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التحليل المقارن متعدد الهدف ومتعددة الخوارزمية للتصميم الأمثل لمحولات التوزيع

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الخلاصـة

يعرض هذا البحث محول التحجيم من ثلاث مراحل باستخدام أربع خوارزميات ذكية وهي البرمجة الهندسية GP، الخوارزمية الجينية GA، محاكاة الصلبة SA والجسيمات طوف الأمثل SOP. وقد استخدمت أربع وظائف موضوعية مستقلة وثمانية القيود. ويظهر تحليل مقارن أجريت على النتائج المتحصل عليها من هذه الخوارزميات الذكية بأن جميع المخرجات من خوارزميات ذكية هي نفسها والنتائج تظهر أن GP هو الأسرع، بينما تتبعها GA و SA و PSO في السرعة. وتمت مقارنة النتائج التكلفة موضوعية مع النتائج التي حصلت عليها (Masood, 2012) وأظهرت بأن التكاليف تم حفظها بالترتيب التالي، 6.4%، 16.32%، 10.63% و 16.79% للخوارزميات GP و GA و SA و PSO على التوالي.

A comparative analysis of multi-objective and multi-algorithm approaches for the optimal design of distribution transformers

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ABSTRACT

This paper presents the sizing of three phase transformer using four intelligent algorithms namely geometric programming, genetic algorithm, simulated annealing, and particle swam optimization. Four independent objective functions and eight constraints were used. The comparative analysis carried out on the results obtained from these intelligent algorithms shows that all the outputs from the intelligent algorithms are the same. The fastness of results shows that geometric programming is the fastest, while genetic algorithm, simulated annealing, and particle swam optimization followed in that order. The output results from the cost objective function were compared with the results obtained by Masood (2012) and it showed that money was saved in the following order, 6.4%, 16.32%, 10.63% and 16.79% respectively for geometric programming, genetic algorithm, simulated annealing, and particle swam optimization.

Keywords: Constraints; minimisation; objective functions; particle swam optimization; simulated annealing.

Abbreviations

Symbol	Abbreviation	Symbol	Abbreviation
M_{fe}	Mass of iron	t_p	Thickness of primary winding
d_{fe}	Density of iron	R_s	Radius of secondary winding
s_f	Stacking factor	t_s	Thickness of secondary winding
d	Diameter of core	g_1	Gap between core and primary winding
H	Window height	J_p	Current density
T	Window width	B	Flux density
M_{cu}	Mass of copper	P	Power output
d_{cu}	Density of copper	I_p	Primary current
h	Height of winding	E_p	Primary voltage
R_p	Radius of primary winding	E_t	Volt/turn

INTRODUCTION

Power transformer is a static electromagnetic device whose main function in the electric power system is to transform power variable from one level to another. Power transformer, if properly designed has efficiency close to 100%. The near 100% efficiency arises from low copper and iron core losses. Being the most important device in electric power system transmission and distribution, it's design and construction need to fulfil the followings:- low power loss, cheapness, efficiency and availability. With these requirements, the theoretical method of power design has given way to an intelligent algorithm approach. These intelligent algorithms are now used to optimize or minimize certain transformer sizing parameters, which leads to a more efficient, cheaper and more reliable transformer. Optimization procedures using intelligent algorithm is now a science of its own, which is deployed in determining the best solution to certain mathematically defined problems, which are often models of physical reality. The optimization procedure used in the design of transformer involves the setting up of objective functions and making output value to reach a maximum or minimum while keeping all the transformer design variables within an acceptable limit or range. Several intelligent algorithms are available for use in the design of three phase transformers

In the literature, several intelligent algorithm optimization procedures have been used by researchers, in order to reduce or minimize the transformer cost, losses and mass or to maximize the efficiency. The intelligent algorithms used as optimization techniques in the design of three phase distribution transformer are:- the genetic algorithm(GA) (Amit et al., 2011; Ravi et al., 2013) , the finite element method (FEM) (Tsili et al., 2005), simulated annealing (SA) (Amit et al., 2011), geometric programming (GP) (Jabbr, 2005) and Mathematica (Masood et al., 2012). These intelligent algorithms give better transformer design variables (parameters) that result in the production of cheaper, efficient and more reliable transformers as compared to the design that uses the theoretical approach.

The problem here is how trustworthy are these intelligent algorithms. Are their results the same when applied on the same optimization functions and constrains? Among the intelligent algorithms, are there some that gives faster and accurate results than the others? Arising from these, this work will make a comparative analysis on some intelligent algorithms written in MATLAB's code (X-S Yang,2010; Boyd et al.,2007; Sendilkumar et al., 2013; Mohammad et al., 2014) such as the genetic algorithm, simulated annealing, geometric programming and particle swarm optimization in the design of a three phase transformer. Four objective functions will be used in the comparative analysis. These are the mass, cost, loss and efficiency. The same optimization constraints will be applied in all cases. The aim is to focus on: (1) the acceptance of the constraints by all the intelligent algorithms, (2) the fastness of the algorithms and (3) the closeness of the outputs.

FORMULATION OF MULTI-OBJECTIVE FUNCTIONS

The objective of the work is to optimally design a three phase distribution transformer, by minimizing the mass of the core and copper used, the cost, the losses and the maximization of the efficiency in the production of transformers. Thus the following objective functions were formulated.

(a) The mass objective function:

Taking the cross section of the limbs and yokes to be the same, the mass of the core is given as

$$M_{fe} = d_{fe}V_C \tag{1}$$

The core volume is expressed as

$$V_C = \pi R_c^2\big(3H + 4(3R_c + T)\big) \tag{2}$$

Substituting Equation (2) into Equation (1) gives,

$$M_{fe} = d_{fe}s_f\pi R_c^2\big(3H + 4(3R_c + T)\big) \tag{3}$$

The mass of copper for both the low and high voltage side is given as (Jabbr, 2005),

$$M_{cu} = 2Md_{cu}h_s\pi\big(R_pt_p + R_st_s\big) \tag{4}$$

Combining Equation (1) and Equation (2) we have,

$$M_{fe+cu} = M_{fe} + M_{cu} = d_{fe}s_f\pi R_c^2\big(3H + 2(3d + 2T)\big) + 6d_{cu}h_s\pi\big(R_pt_p + R_st_s\big) \tag{5}$$

(b) The loss objective function:

There are two types of losses in a transformer, which are:

(1) The copper loss which is a combination of the losses in the primary and secondary windings respectively and this can be written using the expression derived by (Masood et al., 2012)

$$L_{cu} = \rho_{cu}\left(2M\pi(1 + ecf_s)J_s^2R_sh_st_s + (1 + ecf_p)\left(\big(J_spf_s/(\alpha \times pf_p \times t_p)\big)^2 \times \big(2M\pi pf_sR_ph_st_p\big)\right)\right) \tag{6}$$

(2) The iron or core loss, which is made up of the eddy current loss and hysteresis loss and can be expressed as

$$L_{fe} = \rho_{fe} k_u f^{k_1} B^{k_2} s_f V_C \tag{7}$$

which gives,

$$L_{fe} = \rho_{fe} k_u f^{k_1} B^{k_2} s_f \pi R_c^2 \big(3H + 4(3R_c + T)\big) \tag{8}$$

The combination of Equation (4) and Equation (5) gives the total losses in the transformer, which can be expressed as

$$L_T = L_{cu} + L_{fe} = rho_{cu} \left(2M\pi(1 + ecf_s) J_s^2 R_s h_s t_s + \left(1 + ecf_p\right) \left(\left(\frac{J_s pf_s}{(\alpha \times pf_p \times t_p)} \right)^2 \times \right.\right.$$

$$\left.\left. \left(2M\pi pf_s R_p h_s t_p\right) \right) \right) + \rho_{fe} k_u f^{k_1} B^{k_2} s_f \pi R_c^2 \big(3H + 4(3R_c + T)\big) \tag{9}$$

(c) The cost objective function:

Let the cost of the iron be C_{fe} in Naira per kilogram of iron and let the cost of copper per kilogram be C_{cu} then the total cost of material needed for the construction of the transformer is,

$$C_{TM} = C_{fe} M_{fe} + C_{cu} M_{cu}$$

$$= C_{fe} d_{fe} s_f \pi R_c^2 \big(3H + 4(3R_c + T)\big) + 2M C_{cu} d_{cu} h_s \pi \big(R_p t_p + R_s t_s\big) \tag{10}$$

In the design, an assumption is made, which is, that the cost of labour should be 10% of the total cost of material. Then the cost of producing the transformer is:

$$C_T = (1 + 0.01) C_{fe} d_{fe} s_f \pi R_c^2 \big(3H + 4(3R_c + T)\big) + 2M C_{cu} d_{cu} h_s \pi \big(R_p t_p + R_s t_s\big) \tag{11}$$

(d) The efficiency objective function:

The efficiency of a transformer is the ratio of the power output and power input which can be expressed as p.u or percentage. The efficiency expression is:

$$E_{ff} = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + L_T} \tag{12}$$

The total loss L_T is as given in Equation (3) and inserting it in Equation (5) gives,

$$E_{ff} = \frac{P_{out}}{P_{out}+rho_{cu}\left(2M\pi(1+ecf_s)J_s^2 R_s h_s t_s+(1+ecf_p)\left(\left(\frac{Jspf_s}{(\alpha\times pf_p\times t_p)}\right)^2\times(2M\pi pf_s R_p h_s t_p)\right)\right)\ldots +\rho_{fe}k_u f^{k_1}B^{k_2}k_f\pi R_C^2(3H+4(3R_c+T))} \tag{13}$$

Equation (3) is not in the standard form for geometric programming formulation. It has to be converted into the standard form by taking the inverse which now becomes,

$$E_{inv} = \frac{1}{E_{ff}} = \frac{P_{out}+rho_{cu}\left(3M\pi(1+ecf_s)J_s^2 R_s h_s t_s+(1+ecf_p)\left(\left(\frac{Jspf_s}{(\alpha\times pf_p\times t_p)}\right)^2\times(3M\pi pf_s R_p h_s t_p)\right)\right)\ldots +\rho_{fe}k_u f^{k_1}B^{k_2}k_f\pi R_C^2(3H+4(3R_c+T))}{P_{out}} \tag{14}$$

DESIGN CONSTRAINTS

The design constraints to be used include the flux density constraint, the current density constraint and the power transfer constraint. The same constraints will be used for all the objective functions in the design optimization method. There are two types of constraints: the equality and inequality constraints.

1.1. Equality constraint

(a) The flux density constraint

The flux density of the core and the yoke are the same, this arises from the same cross section assumed for both. The flux density expression is given as

$$B = \frac{E_t}{4\sqrt{2}k_f\pi f R_c^2} \tag{14}$$

The only variable in this constraint is that of the core radius. The flux density constraint is an equality constraint and can be expressed as such in the GP, GA, SA and APSO formats.

(b) The current density constraint

The density can be expressed in terms of the window height and window width as

$$J_p = \frac{4E_p I_p}{E_t H T k_w} \tag{15}$$

There are two design variables; the window height and the window width. This constraint is also an equality one.

(c) The power transfer constraint

The power transfer is given as (Jabbr,2005; Masood et al.,2012)

$$P = \sqrt{2}\pi^2 f s_f {R_c}^2 B J_s h_s t_s \tag{16}$$

Other equality constraints used are as stated below:

$$2T = H \tag{17}$$

$$h_s = 0.85H \tag{18}$$

$$J_p = \frac{J_s t_s p_{fs}}{\alpha t_p p_{fp}} \tag{19}$$

1.2. Inequality constraints

The inequality constraints used are relational ones that will provide the desired results. These are:

$$T \geq 2g_1 + 2t_p + 2g + 2t_s + g_o \tag{20}$$

$$R_s \geq R_c + g_1 + \frac{t_s}{2} \tag{21}$$

$$R_p \geq R_s + g_1 + \frac{(t_s + t_p)}{2} \tag{22}$$

DESIGN EXAMPLE

Figure 1 shows some of the output parameters that will be analysed in this work, while Table 1 shows the input parameters. Other additional constant parameters to be used are as shown in Table 2.

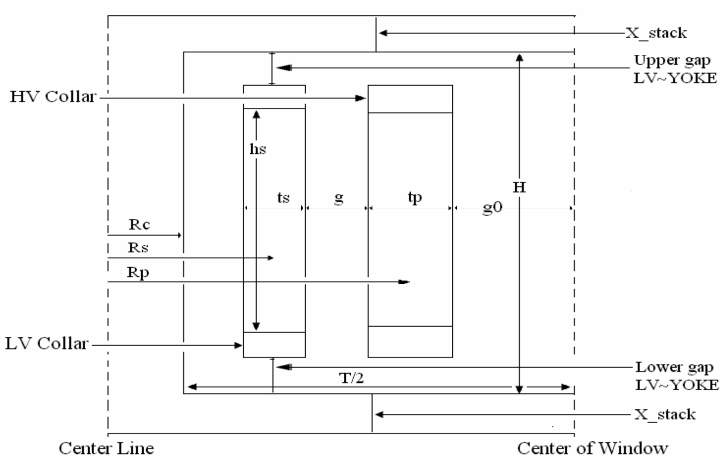

Fig. 1. Transformer output parameters (Masood et al., 2012)

Table 1. Transformer input parameters

Input parameters	**Values**
Power (kVA)	25
Voltage (kV)	11/.415
Frequency (Hz)	50
Space factor (k_w)	0.2
Stacking factor (s_f)	0.95
Density of iron kg/m^3	7650
Density of copper kg/m^3	8900
Flux density Wb/m^2	1.3
Current density Wb/m^2	2.5×10^6

Table 2. Other transformer input parameters

Input parameters	**Values**
Transformer constant (ku)	1.9×10^{-3}
Transformer constant (k_1)	1.24
Transformer constant k_2	2
Fill factor of secondary winding (pfs)	0.5
Fill factor of primary winding (pfp)	0.5
Eddy current factor for LV winding ($ecfs$)	0.1
Eddy current factor for HV winding ($ecfp$)	0.1
Alpha (α)	0.95
Distance between the core and LV winding (g_1)(m)	0.003
Copper wire resistivity (rho) ($ohm - m$)	2.1×10^{-8}
Number of Phases (M)	3
Half of the clearance between the two phases (go)(m)	0.04

Table 3 shows design parameter limits. To obtain an acceptable transformer design, the design variables need to be bound between an upper limit and a lower limit values.

Table 3. Design variable lower and upper limits

Design Variabes	Description	Lower Limit	Upper Limit
x_1	Transformer height (H) (m)	0.05	0.30
x_2	Radius of core $(R_c)(m)$	0.01	0.20
x_3	Radius of Primary Winding $(R_p)(m)$	0.01	0.12
x_4	Radius of Secondary Winding $(R_s)(m)$	0.05	0.10
x_5	Transformer width (T) (m)	0.05	0.30
x_6	Primary Winding height $(h_p)(m)$	0.04	0.30
x_7	Secondary Winding height $(h_s)(m)$	0.04	0.30
x_8	Primary Winding Thickness $(t_p)(m)$	0.15	0.20
x_9	Secondary Winding Thickness $(t_s)(m)$	0.12	0.15
x_{10}	Gap between HV and LV $(g)(m)$	0.08	0.01

RESULTS AND DISCUSSION

Table 4. Transformer design optimization: mass as objective function

Design Variables	GP	GA	SA	PSO
$H(x_1)$ (cm)	23.31	24.35	23.27	21.73
$R_c(x_2)$ (cm)	5.11	4.96	4.96	4.96
$R_p(x_3)$ (cm)	8.16	8.77	7.89	8.31
$R_s(x_4)$ (cm)	5.98	5.89	5.75	7.15
$T(x_5)$ (cm)	11.66	12.01	11.63	11.07
$h_p(x_6)$ (cm)	18.82	19.40	18.75	17.26
$h_s(x_7)$ (cm)	19.82	20.42	18.83	18.24
$t_p(x_8)$ (cm)	1.55	1.50	1.56	1.59
$t_s(x_9)$ (cm)	1.18	1.40	1.20	1.25
$g(x_{10})$ (cm)	0.80	0.80	0.87	0.82
M_{fe+cu} (kg)	137.59	135.35	131.33	127.11

Table 5. Transformer design optimization: loss as an objective function

Design Variables	GP	GA	SA	PSO
$H(x_1)\ (cm)$	24.49	24.44	23.72	23.04
$R_c(x_2)\ (cm)$	4.05	4.96	3.94	3.94
$R_p(x_3)\ (cm)$	8.18	9.00	8.80	9.94
$R_s(x_4)\ (cm)$	5.08	5.89	6.21	7.79
$T(x_5)\ (cm)$	12.24	12.22	11.87	11.61
$h_p(x_6)\ (cm)$	19.78	19.73	19.28	18.44
$h_s(x_7)\ (cm)$	20.82	20.77	20.17	19.46
$t_p(x_8)\ (cm)$	1.55	1.50	1.60	1.59
$t_s(x_9)\ (cm)$	1.47	1.22	1.23	1.30
$g(x_{10})\ (cm)$	0.80	0.81	0.86	0.81
$T_{loss}\ (Watts)$	308.88	373.19	296.61	236.05

Table 6. Transformer design optimization: cost as an objective function

Design Variables	GP	GA	SA	PSO
$H(x_1)\ (cm)$	23.72	23.52	23.50	21.59
$R_c(x_2)\ (cm)$	5.29	4.96	4.96	4.96
$R_p(x_3)\ (cm)$	8.44	8.09	7.84	9.27
$R_s(x_4)\ (cm)$	6.23	5.90	5.64	6.72
$T(x_5)\ (cm)$	11.86	11.76	11.83	11.01
$h_p(x_6)\ (cm)$	19.16	19.00	18.87	17.00
$h_s(x_7)\ (cm)$	20.16	20.00	19.95	18.04
$t_p(x_8)\ (cm)$	1.55	1.50	1.53	1.55
$t_s(x_9)\ (cm)$	1.28	1.27	1.32	1.26
$g(x_{10})\ (cm)$	0.80	0.80	0.82	0.84
$Cost(Naira)$	0.0594×10^6	0.0528×10^6	0.0566×10^6	0.0526×10^6

Table 7. Transformer design optimization: efficiency as an objective function

Design Variables	GP	GA	SA	PSO
$H(x_1)$ (cm)	24.09	24.57	23.82	22.51
$R_c(x_2)$ (cm)	5.49	4.94	4.96	4.96
$R_p(x_3)$ (cm)	8.85	9.00	9.90	8.18
$R_s(x_4)$ (cm)	6.45	6.89	6.45	5.96
$T(x_5)$ (cm)	12.04	12.28	11.96	11.45
$h_p(x_6)$ (cm)	19.45	19.84	19.23	17.78
$h_s(x_7)$ (cm)	20.47	20.88	20.25	18.87
$t_p(x_8)$ (cm)	1.60	1.32	1.55	1.67
$t_s(x_9)$ (cm)	1.32	1.31	1.35	1.34
$g(x_{10})$ (cm)	0.80	0.80	0.81	0.88
η (%)	98.04	98.18	98.42	98.71

The first comparison is to compare the output results obtained from the intelligent algorithms using cost as an objective function with the results (Masood et al., 2012) obtained using Mathematica and cost as an objective function on the design of a transformer with the same parameters as given in Table 1. The cost objective function and constraints used are slightly different from that of Masood et al., (2012). The results are presented in Table 8, and the % differences from that of Masood et al., (2012) are presented in Table 9.

Table 8. Transformer design optimization: comparison with (Masood et al., 2012) results

Design Variables	(Masood)	GP	GA	SA	PSO
$H(x_1)$ (cm)	-	23.72	23.52	23.50	21.59
$R_c(x_2)$ (cm)	4.98	5.29	4.96	4.96	4.96
$R_p(x_3)$ (cm)	8.2278	8.44	8.09	7.84	9.27
$R_s(x_4)$ (cm)	5.8792	6.23	5.90	5.64	6.72
$T(x_5)$ (cm)	-	11.86	11.76	11.83	11.01
$h_p(x_6)$ (cm)	-	19.16	19.00	18.87	17.00
$h_s(x_7)$ (cm)	19.86	20.16	20.00	19.95	18.04
$t_p(x_8)$ (cm)	1.699	1.55	1.50	1.53	1.55
$t_s(x_9)$ (cm)	1.3997	1.28	1.27	1.32	1.26
$g(x_{10})$ (cm)	0.80	0.80	0.80	0.82	0.84
$Cost(10^6 Naira)$	0.057	0.059	0.053	0.0566	0.0526

The % difference will be computed using

$$\left|\frac{Mosood's\ Results - Simulated\ Results}{Mosood's\ Results}\right| \times \frac{100}{1} \tag{23}$$

Table 9. Transformer design optimization: percentage difference with (Masood et al., 2012) results

Design Variables	GP	GA	SA	PSO
$H_w(x_1)$ (%)	-	-	-	-
$R_c(x_2)$ (%)	6.22	0.40	0.40	0.40
$R_p(x_3)$ (%)	2.58	8.17	4.71	12.67
$R_s(x_4)$ (%)	5.97	0.35	4.00	14.30
$W_w(x_5)$ (%)	-	-	-	-
$h_p(x_6)$ (%)	-	-	-	-
$h_s(x_7)$ (%)	1.51	0.70	0.40	0.90
$t_p(x_8)$ (%)	7.13	10.13	8.33	7.13
$t_s(x_9)$ (%)	8.55	9.27	5.69	9.98
$g(x_{10})$ (%)	0.00	0.00	2.50	5.00
$Cost$(%)	3.51	7.02	0.7	7.54

The comparisons using Table 8 and 9 shows that results obtained using the four intelligent algorithms as compared with Masood et al., (2012) are within the acceptable design limits. In terms of cost minimization, if the 10% added to compensate for labour is deducted the intelligent algorithms reduces the cost as compared to Masood et al., (2012) by 6.4%, 16.32%, 10.63% and 16.79% respectively for GP, GA, SA and PSO.

Fastness is a major factor in choosing an intelligent algorithm. Comparison of the number of times an accurate and comparable result occurred, using these intelligent algorithms is as presented in Table 10. From the table, GP gives the fastest and accurate result based on the algorithms and the constraint provided. This means that the algorithms and constraints were all accepted. The G A was next, but it sends an infeasible warning after 20 seconds. Constraints (15) and (16) were removed due to these warnings; accurate results appeared after the constraint removals. The SA and PSO initially give inaccurate results as the relational variables *H* and *T* were as far apart from each other. Only on removal of constraints (15) and (16), relational variables realigned and accurate results started showing.

Table 10. Simulation time

	GP	GA	SA	PSO
Number of times ran	1	2	4	6
Time used (secs)	3	240	480	720

CONCLUSION

A 25 kVA, 11/0.415 kV, 50Hz, 3 phase Transformer has been designed using four intelligent algorithms. Four objective functions namely mass, cost, loss and efficiency, and eight constraints were used on the four intelligent algorithms. The transformer frame parameters designed for were: window height, window width and core diameter. The expected out parameters are the cost, efficiency, losses, and mass of transformer. The results of the transformer parameters obtained from these intelligent algorithms compared favourably within themselves and that of [4]. This work demonstrates that intelligent algorithms, if used with appropriate objective functions and constraints would produce good results.

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