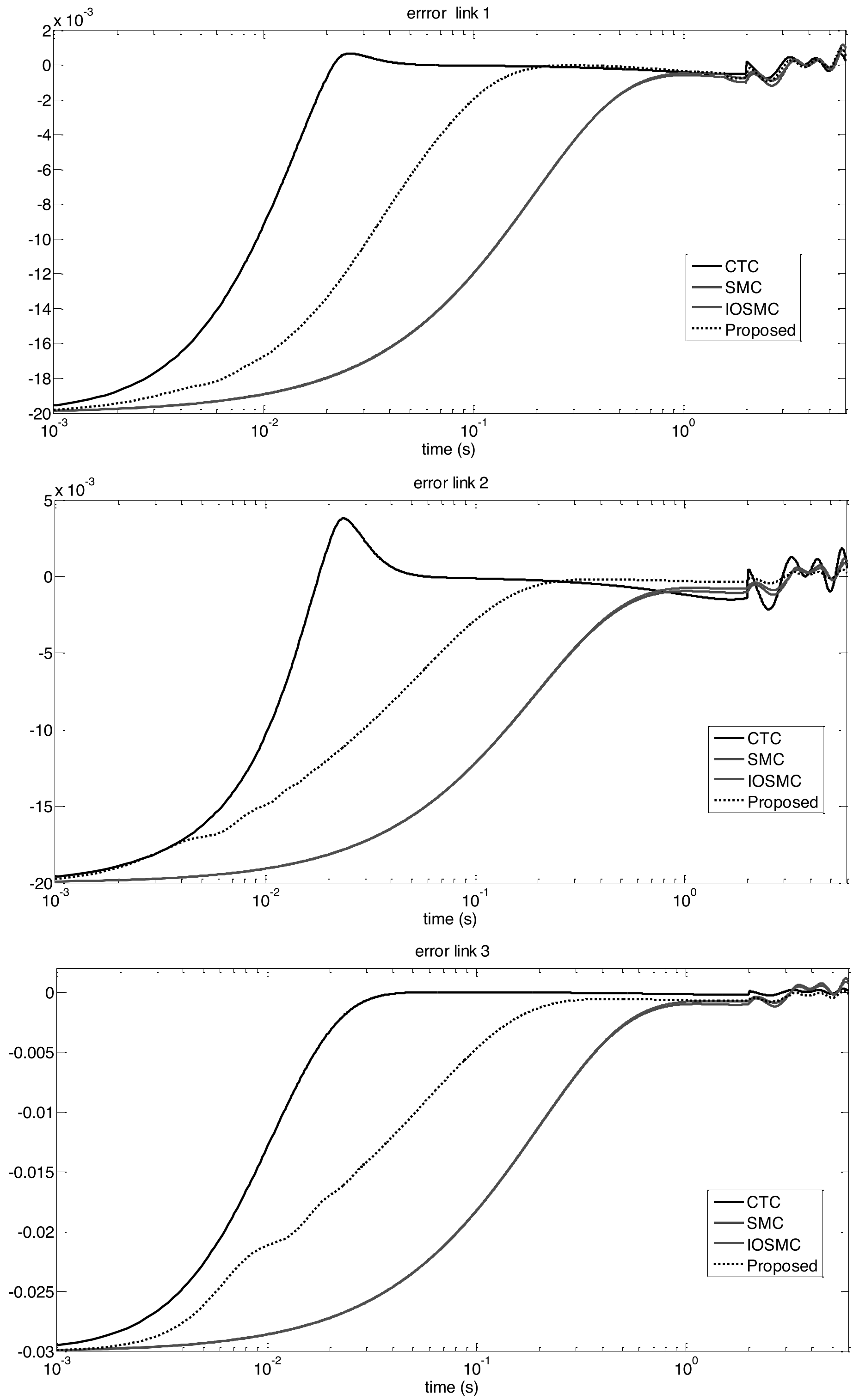

Fig. 3. Position tracking errors for Link 1, Link 2, and Link 3 in radians.

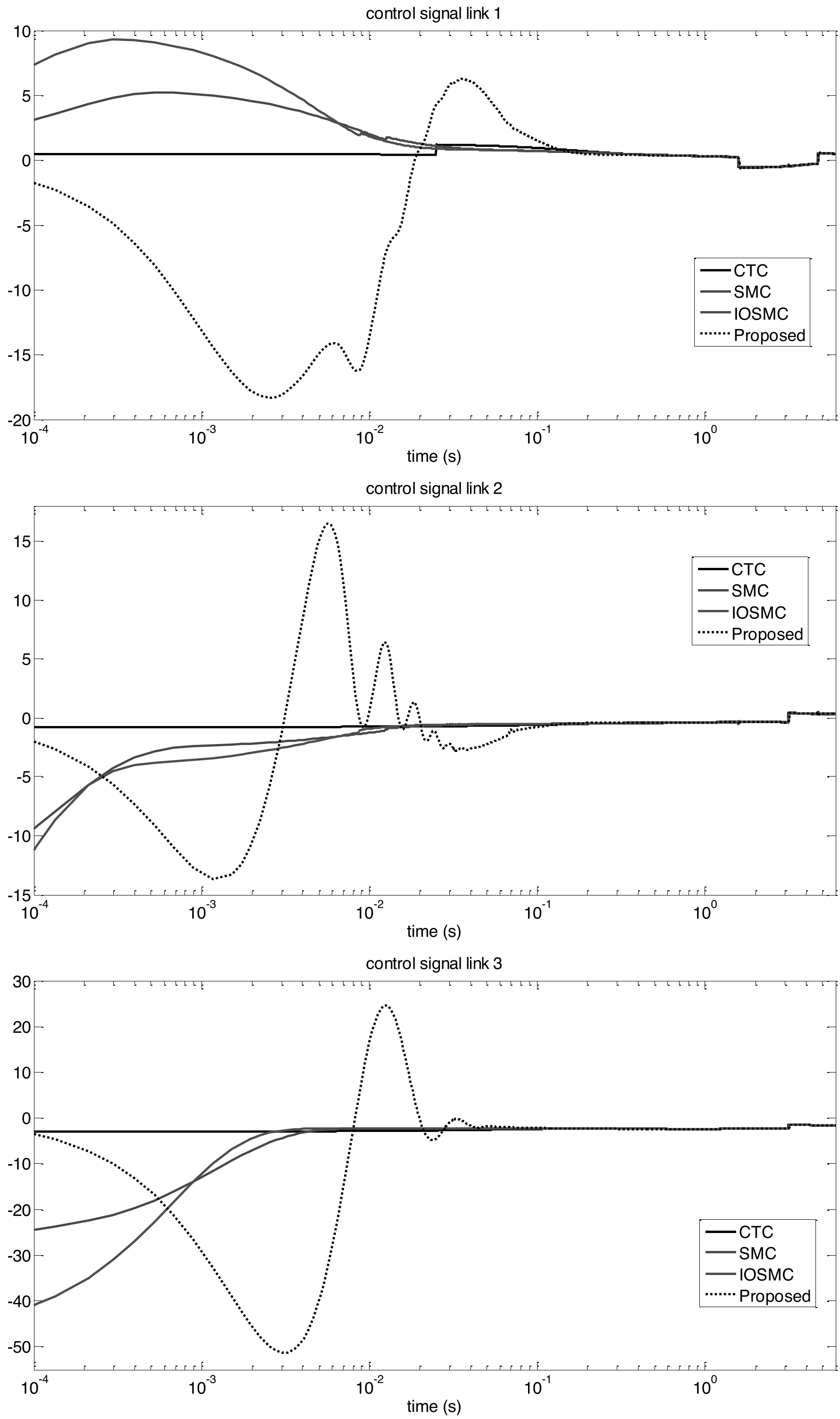

Fig. 4. Voltage inputs for Link 1, Link 2, and Link 3 in volts.

CONCLUSIONS

This paper presents a robust and adaptive model-free controller for trajectory tracking control of a robotic manipulator under model uncertainties and external disturbances. The proposed control scheme is based on SMC and PID controllers. The first term in the proposed control law is used to reduce the tracking error, while the second term reduces the effect of parameter variations and external disturbances and makes the manipulator robust against system parameter variations and external disturbances. The performance of the proposed control method is presented in comparison with well-established control strategies of CTC, SMC, and IOSMC, and simulation tests are executed to verify the superiority of the proposed approach. Performance indices of *IAE* and *ISV* are utilised to give numerical indications of robustness of the proposed system to the adverse effects of model uncertainties and external disturbances. Simulation results demonstrate fast response, high accuracy, strong robustness, and effectiveness of the proposed control scheme compared to CTC, SMC, and IOSMC methods. Adding the simplicity and easy implementation properties, the proposed scheme is promising as a powerful alternative to current methods in trajectory tracking control of robotic manipulators.

REFERENCES

Abdollahi, F., Talebi, H. & Patel, R. 2006. A stable neural network-based observer with application to flexible-joint manipulators. IEEE Transactions on Neural Networks, 17: 118–129.

Amera, A.F., Sallamb, E.A. & Elawadyb, W. M. 2011. Adaptive fuzzy sliding mode control using supervisory fuzzy control for 3 DOF planar robot manipulators. Applied Soft Computing, 11:4943–4953.

Bartolini, G., Ferrara, A., Usai, E. & Utkin, V.I. 2000. On Multi-input Chattering-free Second-order Sliding Mode Control. IEEE Transactions on Automatic Control, 45:1711–1717.

Bingul, Z & Karahan, O. 2012. Fractional PID controllers tuned by evolutionary algorithms for robot trajectory control. TurkJ Electr Eng ComputSci, 20:1123–36.

Chern, T.L. & Wu, Y.C. 1993. Design of brushless DC position servo systems using integral variable structure approach. IEE Proc. Electr. Power Appl., 140:27–34.

Craig, J.J. 2005. Introduction to robotics: mechanics and control. Vol. 3, Pp. 48 - 70. Prentice Hall, Upper Saddle River.

De, J.B. & Banens, J. 1994. Experimental evaluations of robot controllers. Proceedings of the 33rd conference on decision and control, Lake Buena Vista, Fl, USA, 363–368.

Debbarma, S., Saikia L. & Sinha, N. 2014. Automatic generation control using two degree of freedom fractional order PID controller. Electr Power Energy Syst., 58:120–129.

Dehghan, S.A., Danesh, M., Sheikholeslam, F. & Zekri, M. 2015. Adaptive force–environment estimator for manipulators based on adaptive wavelet neural network. Applied Soft Computing, 28:527–540.

Eker, I. 2006. Sliding mode control with PID sliding surface and experimental application to an electromechanical plant. ISA Trans., 45:109–118.

Fallaha, C.J., Saad, M., Kanaan, H.Y. & Kamal, A. H. 2011. Sliding-mode robot control with exponential reaching law. IEEE Trans. Ind. Electron., 58:600–610.

Fateh, M. M. & Fateh, S. 2012. Decentralized direct adaptive fuzzy control of robots using voltage control strategy. Nonlinear Dyn., 70:1919–1930.

Ghosh, A., Krishan, T., Tejaswy, P., Mandal, A., Pradhan, J. & Ranasingh, S. 2014. Design and implementation of a 2-DOF PID compensation for magnetic levitation systems. ISA Transactions, 53:1216–1222.

Ghosh, B.B., Sarkar, B.K. & Saha, R. 2015. Real time performance analysis of different combinations of fuzzy–PID and bias controllers for a two degree of freedom electro hydraulic parallel manipulator. Robotics and Computer-Integrated Manufacturing, 34:62–69.

Ho, H.F., Wong, Y.K. & Rad, A.B. 2007. Robust fuzzy tracking control for robotic manipulators. Simul. Modell. Pract. Theory, 15:801–816.

Hoseini, S.M., Farrokhi, M. & Koshkouei, A.J. 2008. Robust adaptive control of uncertain non-linear systems using neural networks. International Journal of Control, 81:1319 - 1330 .

Huang, C.Q., Xie, L.F. & Liu, Y.L. 2012. PD plus error-dependent integral nonlinear controllers for robot manipulators with an uncertain Jacobian matrix. ISA Transactions, 51:792–800.

Islam, S. & Liu, X.P. 2011. Robust sliding mode control for robot manipulators. IEEE Trans. Ind. Electron., 58:2444–2453.

Laghrouche, S., Plestan, F. & Glumineau, A. 2007. Higher Order Sliding Mode Control Based on Integral Sliding Surface. Automatica, 43:531–537.

Levant, A. 2007. Principles of 2-sliding Mode Design. Automatica, 43:576–586.

Liu, F., Liang, L. & Gao, J. 2014. Fuzzy PID Control of Space Manipulator for Both Ground Alignment and Space Applications. International Journal of Automation and Computing, 11:353 - 360.

Liu, H. & Zhang, T. 2013. Neural network-based robust finite-time control for robotic manipulators considering actuator dynamics. Robotics and Computer-Integrated Manufacturing, 29(2), 301 - 308.

Plestan, F., Glumineau, A. & Laghrouche, S. 2008. A New Algorithm for High-order Sliding Mode Control. International Journal of Robust and Nonlinear Control, 18:441– 453.

Richa, S., Rana, P. & Vineet, K. 2014. Performance analysis of fractional order fuzzy PID controllers applied to a robotic manipulator. Expert Systems with Applications, 41:4274–4289.

Roopaei, M. & Jahromi, M.Z. 2009. Chattering-free fuzzy sliding mode control in MIMO uncertain systems. Nonlinear Analysis, 71:4430–4437.

Sahu, R.K., Panda, S. & Rout, U.K. 2013. DE optimized parallel 2-DOF PID controller for load frequency control of power system with governor dead-band nonlinearity. Electr Power Energy Syst., 49:19–33.

Schilling, R.J. 2003. Fundamentals of Robotics Analysis & Control. Prentice-Hall, New Delhi.

Sharma, R., Gaur, P. & Mittal, A.P. 2015. Performance analysis of two-degree of freedom fractional order PID controllers for robotic manipulator with payload. ISA Transaction, 58:279–291.

Slotine, J. J. & Li, W. 1991. Applied nonlinear control (Vol. 199, No. 1). Prentice-Hall, Englewood Cliffs, NJ.

Slotine, J.J. & Li, W. 1987. On the adaptive control of robot manipulators. The International Journal of Robotics Research, 6:49–59.

Tarn, T.J., Bejczy, A.K., Marth, G.T. & Ramadorai, A.K. 1993. Performance comparison of four manipulator servo schemes. IEEE Control Systems, 13:22–29.

Tseng, M.-L. & Chen, M. S. 2010. Chattering reduction of sliding mode control by low-pass filtering the control signal. Asian Journal of Control, 12(3):392–398.

Vega, V.P., Arimoto, S., Liu, Y., Hirzinger, G. & Akella, P. 2003. Dynamic Sliding PID Control for Tracking of Robot Manipulators: Theory and Experiments. IEEE Transactions on Robotics and Automation, 19:967–976.

Wai, R.J. 2007. Fuzzy sliding-mode control using adaptive tuning technique. IEEE Trans. Ind. Electron., 54:586–594.

Wai, R.J., Tu, C.Y. & Hsieh, K.Y. 2004. Adaptive tracking control of robot manipulator. International Journal of System Science, 35(11):615- 27.

Wang Y.N., Mai, T.L. & Mao, J. X. 2014. Adaptive motion/force control strategy for non-holonomic mobile manipulator robot using recurrent fuzzy wavelet neural networks. Engineering Applications of Artificial Intelligence, 34:137–153.

Ye, J. 2014. Compound control of a compound cosine function neural network and PD for manipulators. International Journal of Control, 87:2118–2129.

Submitted: 22/12/2016
Revised : 21/04/2017
Accepted : 24/09/2017

Journal of Engg. Research **Vol. 6 No. (3) September 2018 pp.** *188*

السيطرة المتينة لتتبع مسار أنظمة الروبوت اعتماداً على الطرق PID–SMC

تولكاي كارا* و علي حسين مري **

*جامعة غازي عنتاب، قسم الهندسة الكهربائية والالكترونية، غازي عنتاب، تركيا

** جامعة بغداد، كلية الخوارزمي للهندسة، بغداد، العراق

الخلاصة

يقدم هذا البحث مسيطر متين اعتماداً على المسيطر التقليدي PID والسيطرة ذات النمط الانزلاقي (SMC) لحل مشكلة تتبع المسار لأنظمة الروبوتات اللا خطية. وقد أُخذ في الاعتبار معادلات النموذج الرياضي للمحركات للحصول على دقة عالية في تتبع المسار. استُخدم في هذا البحث مرشح للترددات الواطئة لغرض التخلص من الترددات العالية وكذلك الضوضاء. الطريقة المقترحة تجمع بين صفات البساطة التي يتميز بها المسيطر التقليدي PID وكذلك سمة القوة والمتانة التي يتميز بها نظام SMC. وبخلاف معظم المسيطرات المتينة الاخرى، في هذا البحث لا توجد حاجة لمعرفة النموذج الرياضي لأنظمة الروبوت وإنما المهم هو معرفة الحد الأعلى للاضطرابات في النموذج الرياضي. استُخدمت نظرية لابنوف لإثبات استقراريه الطريقة المقترحة في حالة التعرض إلى ازعاجات خارجية وكذلك في حالة الاضطرابات في النموذج الرياضي. تمت مقارنة أداء الطريقة المقترحة مع طرق أخرى. ولقد اثبتت الاختبارات متانة الطريقة المقترحة بوضوح.