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# تحديد مصدر الاختلاف من عملية التصنيع متعددة المراحل المتميز من خلال التحليل وتيار الاختلاف: دراسة حالة في صناعة السيارات

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## الخلاصــة

إن مشكلة جودة المنتج هي مسألة حاسمة لعمليات التصنيع متعددة المراحل وخاصة فى خطوط الإنتاج المستمرحيث يتم قياس خصائص الجودة في نهاية عملية التصنيع وبالتالي فإنه من المهم خفض عملية الاختلاف من خلال تحديد مصادرها والقضاء على أسبابها. في هذا الصدد، هذا البحث يقدم طريقة جديد في تحديد مصدرالاختلاف في تصنيع متعدد المراحل من خلال طريقة فيشرالخطية وطريقة تيارالاختلاف. يستخدم التحليل المتميز الخطي لفصل اختلاف خصائص الجودة من خلال مراحل عمليات التصنيع المختلفة في حين يتم استخدام التيار من طريقة الاختلاف لنمذجة انتشار الاختلاف في عمليات التصنيع متعددة المراحل. وأخيرا، يتم تعيين الانحراف المتوقع في التحليل من أجل تحديد مصدر الاختلاف مع عرض حالة توضيحية. خلاصة البحث تبين بأن الطريقة المقترحة تحسن من تشخيص أخطاء خطوط الإنتاج مستمرة في عمليات التصنيع متعددة المراحل.

# Variation source identification of multistage manufacturing processes through discriminant analysis and stream of variation methodology: a case study in automotive industry

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## ABSTRACT

Product quality problem is a critical issue for multistage manufacturing processes, especially in continuous production lines whereby quality characteristics are measured at the end of the line. Therefore, it is important to reduce process variation by identifying its sources and eliminating its causes. In this regard, a novel approach, to identify the source of variation in multistage manufacturing processes through integration of the Fisher's linear discriminant analysis and the stream of variation methodology, is proposed. Linear discriminant analysis is used to separate the variation of quality characteristics through the different stages of the manufacturing processes while the stream of variation methodology is used for variation propagation modeling in multistage manufacturing processes. Finally, the future deviation is assigned into the analysis, in order to identify the source of variation. With an illustrative case study, it is concluded that the proposed approach improves fault diagnosis of continuous production lines in multistage manufacturing processes.

**Keywords:** Discriminant analysis; machining process; state space model; stream of variation; variation source identification.

## INTRODUCTION

Variations in product characteristics are one of the most important problems in quality control and improvement for multistage manufacturing processes (MMPs). Most of the developed quality control and improvement studies in MMPs can be classified into variation propagation modeling, monitoring and diagnosis, and quality-oriented design optimization. Jin & Shi (1999) proposed a state space model, based on fundamental physical laws that can be used as a physical model for modeling variation propagation.

This state space model that has been used to model variation transmission between stages in various multiple manufacturing processes is proposed by Xie *et al.* (2012) and Liu *et al.* (2009) for assembly processes and by Loose *et al.* (2007); Djurdjanovic & Ni (2003) and Abellan-Nebot *et al.* (2012) for machining process. In monitoring and diagnosis areas of quality improvement study, that constitutes the research area of this paper, the main technique that can be used for process monitoring and fault diagnosis of MMPs is statistical process control. Hawkins (1993) proposed a regression-adjusted chart for monitoring correlated quality characteristics. An exponential weighted moving average scheme has also been investigated by Xiang & Tsung (2008) that is a monitoring method for MMPs. Nevertheless, up to now, one of the main problems to applying these methods is how to realize the faulty stage. When a fault is detected at a stage, it might be due to a change in previous stages.

Variation source identification of quality characteristics at the production line is a critical issue, especially when it must continuously work. In large industries, most of the production lines and their subsets are continuously working. An example of these production lines is machining process and assembly stations of the automotive industry. Another technique that can be used for process monitoring and fault diagnosis is the root cause identification for MMPs. This technique can be classified into two categories: (1) statistical-estimation-based method; and (2) pattern-matching-based method. In the first method, mathematical models that link the system error and the system quality measurements together are treated as a linear mixed model. By this procedure, the ordinary least square technique can be used to estimate the process error and to provide confidence intervals. Zhou *et al.* (2004) employed a maximum likelihood estimator and Ding *et al.* (2005) compared different methods to estimate variances. In the pattern-matching-based methods, the linear pattern is defined between signatures of potential errors and symptoms of the present errors. Signatures can be defined based on the model, and symptoms can be extracted from measurement data. Several pattern matching studies that have been reported in literature are listed as Li *et al.* (2007); Huang & Kong (2008) and Zeng & Zhou (2008). Although extensive research work has been performed in order to identify the source of variation in MMPs, most of which has been focusing on fault diagnosis between stages. However, to improve product quality, it is desirable to develop a fault diagnosis methodology to identify the source of variation for continuous manufacturing processes. Most of current practices in industries are stage-by-stage inspections. If the quality characteristics are measured at the end of the production line, a specific study is needed to find the faulty stage. In this study, a novel approach is proposed to variation source identification in multistage manufacturing processes based on discriminant analysis. In this regard, the proposed method separates product quality measurements in multistage machining processes. Then, faulty stage is determined by allocating future deviation to its classified source.

An illustrative diagram that contains the research framework of quality control and improvement in multistage systems is shown in Figure 1. The proposed method is a monitoring and diagnosis based methodology, which proceeds to identify the source of variation based on statistical estimation methods. For more information about the basic framework of quality control and improvement research in multistage systems see Shi & Zhou (2009).

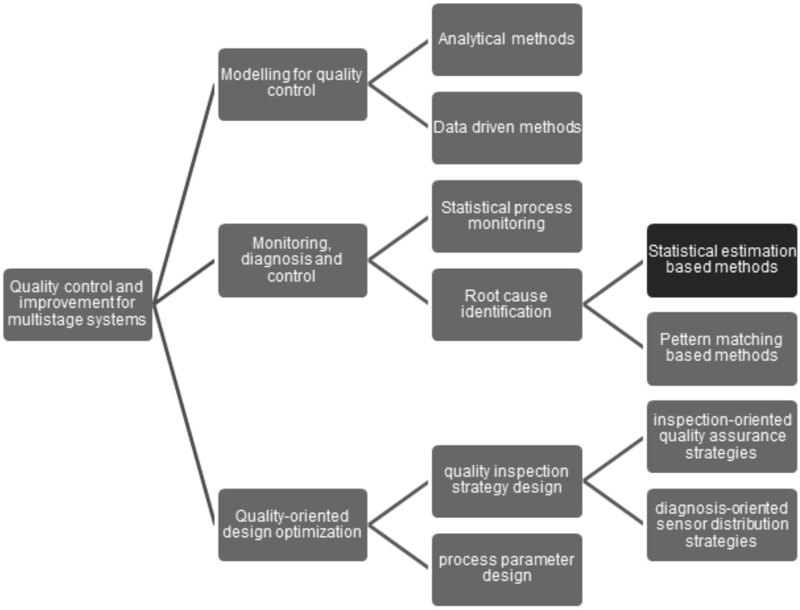


**Fig. 1.** Research framework of quality control and improvement in multistage systems

The remainder of this study is organized as follows. The stream of variation methodology was first explained. Then discriminant analysis for MMPs is discussed, where its function in the stream of variation methodology is introduced. Afterward, fault diagnosis of multistage manufacturing process is explained through discriminant analysis for deviation of product quality measurements. Finally, a case study is provided to demonstrate the applicability of proposed method and conclusions are subsequently presented.

## STREAM OF VARIATION METHODOLOGY

Most of modern manufacturing processes such as automotive, aerospace, appliance, aircraft and shipbuilding are multistage systems. These processes consist of multiple stages or stations that produce a final product. Each stage introduces variation and consequently this variation propagates through the production process. These variations not only are propagated along a series of stages but also are accumulated at the end of the stages. Hence, variation at the final stage could be affected by the fault on previous stages. This leads to a specific strategy, namely, stream of variation methodology for modeling, analysis, and control of product quality and productivity improvement in MMPs. This methodology integrates multivariate statistics, control theory, and manufacturing knowledge into a mathematical model to describe the propagation of quality information in MMPs. The most popular model used for transmitted variation between stages in a multistage system is the state space model as shown in Figure 2, first proposed by Jin & Shi (1999).

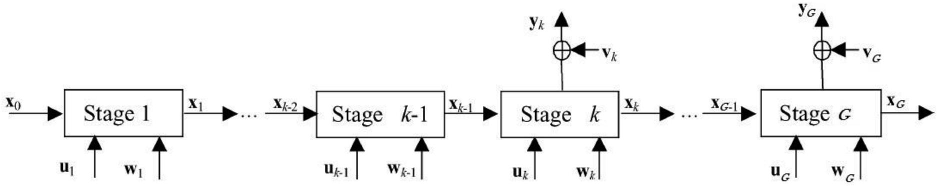


**Fig. 2.** Diagram of a *G*-stage process

This physical model can be set up in a *G*-stage process as the following two Equations:

$$\boldsymbol{x}_j = \boldsymbol{A}_{j-1}\boldsymbol{x}_{j-1} + \boldsymbol{B}_j\boldsymbol{u}_j + \boldsymbol{e}_j \tag{1}$$

$$\boldsymbol{y}_j = \boldsymbol{C}_j\boldsymbol{x}_j + \boldsymbol{v}_j$$

Where, $\boldsymbol{x}_j$ and $\boldsymbol{u}_j$ are the vectors representing the deviation of quality characteristics and process faults at station $j$, respectively and $j$ is the stage index, $j = 1,2, \dots, g$. The vector $\boldsymbol{x}_{j-1}$ denotes the deviation of quality characteristics which is accumulated in stages 1, 2, 3,…, $j$-1, where natural variation and un-modeled errors in the process are represented by vector $\boldsymbol{e}_j$ . It should be noted that the product quality measurements collected in $\boldsymbol{y}_j$ and $\boldsymbol{v}_j$ denotes a noise term due to measurement. $\boldsymbol{A}_{j-1}$ is a dynamic matrix that maps product quality variations between stages $j$ and $j$-1, and $\boldsymbol{B}_j$ is an input matrix for process errors at stage $j$. Also $\boldsymbol{C}_j$ is ameasurement matrix that maps quality characteristics to product quality measurements. Interested readers are referred to Shi (2007) and Ceglarek *et al.* (2004) for further discussion on the state space model and interpretation of system matrices. In this study, “process fault”, “process error”, and “variation source” can be interchangeably used with each other. Also the key quality characteristics of the product are dimensional deviation of key features and the process faults are fixture errors. The modeling of fixture-related variation propagation using the state space model in an assembly process was addressed by Jin & Shi (1999); Ding *et al.* (2000) and Camelio *et al.* (2003). Stream of variation modeling for multistage machining processes was also investigated by Djurdjanovic & Ni (2001); Huang *et al.* (2000) and Loose *et al.* (2007).

## PROPOSED APPROACH TO FAULT DIAGNOSIS FOR MMPS

In multistage systems, quality managers always explore techniques to achieve a distinction between variation sources. Because of existing correlations between stages, it is hard to access the analytical model that can separate the variation source of product characteristics. Statistical tools such as applied multivariate analysis can be used for variation source identification. Multi-group discriminant analysis is a popular technique that involves partitioning the variable space into two or more mutually exclusive regions. In this section, an approach is proposed based on multi-group discriminant analysis that estimates Fisher's linear discriminant function to provide the best separation between process errors of the stages. There are three assumptions in the proposed method: (1) data come from a multivariate normal distribution; (2) covariance matrices across groups are equal; and also (3) misclassification costs and prior probabilities are equal. Violation of this assumption affects both the classification results and the statistical significance tests in discriminant analysis. More information about discriminant analysis can be found on Sharma (1996). As introduced in the previous section, the state space model defined in Equation (1) can be converted to a linear model as:

$$\boldsymbol{y}_j = \sum_{i=1}^{j} \boldsymbol{C}_j \boldsymbol{\Phi}_{j,i} \boldsymbol{B}_i \boldsymbol{u}_i + \boldsymbol{C}_j \boldsymbol{\Phi}_{j,0} \boldsymbol{x}_0 + \boldsymbol{v}_j \quad ,\{j\} \subset \{1,2,\dots,g\} \tag{2}$$

Where, $\boldsymbol{\Phi}_{j,i}$ is the state transition matrix, $\boldsymbol{\Phi}_{j,i} = \boldsymbol{A}_{j-1}\boldsymbol{A}_{j-2} \dots \boldsymbol{A}_i$ for $j>i$, and $\boldsymbol{\Phi}_{j,j} = 1$ . Without loss of generality, the effect of initial error $\boldsymbol{x}_0$ is neglected (Liu, 2010). Equation (2) can be converted to an extended model taking into account the measurements on all the stages. It expresses quality vectors as linear combinations of fault patterns defined in matrix $\boldsymbol{\Gamma}$ as in the following Equations:

$$\boldsymbol{Y}^{\mathrm{T}} = \boldsymbol{\Gamma}\boldsymbol{U}^{\boldsymbol{T}} + \boldsymbol{V}^{\mathrm{T}} \tag{3}$$

$$\begin{pmatrix} \boldsymbol{y}_1^{\mathrm{T}} \\ \boldsymbol{y}_2^{\mathrm{T}} \\ \vdots \\ \boldsymbol{y}_{\mathrm{g}}^{\mathrm{T}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{C}_1\boldsymbol{B}_1 & \cdots & 0 \\ \boldsymbol{C}_2\boldsymbol{\Phi}_{2,1}\boldsymbol{B}_1 & & 0 \\ \vdots & \ddots & \vdots \\ \boldsymbol{C}_{\mathrm{g}}\boldsymbol{\Phi}_{g,1}\boldsymbol{B}_1 & \cdots & \boldsymbol{C}_{\mathrm{g}}\boldsymbol{B}_{\mathrm{g}} \end{pmatrix} \begin{pmatrix} \boldsymbol{u}_1^{\mathrm{T}} \\ \boldsymbol{u}_2^{\mathrm{T}} \\ \vdots \\ \boldsymbol{u}_{\mathrm{g}}^{\mathrm{T}} \end{pmatrix} + \begin{pmatrix} \boldsymbol{v}_1^{\mathrm{T}} \\ \boldsymbol{v}_2^{\mathrm{T}} \\ \vdots \\ \boldsymbol{v}_{\mathrm{g}}^{\mathrm{T}} \end{pmatrix} \tag{4}$$

Where, $\boldsymbol{Y}^{\mathrm{T}}$, $\boldsymbol{U}^{\boldsymbol{T}}$, and $\boldsymbol{\Gamma}$ can be derived according to Equation (2). Let $\boldsymbol{U}^{\boldsymbol{T}}$ be the fault vector that represents the process faults of *p* quality characteristics at *g* stages, whose covariance matrix is given by $\boldsymbol{\Sigma}$ and the total sum of squares and cross products (SSCP) matrix by $\boldsymbol{T}$. If $\boldsymbol{\alpha}$ be a $p\times 1$ vector of weights, then discriminant function for product quality measurements will be given by:

$$\boldsymbol{\xi} = \boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{y} \tag{5}$$

The maximum number of discriminant functions that can be computed is the minimum of $g$-1 or $p$, where $g$ is the number of stages and $p$ is the number of product characteristics. Since $\boldsymbol{T}=\boldsymbol{B}+\boldsymbol{W}$, where $\boldsymbol{B}$ and $\boldsymbol{W}$ are, respectively, the between groups and within group SSCP matrices for $p$ quality characteristics. The discriminant functions for the stream of variations that are linear combinations of product characteristics are estimated such that the following ratio for the discriminant scores is maximum:

$$\boldsymbol{\lambda} = \frac{\boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{\alpha}}{\boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{W}\boldsymbol{\alpha}} \tag{6}$$

The vector of weight can be obtained by differentiating $\boldsymbol{\lambda}$ with respect to $\boldsymbol{\alpha}$, and equating to zero. Where, $\boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{\alpha}$ and $\boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{W}\boldsymbol{\alpha}$ are, respectively, between-groups and within-group sum of squares for discriminant scores $\boldsymbol{\xi}$. Then the vector of weights $\boldsymbol{\alpha}$ that maximizes Equation (6) is given by $\boldsymbol{\alpha}_1 = \boldsymbol{s}_1$. If $\boldsymbol{y}^T = [y_1, y_2, \dots, y_p]^{\mathrm{T}}$ be a $p\times 1$ vector of product quality measurements for $p$ characteristics, then the linear combination $\boldsymbol{\alpha}_{\boldsymbol{1}}^{\boldsymbol{T}}\boldsymbol{y}$ will called the first discriminant function and continuing, $\boldsymbol{\alpha}_{\boldsymbol{j}}^{\boldsymbol{T}}\boldsymbol{y} = \boldsymbol{s}_{\boldsymbol{j}}^{\boldsymbol{T}}\boldsymbol{y}$ is the $j$ the discriminant function for product quality measurements which can be estimated in a linear relationship as follow:

$$\boldsymbol{z}_j = s_{j1}y_1 + s_{j2}y_2 + \cdots + s_{jp}y_p \quad ,j = 1, \dots, m \tag{7}$$

Therefore, $m$ is the maximum number of discriminant functions that separates the variation space into ($m$+1) regions. These regions represent the differences among the variation sources at stages geometrically. Adequacy of this representation can be assessed by statistical significant test of each discriminant function (Sharma, 1996). Also the test statistics for evaluating the statistical significance of the classification rate is proposed by Huberty (1984). In order to validate the discriminant function, the external validity needs to be examined. The case study provides results from the experimental validation of the proposed method. The final object of this study is to allocate future deviation of the product quality measurements at the end of the production line into one of the stages that is the variation source of process error. This can be done based on discriminant score of measured characteristics. First, the space of variation source is divided into mutually exclusive regions according to its eigenvalues, then, future deviation is allocated into a stage in which it falls. As mentioned in the previous section, all product quality measurements come from a multivariate normal distribution. Therefore, $\boldsymbol{y}_{\mathrm{j}}$ represents the $p$-variate deviation of product characteristics at stage $j$ and follows a $p$-variate normal distribution with mean $\boldsymbol{\mu}_{\boldsymbol{j}}$ and common covariance $\boldsymbol{\Sigma}$, i.e. $\boldsymbol{y}_{\mathrm{j}} \sim N_p(\boldsymbol{\mu}_{\boldsymbol{j}}, \boldsymbol{\Sigma})$. The joint density function of any stage $j$ for a $p\times 1$ vector of $\boldsymbol{y}$ is given by

$$f_j(\boldsymbol{y}) = (2\pi)^{-\frac{P}{2}}\left|\boldsymbol{\Sigma}\right|^{-\frac{1}{2}}e^{-\frac{1}{2}(\mathrm{y}-\boldsymbol{\mu}_j)^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\mathrm{y}-\boldsymbol{\mu}_j)} \tag{11}$$

Classification rule can be obtained by minimizing total cost of misclassification (TCM). Since the misclassification costs and priors probabilities are equal, it can be concluded that the vector $\boldsymbol{y}$ assigns to the region $j$ if

$$(\boldsymbol{\mu}_j - \boldsymbol{\mu}_i)'\boldsymbol{\Sigma}^{-1}\boldsymbol{y} \geq 0.5(\boldsymbol{\mu}_j - \boldsymbol{\mu}_i)^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}_j + \boldsymbol{\mu}_i) \text{ for all } j \neq i \quad (12)$$

Now it is possible to allocate the vector of measured deviation at the end of the production line into the source of variation. As technology for automatically continuous process becomes less expensive and more widely used by manufacturing industries such as automotive, aerospace, appliance and electronics, there are many applications in which this diagnostic method could be used to identify the source of variation. In the next section, the application of the proposed method is shown below using a case study.

## AN ILLUSTRATIVE CASE STUDY

In order to demonstrate the applicability of the proposed method, a case study that involves four-stage machining process is conducted at production line of the Nasir-Kyung Corporation;  an automotive part supplier of Saipa manufacturing group. The production of this corporation is the automobile engine connecting rod, which is shown with its key datum features in Figure 3.

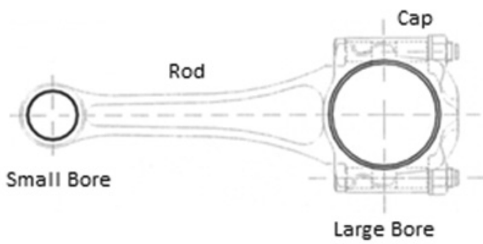


**Fig. 3.** The work piece: a Connecting Rod

The quality of the connecting rod considerably affects the power and the velocity of the automotive engine. The production line of this product is a U-shaped multistage machining process that works continuously. Therefore, it is not possible to take any quality measurement between stages. For this work piece, there are four variation sources considered at four machining processes. These machining processes are named as small bore machining, bolt seat machining, groove machining, and bolt-hole machining. The wok piece is clamped on the cube that is fixed by a fixture. The used fixture scheme in this case study is the 3-2-1 fixture scheme. Modeling of the variation propagation is focused on the one of the key features of the connecting rod. This key feature is small bore diameter that is also considered as the datum feature in the fixture setup. The nominal position and orientation of the datum feature w.r.t the reference coordinate system is listed in Table 1. In this case study, distance and angle are in unit millimeter and radian, respectively.

**Table 1.** Nominal position and orientation of the feature

| Feature Name | Euler Angle | Translational Vector | Importance | Spec. |
|---|---|---|---|---|
| Small Bore | [0, 0, 0] | [102, 98, 178] | A | 21.84~21.89 |

The origin of the reference coordinate is the intersection point between the centerline of small bore and the upper plane of the cube fixture. In the 3-2-1 fixture scheme, the work piece position is located by six locators. To develop the state space model, the nominal locations of the fixture locators are needed w.r.t the datum feature. These values are listed in Table 2.

**Table 2.** Nominal locations of the fixture locators

| Primary Datum | Secondary Datum | Tertiary Datum |
|---|---|---|
| [84, 90, 0] | [0, 78, 80] | [92, 0, 108] |
| [88, 180, 0] | [0, 88, 182] | |
| [184, 142, 0] | | |

The nominal fixture locations are given in the coordinate systems attached to the corresponding datum feature. With this known information, the state space model can be established to link the datum induced errors and fixture errors with the product quality measurements. For the datum feature, the relevant product characteristics are its orientation and the distance from the reference coordinate of the datum feature plane. The deviation of the orientation is denoted by $[\delta\phi, \delta\theta, \delta\psi]$ and the deviation of the distance is denoted by $[\delta x, \delta y, \delta z]$. The overall dimensional error of product characteristics is described by combining error sources together in a $6\times1$ vector. The model noise and measurement noise is neglected and the initial state vector sets as zero. For applying the proposed method, 40 work-in-process products in the off-line mode are randomly selected as the training data. In order to model variation propagation, the linear model is built based on the measured deviation of these products at the off-line mode, and stream of variation is modeled at four machining stages by the state space equations. Subsequently, discriminant functions are estimated based on product quality measurements. In order to identify the source of variation for future deviation, the quality characteristics of the faulty product are first measured at the end of the production line. Then, the measured deviations are allocated into the source of variation by classification rule. The deviation value, the predicted group membership, and the discriminant score of six product characteristics are listed in Table 3.

**Table 3.** Predicted group membership for future deviation

| characteristic | deviation | Predicted group membership | Discriminant score |
|---|---|---|---|
| $\delta x$ | -1.7600 | # 1 | -20.2675 |
| $\delta y$ | 1.8400 | # 1 | 3.6143 |
| $\delta z$ | 0.0000 | - | 0.0000 |
| $\delta\phi$ | 0.0000 | - | 0.0000 |
| $\delta\theta$ | 0.0000 | - | 0.0000 |
| $\delta\psi$ | 0.0200 | # 1 | 21.2146 |

It can be concluded that the deviation of the product characteristics belongs to the stage number One. Subsequently, small bore machining is the faulty stage. As the deviation is in the 3-D plane, three parameters are required to completely characterize the deviation of the final product. Those are deviation in $x$ and $y$ directions and the rotation along the $z$ axis. The faulty product at the end of the machining process is deviated -1.76 mm and 1.84 mm respectively in $x$ and $y$ direction and 0.02 radian in ψ orientation too. Figure 4 presents a view of the work piece deviating from its nominal position.

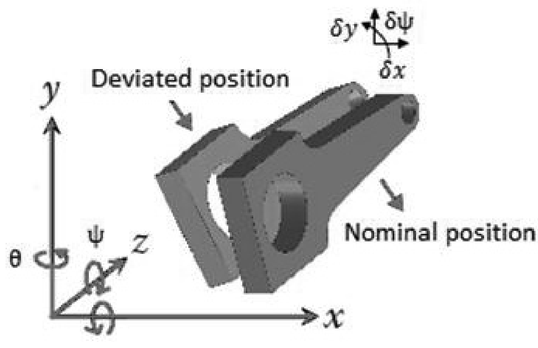


**Fig. 4.** The work piece deviating from its nominal position

Figure 5 shows a clear visualization for the deviations of characteristics w.r.t. each other in which the graphic views of this visualization are illustrated by contour plots. In Figure 5 (left), the values for $\delta x$ and $\delta y$ are represented on the x- and y-axes, while the values for the grouped variable are represented by shaded regions, called contours. The contour lines that form the boundaries of deviation regions connect points with equal values. This leads to range of deviations for stage four to vary between 0.03 mm to 0.09 mm and also from 0.12 mm to 0.14 mm on the x-axis and from 0 to - 0.04 mm on the y- axis. In Figure 5 (right), the values for $\delta x$, $\delta y$, and $\delta\psi$ are represented on three axes. It can be concluded that the highest deviation in the ψ orientation is found where $\delta x$ varies from 0.04 mm to 0.07 mm and also $\delta y$ varies from 0 to - 0.02 mm. As $\delta x$ and $\delta y$ move away to larger values, the values for $\delta\psi$ increase steadily.

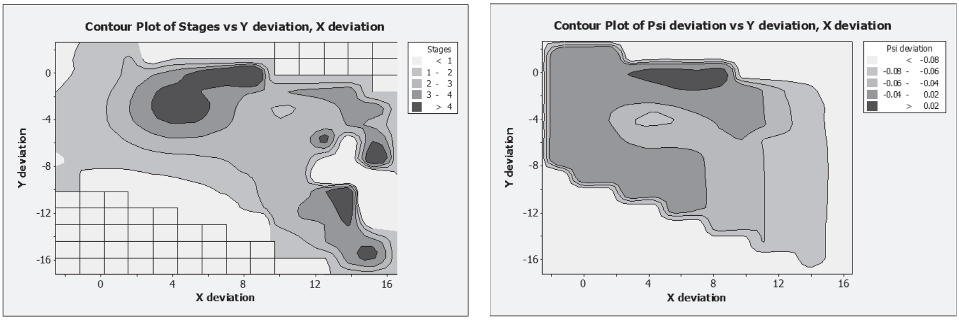


**Fig. 5.** Contour plot of X and Y deviations vs Stages (left) and Psi deviation (right)

To validate the proposed method, a work piece is machined in the test bed with current machining process under prior knowledge about the faulty stage (Shi, 2007). The training data that is obtained from work-in-process products at the off-line mode is provided the empirical range for fixture error in the *x* direction. Based on this information, proper value of error can be considered to add to the fixture locator in connecting rod machining process. The empirical range of fixture error for the connecting rod machining process is listed in Table 4.

**Table 4.** Empirical range of fixture error

| Small Bore machining | Bolt Seat machining | Groove machining | Bolt Hole machining |
|---|---|---|---|
| Stage 1 | Stage 2 | Stage 3 | Stage 4 |
| 0.10 ~ 0.90 | 0.16 ~ 0.50 | 0.60 ~ 0.70 | 0.76 ~ 0.99 |

In the machining, a fixture error is intentionally added to the process at stage one; the input to the model that correspond to the fixture error is $\boldsymbol{u}_{f,1} = [0\ 0\ 0\ 0.6\ 0\ 0]^T$. The nonzero value in the vector of process error expresses that the locator is deviated from its nominal position in the *x* direction by 0.6 mm. The work piece is measured at the end of the machining process and the deviation of product characteristics is obtained. Based on discriminant analysis for each characteristic, group membership of deviations is predicted. The deviation, the predicted group membership, and the discriminant score of product characteristics in the test bed are listed in Table 5.

**Table 5.** Predicted group membership for the intentionally added deviation

| characteristic | deviation | Predicted group membership | Discriminant score |
|---|---|---|---|
| $\delta x$ | -5.2800 | # 1 | -27.0800 |
| $\delta y$ | 5.5200 | # 1 | 4.8756 |
| $\delta z$ | 0.0000 | - | 0.0000 |
| $\delta\phi$ | 0.0000 | - | 0.0000 |
| $\delta\theta$ | 0.0000 | - | 0.0000 |
| $\delta\psi$ | 0.0600 | # 1 | 34.6877 |

Since the results of the test-bed indicate stage number one is the faulty stage, it can be concluded that the proposed method is sensitive to variation source identification. As previously mentioned, the faulty stage is the first process in the case study. Therefore, small bore machining is important process that strongly influences the dimensional quality of the connecting rod. The importance of this machining process is highlighted by the fact that the dimensional variation of small bore lead to instability of connecting rod at the fixture location in the next process. This instability injures machining tools and decreases the capability of the process.

## CONCLUDING REMARKS

Based on this current research, there is an identified gap and a need for more research to study the complexity of MMPs and product quality problems with particular emphasis on variation source identification. Challenges can be addressed based on detection of faulty stage during production phase, rapid root cause tracking and problem solving during production stages. Statistical process control is one of the most primary techniques that can be used for monitoring and fault diagnosis in MMPs. However, stage-by-stage inspection, high alarm rates, and recognition of the faulty stage are the main problems in applying these methods. Therefore, it is desirable to develop a fault diagnosis methodology to identify the source of variation especially for continuous production lines. In this study, a novel approach was proposed to diagnose faults in multistage manufacturing processes based on stream of variation methodology and discriminant analysis. This approach separates process error of product characteristics and facilitates the identification of faulty stage in continuous multistage manufacturing processes. The proposed method first constituted state space model, then discriminant function was estimated based on the measurements of the quality characteristics during production phases separated stream of process errors between stages geometrically.

Consequently, future deviation at the end of the production line can be assigned to the region of variation that corresponds to the faulty stage. The effectiveness of this method has been demonstrated using a case study from automotive industry; the Nasir-Kyung Corporation that is one of the prosperous suppliers of Saipa manufacturing group in Iran. The results showed that variation sources in multistage connecting rod machining process could be simply identified. Furthermore, a quantitative comparison between the real case and simulated case confirmed the recognition of the faulty stage with the same predicted group membership. The proposed diagnostic method is widely applicable in large manufacturing industries such as automotive, aerospace, appliance, and the like.

The proposed diagnostic methodology is limited to between-stage variations. The development of the proposed method that separates within-stage variations in multistage manufacturing process would be a valuable contribution. New research is needed to forecast the trend of the propagated variation employing the statistical tools such as time series analysis.

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