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المرحلة الأولى رصد داخل ملف
التعريف للمتغيرات المتعددة المترابطة بالملامح الخطية

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الخـلاصـة

وباعتبارها واحدة من أهم أفرع التخصصات في مراقبة العمليات الإحصائية (SPM)، فقد اجتذب رصد الملف متعدد المتغيرات الانتباه في السنوات الأخيرة. وقد أجريت معظم البحوث على تحليل الملف متعدد المتغيرات في إطار افتراض استقلالية قيم الاستجابة. ومع ذلك، يتم انتهاك افتراض الاستقلال في العديد من التطبيقات الحقيقية، مثل عندما يتم جمع الملاحظات في فترات زمنية قصيرة. في هذه الورقة، نركز على مراقبة المرحلة الأولى من الملفات متعددة المتغيرات عندما تكون قيم الاستجابة المتتالية داخل كل ملف مرتبطة تلقائيا وتتبع المتوسط المتحرك للانحدار الذاتي (نموذج ARMA(1،1)). أولا، يتم تطبيق طريقة التحويل للقضاء على تأثير الارتباط الذاتي. ثم يتم استخدام نهجين، ((T^2 و (Wilks)، للتحقق من استقرار العملية تحت مقادير مختلفة من التحولات ومتغيرات مختلفة من نموذج ((ARMA(1،1). ويطبق مثال عددي يستند إلى دراسات المحاكاة لتقييم أداء مخططات التحكم المطبقة في وجود الارتباط الذاتي داخل النطاق من حيث معيار احتمال الإشارة. وأظهرت النتائج أن (WILKS) لامدا يتفوق في الغالب على الرسم البياني (T^2) في جميع الحالات خارج السيطرة.

Phase I monitoring of within-profile autocorrelated multivariate linear profiles

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ABSTRACT

As one of the most important subareas of statistical process monitoring (SPM), multivariate profile monitoring has attracted attention in recent years. Most researches on multivariate profile analysis have been carried out under the independency assumption of response values. However, the independency assumption is violated in many real applications, such as when the observations are gathered in short time intervals. In this paper, we focus on Phase I monitoring of multivariate profiles when the consecutive response values within each profile are autocorrelated and follow the autoregressive-moving average (ARMA(1,1)) model. First, a transformation method is applied to eliminate the effect of autocorrelation. Then, two approaches, T^2 and Wilks' lambda, are used to check the stability of the process under different magnitudes of shifts and different parameters of the ARMA(1,1) model. A numerical example based on simulation studies is applied to evaluate the performance of the applied control charts in the presence of within-profile autocorrelation in terms of signal probability criterion. The results show that Wilks' lambda outperforms the T^2 chart in almost all out-of-control situations.

Keywords: Multivariate simple linear profile; Phase I; signal probability; statistical process monitoring (SPM); within-profile autocorrelation.

INTRODUCTION

Control charts are one of the most effective tools for reducing process variations by distinguishing between common and assignable causes. In many process monitoring applications, the quality of the product or process is characterized by univariate or correlated multivariate quality characteristics. Shewhart control charts such as and , as well as memory-based charts such as exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) control charts, are used to monitor processes with univariate quality characteristics. However, using univariate control charts for each quality characteristic separately in monitoring multivariate process leads to misleading results because of ignoring the correlation structure among quality characteristics. In such situations, multivariate charts such as T^2, multivariate exponentially weighted moving average (MEWMA), and multivariate cumulative sum (MCUSUM) are used to take into account the correlation structure. For more information about classic multivariate monitoring schemes, refer to Hotelling (1947), Murphy (1987), Tracy et al. (1992), Vasilopoulos & Stamboulis (1978), and Mason & Young (2002) for example.

In some industrial processes, the quality of the products is characterized by a relationship between a response variable and one or more explanatory variables, called a profile. The most common profiles in the case of single-response variables based on the type of relationship between response and explanatory variable(s) are simple linear profiles, polynomial profiles, multiple profiles, and generalized linear model- (GLM-) based profiles. Profiles like these, with a single-response variable, are referred to as “univariate profiles”. Many researchers have developed approaches to monitor various univariate profiles in Phases I and II. The purpose of Phase I profile monitoring is to estimate the in-control values of the regression parameters from historical process data. In Phase II profile monitoring, quality engineers focus on detecting changes in profile parameters based on the results of Phase I analysis. Most efforts in the profile monitoring area have been devoted to Phase I. Phase I monitoring of profiles has been addressed by many authors; see Mahmoud & Woodall (2004) and Farahani et al. (2014) for example. For more examples of monitoring profiles in Phase II, see Saghaei et al. (2009), Zhang et al. (2009), Zou et al. (2012), Amiri et al. (2013), and Noorossana et al. (2015). See the review paper provided by Woodall (2007) for detailed information on profile monitoring approaches.

In some situations, several correlated response variables should be modeled as a set of linear functions of the explanatory variable(s). Such profiles are referred to as “multivariate profiles”. Most researches in the area of profile monitoring have been devoted to univariate profiles. However, in recent years, several studies of monitoring multivariate profiles have been carried out; see, for example, Noorossana et al. (2010a), Noorossana et al. (2010b), and Eyvazian et al. (2011).

In all of the studies to date on univariate and multivariate cases, the response values are assumed to be independently and identically distributed normal random variables. However, in some situations, the independency assumption of response values is violated. In the first research in this area, Soleimani et al. (2009) studied Phase II monitoring of simple linear regression in the case of within-profile autocorrelation. Monitoring autocorrelated profiles in the univariate case was also investigated by Noorossana et al. (2008), Jensen & Birch (2009), Noorossana et al. (2010c), Amiri et al. (2010), Abdel-Salam et al. (2013), Keramatpour et al. (2013), Narvand et al. (2013), Soleimani et al. (2013a), Keramatpour et al. (2014), Soleimani & Hadizadeh (2014), Zhang et al. (2014), and Khedmati & Niaki (2015).

Most studies of monitoring autocorrelated profiles have considered a single-response variable. To the best of our knowledge, there are only two studies of monitoring multivariate autocorrelated profiles. Soleimani et al. (2013b) monitored multivariate simple linear profiles in Phase II in the case of within-profile autocorrelation. They proposed a remedial approach based on a transformation method to eliminate the autocorrelation structure within multivariate profiles. They also presented a case study to show the application of their proposed model in real practice. Soleimani & Noorossana (2014) studied Phase II monitoring of multivariate simple linear profiles in the presence of between-profile autocorrelation. They proposed three time-series-based methods to eliminate the effect of autocorrelation. Both studies aimed at monitoring multivariate autocorrelated profiles in Phase II.

To motivate our study, we refer to a real example in the body shop of an automotive industrial corporation. In this example, which is provided by Noorossana et al. (2010a), the relationship between four correlated response variables and one explanatory variable is expressed by a linear function. The explanatory variable is the nominal force of the press machine, which is exerted by cylinders on metal plates. The response variables are the values of the real forces collected from the four cylinders of the press, which are measured by a programmable logic controller. The response values within each profile are assumed to be independent from each other. However, sometimes the successive observations within each profile are collected at short time intervals. In such situations, the independency assumption of the response values within each profile is violated. Hence, it is important to take into account the within-profile autocorrelation structure to monitor autocorrelated multivariate profiles in Phase I, a topic which is neglected in the area of profile monitoring.

Hence, in this paper, we take into account within-profile autocorrelation to monitor multivariate linear profiles in Phase I. For this purpose, first, a transformation method is presented to eliminate the within-profile autocorrelation. Then, two control charts, Hotelling's T^2 and Wilks' lambda/M, are proposed for Phase I monitoring of multivariate simple linear profiles where the response values within each profile are autocorrelated and follow the autoregressive-moving average (ARMA(1,1)) model.

The rest of this paper is organized as follows. In Section 2, the multivariate simple linear profile model when the observations within each profile are autocorrelated is discussed. In Section 3, two monitoring approaches based on transformation of autocorrelated response values are extended. In Section 4, the performance of the extended approaches is evaluated and compared through a numerical example. In Section 5, conclusions and recommendations for future study are discussed.

SIMPLE LINEAR PROFILE IN THE PRESENCE OF WITHIN-PROFILE AUTOCORRELATION

Assume that the p-dimensional observations of the kth $k=1,\ldots,m$ profile are collected as follows:

$$\begin{pmatrix} x_1 & y_{11k} & y_{12k} & \cdots & y_{1pk} \\ x_2 & y_{21k} & y_{22k} & \cdots & y_{2pk} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_n & y_{n1k} & y_{n2k} & \cdots & y_{npk} \end{pmatrix}, \tag{1}$$

where $y_{ijk}; i=1,\ldots,n,\ j=1,\ldots,p,\ k=1,\ldots,m$ is the jth response variable at ith treatment of the kth profile. For the ith treatment of the kth profile, the following model is used to relate the multivariate response variables with the explanatory variable when the process is statistically in control:

$$\mathbf{y_{ik}} = \boldsymbol{\beta_0} + x_i\boldsymbol{\beta_1} + \boldsymbol{\varepsilon_{ik}}; i = 1, 2, \ldots, n, \tag{2}$$

where x_i is the value of the explanatory variable at the ith observation, $\mathbf{y_{ik}}$ is a $1\times p$ vector of the response variables, $\boldsymbol{\beta_0}$ and $\boldsymbol{\beta_1}$ are $1\times p$ vectors of the intercept and slope parameters, and $\boldsymbol{\varepsilon_{ik}}$ is a $1\times p$ vector of the error terms with the following distribution:

$$\boldsymbol{\varepsilon_{ik}} \sim MVN(\mathbf{0}, \boldsymbol{\Sigma}) \tag{3}$$

The matrix form of Equation (2) is given as

$$\mathbf{Y}_k = \mathbf{XB} + \mathbf{E_k}; k = 1,2,...,m \tag{4}$$

In Equation (4), $\mathbf{B} = (\boldsymbol{\beta}_0, \boldsymbol{\beta}_1)^T$ is a $2 \times p$ matrix of the regression model, and $\mathbf{E_k} = (\boldsymbol{\varepsilon}_{1k}, \boldsymbol{\varepsilon}_{2k}, ..., \boldsymbol{\varepsilon}_{nk})^T$ is an $n \times p$ matrix of error terms in the kth profile. In the case of independency, the rows of $\mathbf{E_k}$ are independent from each other, while the elements of each row are correlated. One of the most basic assumptions of the least squares method is the independency assumption for error terms. Hence, using the least squares method to estimate the regression parameters in the presence of within-profile autocorrelation leads to misleading results. Here, we suppose that the error terms in the successive multivariate simple profiles are autocorrelated. Therefore, the multivariate simple linear profile model in the presence of within-profile autocorrelation according to an autoregressive-moving average or ARMA(1,1) process is given as follows:

$$\begin{aligned} \mathbf{y_{ik}} &= \boldsymbol{\beta}_0 + x_i\boldsymbol{\beta}_1 + \boldsymbol{\varepsilon}_{\mathbf{ik}}; i = 1,2,...,n \; k = 1,2,... \\ \boldsymbol{\varepsilon}_{\mathbf{ik}} &= \boldsymbol{\varepsilon}_{\mathbf{i-1,k}}\boldsymbol{\varphi} + \mathbf{u_{ik}} - \mathbf{u_{i-1,k}}\boldsymbol{\theta} \end{aligned} \tag{5}$$

It can be proven that this structure leads to a similar autocorrelation structure (ARMA(1,1)) within observations of each profile as follows (Soleimani et al., 2013b):

$$\mathbf{y_{ik}} - (\boldsymbol{\beta}_0 + x_i\boldsymbol{\beta}_1) = [\mathbf{y_{(i-1)k}} - (\boldsymbol{\beta}_0 + x_{i-1}\boldsymbol{\beta}_1)]\boldsymbol{\varphi} + \mathbf{u_{ik}} - \mathbf{u_{(i-1)k}}\boldsymbol{\theta}, \tag{6}$$

where $\mathbf{u_{ik}}$ is a $1 \times p$ vector of the independent error terms that follows $\mathbf{u_{ik}} \sim MVN(\mathbf{0}, \boldsymbol{\Sigma}_{\mathbf{U}})$, and $\boldsymbol{\varphi}$ and $\boldsymbol{\theta}$ are both $(p \times p)$ diagonal matrices. For the sake of simplicity, the diagonal elements for both $\boldsymbol{\varphi}$ and $\boldsymbol{\theta}$ are considered equal. Hence,

$$\boldsymbol{\varphi} = \begin{bmatrix} \phi & 0 & \cdots & 0 \\ 0 & \phi & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \phi \end{bmatrix}, \boldsymbol{\theta} = \begin{bmatrix} \theta & 0 & \cdots & 0 \\ 0 & \theta & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \theta \end{bmatrix}. \tag{7}$$

Here, without loss of generality, we suppose that the independency assumption of within-profile observations is violated, according to the following equation:

$$\begin{pmatrix} Cov(y_{i1}, y_{i'1}) & Cov(y_{i1}, y_{i'2}) & \cdots & Cov(y_{i1}, y_{i'p}) \\ Cov(y_{i2}, y_{i'1}) & Cov(y_{i2}, y_{i'2}) & \cdots & Cov(y_{i2}, y_{i'p}) \\ \vdots & \vdots & \vdots & \vdots \\ Cov(y_{ip}, y_{i'1}) & Cov(y_{ip}, y_{i'2}) & \cdots & Cov(y_{ip}, y_{i'p}) \end{pmatrix} = \begin{pmatrix} v_1 & 0 & \cdots & 0 \\ 0 & v_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & v_p \end{pmatrix}, \tag{8}$$

where

$$v_j = [(1-\phi\theta)(\phi-\theta)/(1-\phi^2)]\sigma_j^2; j = 1,..., p \tag{9}$$

Note that i and i' in Equation (8) are two adjacent treatments. For more elaboration, the explained autocorrelation structure of a bivariate profile with n=4 treatments is written as follows:

$$\begin{bmatrix} \varepsilon_{11k} & \varepsilon_{12k} \\ \varepsilon_{21k} & \varepsilon_{22k} \\ \varepsilon_{31k} & \varepsilon_{32k} \\ \varepsilon_{41k} & \varepsilon_{42k} \end{bmatrix} = \begin{bmatrix} - & - \\ \varepsilon_{11k} & \varepsilon_{12k} \\ \varepsilon_{21k} & \varepsilon_{22k} \\ \varepsilon_{31k} & \varepsilon_{31k} \end{bmatrix} \times \begin{bmatrix} \phi & 0 \\ 0 & \phi \end{bmatrix} + \begin{bmatrix} u_{11k} & u_{12k} \\ u_{21k} & u_{22k} \\ u_{31k} & u_{32k} \\ u_{41k} & u_{42k} \end{bmatrix} - \begin{bmatrix} - & - \\ u_{11k} & u_{12k} \\ u_{21k} & u_{22k} \\ u_{31k} & u_{32k} \end{bmatrix} \times \begin{bmatrix} \theta & 0 \\ 0 & \theta \end{bmatrix} \tag{10}$$

It is pointed out in the literature on statistical process monitoring that autocorrelation affects the performance of different control charts. Therefore, the transformation presented by Golnabi & Houshmand (1999) is extended and utilized to remove the within-profile autocorrelation structure. For the kth profile, this transformation is given as follows:

$$\mathbf{Y}_k' = \boldsymbol{\gamma}^{-1/2}(\mathbf{Y} - \mathrm{E}(\mathbf{Y})) \tag{11}$$

where E$(\mathbf{Y})$ $=\mathbf{XB}$ and $\boldsymbol{\gamma}$ is the covariance matrix of each profile that can be obtained for an ARMA(1,1) process as follows:

$$\boldsymbol{\gamma} = \begin{pmatrix} \frac{(1+\theta^2-2\phi\theta)}{(1-\phi^2)}\sigma_u^2 & \frac{(\phi-\theta)(1-\phi\theta)}{(1-\phi^2)}\sigma_u^2 & \frac{\phi(\phi-\theta)(1-\phi\theta)}{(1-\phi^2)}\sigma_u^2 & \frac{\phi^2(\phi-\theta)(1-\phi\theta)}{(1-\phi^2)}\sigma_u^2 \\ \frac{(\phi-\theta)(1-\phi\theta)}{(1-\phi^2)}\sigma_u^2 & \frac{(1+\theta^2-2\phi\theta)}{(1-\phi^2)}\sigma_u^2 & \frac{(\phi-\theta)(1-\phi\theta)}{(1-\phi^2)}\sigma_u^2 & \frac{\phi(\phi-\theta)(1-\phi\theta)}{(1-\phi^2)}\sigma_u^2 \\ \frac{\phi(\phi-\theta)(1-\phi\theta)}{(1-\phi^2)}\sigma_u^2 & \frac{(\phi-\theta)(1-\phi\theta)}{(1-\phi^2)}\sigma_u^2 & \frac{(1+\theta^2-2\phi\theta)}{(1-\phi^2)}\sigma_u^2 & \frac{(\phi-\theta)(1-\phi\theta)}{(1-\phi^2)}\sigma_u^2 \\ \frac{\phi^2(\phi-\theta)(1-\phi\theta)}{(1-\phi^2)}\sigma_u^2 & \frac{\phi(\phi-\theta)(1-\phi\theta)}{(1-\phi^2)}\sigma_u^2 & \frac{(\phi-\theta)(1-\phi\theta)}{(1-\phi^2)}\sigma_u^2 & \frac{(1+\theta^2-2\phi\theta)}{(1-\phi^2)}\sigma_u^2 \end{pmatrix} \tag{12}$$

After using the transformation in Equation (11), the following model is obtained when the process is statistically in control:

$$\mathbf{y}_{\mathbf{ik}}' = \boldsymbol{\alpha}_{\mathbf{0}} + x_i\boldsymbol{\alpha}_{\mathbf{1}} + \boldsymbol{\varepsilon}_{\mathbf{ik}}; i = 1,2,\ldots,n\ k = 1,2,\ldots. \tag{13}$$

MONITORING APPROACHES

In this section, two methods, Hotelling's T^2 and Wilks' lambda/M, are extended to monitor multivariate simple linear profiles in the presence of within-profile autocorrelation in Phase I.

3.1. Hotelling's T^2 method

To construct the T^2 statistic, first, the estimation of matrix $\boldsymbol{\alpha} = (\boldsymbol{\alpha}_0, \boldsymbol{\alpha}_1)^T$ (a $1\times 2p$ matrix) using the least squares method is obtained based on the transformed response values as follows:

$$\hat{\boldsymbol{\alpha}}_{\mathbf{k}} = (\hat{\boldsymbol{\alpha}}_{\mathbf{0k}}, \hat{\boldsymbol{\alpha}}_{\mathbf{1k}})^T = (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{Y}_k', \tag{14}$$

The elements of $\hat{\boldsymbol{\alpha}}_{\mathbf{k}}$ can also be estimated as

$$\hat{\boldsymbol{\alpha}}_{\mathbf{k}} = (\hat{\boldsymbol{\alpha}}_{\mathbf{0k}}, \hat{\boldsymbol{\alpha}}_{\mathbf{1k}})^T = (\mathbf{X}^{\mathbf{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{Y}'_k, \tag{15}$$

where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, $S_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2$, $S_{xy'(j)} = \sum_{i=1}^{n}(x_i - \bar{x})y'_{ijk}$, $\bar{y}'_{.jk} = \frac{1}{n}\sum_{i=1}^{n} y'_{ijk}$ and $\hat{\alpha}_{1jk} = \frac{S_{xy'(j)}}{S_{xx}}$.

Then, for the kth $k = 1,2,...,m$ profile, the extended T^2 statistic to monitor the autocorrelated multivariate simple linear profile in Phase I is obtained as follows:

$$T_k^2 = (\hat{\boldsymbol{\alpha}}_{\mathbf{k}} - \bar{\hat{\boldsymbol{\alpha}}})^T \mathbf{S}_{\hat{\boldsymbol{\alpha}}}^{-1} (\hat{\boldsymbol{\alpha}}_{\mathbf{k}} - \bar{\hat{\boldsymbol{\alpha}}}), \tag{16}$$

where the mean estimate of the regression parameters is obtained as

$$\bar{\hat{\boldsymbol{\alpha}}}^{\mathbf{T}} = \frac{1}{m}\sum_{k=1}^{m} \hat{\boldsymbol{\alpha}}_{\mathbf{k}}^T. \tag{17}$$

To compute matrix $\mathbf{S}_{\hat{\boldsymbol{\alpha}}}$, first, matrix $\hat{\mathbf{V}} = \left[\hat{\mathbf{v}}_1, \hat{\mathbf{v}}_2, ..., \hat{\mathbf{v}}_{m-1}\right]^T$ is constructed, in which

$$\hat{\mathbf{v}}_k = \hat{\boldsymbol{\alpha}}_{k+1}^T - \hat{\boldsymbol{\alpha}}_k^{\ T}; k = 1,2,...,m-1. \tag{18}$$

Finally, the estimate of a covariance matrix for the regression parameters is derived as

$$\mathbf{S}_{\hat{\boldsymbol{\alpha}}} = \frac{\hat{\mathbf{V}}^{\mathbf{T}} \times \hat{\mathbf{V}}}{2 \times (m-1)}. \tag{19}$$

3.2. Wilks' lambda/M method

Noorossana et al. (2010) used the Wilks' lambda/M method to monitor multivariate simple linear profiles. Here, we extend this approach and utilize it to monitor multivariate simple linear profiles in the case of within-profile autocorrelation. Suppose that there are a total of m profiles, each of size n. To extend the Wilks' lambda statistic, first, we merge all m samples and construct a single sample of size mn . Then the m-1 indicator variables are defined as follows:

$$Z_{ij} = \begin{cases} 1 & \text{if } i\text{th observation belongs to } j\text{th sample} \\ 0 & \text{Otherwise} \end{cases} ; i = 1,...,mn; j = 1,...,m-1. \tag{20}$$

The mth profile is considered as the reference sample, and then for the ith; $i = 1,...,mn$ observation of the pooled profiles, we have

$$\mathbf{y}'_{\mathbf{i}} = \boldsymbol{\alpha}_{\mathbf{0}} + \boldsymbol{\alpha}_{\mathbf{1}}X_i + \boldsymbol{\alpha}_{\mathbf{01}}Z_{1i} + \boldsymbol{\alpha}_{\mathbf{02}}Z_{2i} + ... + \boldsymbol{\alpha}_{\mathbf{0m'}}Z_{m'i} + \boldsymbol{\alpha}_{\mathbf{11}}Z_{1i}X_i + \boldsymbol{\alpha}_{\mathbf{12}}Z_{2i}X_i + ... + \boldsymbol{\alpha}_{\mathbf{1m'}}Z_{m'i}X_i + \boldsymbol{\varepsilon}_{\mathbf{i}}, \tag{21}$$

where $m' = m - 1$. Then to check the stability of the process, the following hypothesis is considered:

$$\begin{aligned} &H_0 : \boldsymbol{\alpha}_{01} = ... = \boldsymbol{\alpha}_{02} = \boldsymbol{\alpha}_{0m'} = \boldsymbol{\alpha}_{11} = \boldsymbol{\alpha}_{12} = ... = \boldsymbol{\alpha}_{1m'} = \mathbf{0} \\ &H_1 : \text{otherwise} \end{aligned} \tag{22}$$

The reduced regression model under the null hypothesis is given as follows:

$$\mathbf{y}'_{\mathbf{i}} = \boldsymbol{\alpha}_{\mathbf{0}} + \boldsymbol{\alpha}_{\mathbf{1}} x_i + \varepsilon_i ; i = 1,...,mn. \tag{23}$$

The extended Wilks' lambda statistic is obtained as

$$\Lambda = \frac{| \mathbf{Y}'^{\mathbf{T}}\mathbf{Y}' - \hat{\boldsymbol{\alpha}}_{\mathbf{f}}^{\mathbf{T}}\mathbf{X}_{\mathbf{f}}^{\mathbf{T}}\mathbf{Y}' |}{| \mathbf{Y}'^{\mathbf{T}}\mathbf{Y}' - \hat{\boldsymbol{\alpha}}_{\mathbf{r}}^{\mathbf{T}}\mathbf{X}_{\mathbf{r}}^{\mathbf{T}}\mathbf{Y}' |}, \tag{24}$$

where $\mathbf{Y}'$ is an $mn \times p$ matrix of transformed response values for the merged samples, $\mathbf{X}_{\mathbf{f}}$ is an $mn \times 2m$ matrix of the independent variables for the full model, and $\mathbf{X}_{\mathbf{r}}$ is an $mn \times 2$ matrix of the independent variables for the reduced model. Note that $\hat{\boldsymbol{\alpha}}_f$ is a $2m \times p$ matrix of the estimated regression parameters for the full model, while $\hat{\boldsymbol{\alpha}}_r$ is a $2 \times p$ matrix of the estimated regression parameters for the reduced model. The extended Wilks' lambda chart triggers an out-of-control signal when $\Lambda > UCL$, where UCL is set such that the desired probability value for a Type I error is obtained.

To monitor the covariance matrix, first, the unbiased estimator of the covariance matrix for the kth sample is computed as follows:

$$\mathbf{S}_{\mathbf{k}} = \frac{\mathbf{Y}_{\mathbf{k}}'^{\mathbf{T}}\mathbf{Y}_{\mathbf{k}}' - \hat{\boldsymbol{\alpha}}_{\mathbf{k}}^{\mathbf{T}}\mathbf{X}^{\mathbf{T}}\mathbf{Y}_k'}{n-2}, k = 1,...,m \tag{25}$$

where $\mathbf{Y}_{\mathbf{k}}'$ is an $n \times p$ matrix of transformed response values in the kth profile, $\mathbf{X}$ is an $n \times 2$ matrix of the explanatory variables, and $\hat{\boldsymbol{\alpha}}_{\mathbf{k}}$ is the estimate of $\boldsymbol{\alpha}$ at the kth profile. To obtain the statistic for checking the stability of the covariance matrix, the value of M is computed as follows:

$$M = \frac{| \mathbf{S}_1 |^{(n-2)/2} | \mathbf{S}_2 |^{(n-2)/2} \ldots | \mathbf{S}_m |^{(n-2)/2}}{| \mathbf{S}_{pl} |^{m(n-2)/2}} = \frac{\left(| \mathbf{S}_1 | \times | \mathbf{S}_2 | \times \ldots \times | \mathbf{S}_m |\right)^{(n-2)/2}}{| \mathbf{S}_{pl} |^{m(n-2)/2}} \tag{26}$$

where

$$\mathrm{S}_{pl} = \frac{\sum_{k=1}^{m} (n-2)\mathbf{S}_{\mathbf{k}}}{\sum_{k=1}^{m} (n-2)} = \frac{\sum_{k=1}^{m} \mathbf{S}_{\mathbf{k}}}{m} \tag{27}$$

The values of the M statistic are obtained in the range of $[0,1]$. The chart triggers an out-of-control signal when $M < CV$, in which CV is the threshold of the control chart that is set such that a desired probability of Type I error is obtained. Finally, the M statistic is used in conjunction with the Wilks' lambda statistic. The procedure triggers a signal when at least one of these statistics shows an out-of-control situation.

PERFORMANCE EVALUATION

In this section, the performance of the proposed methods to monitor multivariate simple linear profiles in the case of within-profile autocorrelation in Phase I is investigated and compared through a numerical example. Consider a bivariate simple linear profile as $y_1 = 3 + 2x + \varepsilon_1$ and $y_2 = 2 + x + \varepsilon_2$, where $n = 4$ and $m = 20$. To compare the performance of the proposed methods in Phase I, the overall probability of Type I errors for each is set approximately equal to 0.05. Note that, in Wilks' lambda/M (WL/M) method, to obtain an overall probability of Type I error equal to 0.05, the value of α for each statistic is considered equal to 0.025. Then, the power of these charts to detect different out-of-control scenarios is compared through 10,000 simulation runs in terms of signal probability criterion. The design matrix of the explanatory variables is considered as $\mathbf{X} = [2,4,6,8]^T$. . It is also assumed that $\mathbf{\Sigma}_U = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$ The following tables show the values of signal probability considering different step shifts taking place after sample τ for $\tau = 10,13,16$ under different scenarios of autocorrelation parameters, that is, (ϕ,θ). Table 1 shows the signal probability values of both methods under different step shifts from β_{01} to $\beta_{01} + \lambda_0\sigma_1$ under different values of parameter λ_0. Based on the results in Table 1, as the magnitude of step change in β_{01} increases, the power of the both methods to detect out-of-control situations increases. The results of Table 1 also confirm that the extended Wilks' lambda/M method outperforms the T^2 chart for all step changes occurring in β_{01} under all values of parameter τ. The values of signal probability for both methods under different step shifts from β_{11} to $\beta_{11} + \delta_1\sigma_1$ are summarized in Table 2, which shows results similar to those of Table 1. In other words, as expected, increases in the value of parameter δ_1 lead to increases in the value of signal probability for both methods under different values of (φ,θ). We can also observe that the capability of the extended Wilks' lambda/M method to detect shifts in parameter β_{11} is better than that of the T^2 chart for all values of parameter τ. Table 3 contains the values for signal probability when σ_1 changes to $\gamma_1\sigma_1$. The results of Table 3 show that when $\tau = 10$, , the performance of the extended T^2 chart is better than that of the Wilks' lambda/M method under small shifts. However, for medium to large shifts, the Wilks' lambda/M method outperforms the T^2 method. Moreover, when $\tau = 13$ and $\tau = 16$, the signal probability values obtained by the Wilks' lambda/M method are larger than those from the T^2 chart under almost all step changes.

Table 1. Probability of an out-of-control signal under the sustained shifts from β_{01} to $\beta_{01}+\lambda_0\sigma_1$.

		λ_0									
		0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
$\tau=10$											
$\varphi=0,\theta=0$	T^2	0.059	0.086	0.127	0.178	0.256	0.329	0.394	0.473	0.539	0.585
	WL/M	0.061	0.094	0.131	0.186	0.298	0.351	0.423	0.511	0.578	0.623
$\varphi=0.9,\theta=0.1$	T^2	0.049	0.061	0.069	0.082	0.102	0.122	0.146	0.183	0.216	0.253
	WL/M	0.051	0.066	0.076	0.093	0.109	0.132	0.155	0.198	0.220	0.280
$\varphi=0.1,\theta=0.9$	T^2	0.095	0.209	0.377	0.530	0.634	0.714	0.762	0.802	0.824	0.858
	WL/M	0.108	0.254	0.398	0.587	0.687	0.754	0.821	0.882	0.911	0.962
$\tau=13$											
$\varphi=0,\theta=0$	T^2	0.061	0.081	0.119	0.180	0.256	0.348	0.434	0.522	0.594	0.661
	WL/M	0.061	0.092	0.126	0.201	0.289	0.392	0.472	0.572	0.622	0.710
$\varphi=0.9,\theta=0.1$	T^2	0.052	0.060	0.070	0.084	0.102	0.126	0.162	0.189	0.223	0.259
	WL/M	0.055	0.070	0.082	0.099	0.120	0.137	0.182	0.210	0.243	0.286
$\varphi=0.1,\theta=0.9$	T^2	0.094	0.223	0.420	0.597	0.733	0.813	0.876	0.916	0.930	0.955
	WL/M	0.111	0.276	0.490	0.623	0.789	0.841	0.890	0.923	0.955	0.976
$\tau=16$											
$\varphi=0,\theta=0$	T^2	0.058	0.083	0.116	0.180	0.258	0.362	0.473	0.567	0.663	0.750
	WL/M	0.062	0.090	0.132	0.200	0.280	0.392	0.492	0.589	0.671	0.762
$\varphi=0.9,\theta=0.1$	T^2	0.049	0.057	0.068	0.080	0.094	0.115	0.144	0.180	0.215	0.255
	WL/M	0.051	0.063	0.076	0.095	0.101	0.130	0.154	0.198	0.225	0.267
$\varphi=0.1,\theta=0.9$	T^2	0.077	0.212	0.445	0.672	0.817	0.909	0.960	0.981	0.992	0.996
	WL/M	0.088	0.234	0.470	0.683	0.880	0.920	0.985	1	1	1

Table 2. Probability of an out-of-control signal under the sustained shifts from β_{11} to $\beta_{11}+\delta_1\sigma_1$.

		δ_1									
		0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
$\tau=10$											
$\varphi=0,\theta=0$	T^2	0.055	0.064	0.085	0.113	0.148	0.189	0.235	0.292	0.335	0.386
	WL/M	0.063	0.075	0.093	0.123	0.158	0.192	0.255	0.302	0.351	0.410
$\varphi=0.9,\theta=0.1$	T^2	0.060	0.057	0.063	0.065	0.076	0.081	0.095	0.102	0.112	0.131
	WL/M	0.066	0.065	0.071	0.077	0.089	0.091	0.105	0.302	0.341	0.395
$\varphi=0.1,\theta=0.9$	T^2	0.067	0.109	0.179	0.264	0.365	0.456	0.539	0.604	0.654	0.710
	WL/M	0.080	0.129	0.193	0.301	0.398	0.471	0.587	0.623	0.671	0.725
$\tau=13$											
$\varphi=0,\theta=0$	T^2	0.060	0.065	0.087	0.110	0.145	0.195	0.249	0.307	0.363	0.423
	WL/M	0.060	0.071	0.088	0.120	0.155	0.200	0.267	0.310	0.377	0.439
$\varphi=0.9,\theta=0.1$	T^2	0.050	0.063	0.063	0.065	0.076	0.081	0.095	0.102	0.112	0.131
	WL/M	0.052	0.065	0.070	0.075	0.080	0.085	0.098	0.110	0.118	0.140
$\varphi=0.1,\theta=0.9$	T^2	0.061	0.109	0.179	0.281	0.387	0.504	0.601	0.691	0.752	0.806
	WL/M	0.060	0.112	0.182	0.300	0.400	0.521	0.650	0.710	0.776	0.820
$\tau=16$											
$\varphi=0,\theta=0$	T^2	0.051	0.064	0.074	0.103	0.135	0.176	0.243	0.309	0.377	0.451
	WL/M	0.050	0.080	0.122	0.140	0.193	0.287	0.325	0.395	0.412	0.476
$\varphi=0.9,\theta=0.1$	T^2	0.052	0.054	0.056	0.059	0.066	0.078	0.085	0.095	0.105	0.120
	WL/M	0.057	0.057	0.060	0.066	0.091	0.109	0.115	0.121	0.127	0.138
$\varphi=0.1,\theta=0.9$	T^2	0.060	0.102	0.167	0.267	0.410	0.559	0.678	0.776	0.842	0.895
	WL/M	0.070	0.127	0.183	0.296	0.571	0.703	0.792	0.811	0.846	0.908

Table 3. Probability of an out-of-control signal under the sustained shifts from σ_1 to $\gamma_1\sigma_1$.

		γ_1									
		1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
$\tau=10$											
$\varphi=0,\theta=0$	T^2	0.055	0.063	0.069	0.084	0.091	0.100	0.116	0.121	0.127	0.133
	WL/M	0.050	0.061	0.065	0.092	0.108	0.118	0.144	0.156	0.165	0.171
$\varphi=0.9,\theta=0.1$	T^2	0.057	0.062	0.071	0.083	0.098	0.100	0.108	0.118	0.128	0.143
	WL/M	0.050	0.057	0.062	0.090	0.113	0.122	0.140	0.150	0.169	0.184
$\varphi=0.1,\theta=0.9$	T^2	0.052	0.062	0.072	0.077	0.094	0.099	0.112	0.116	0.130	0.136
	WL/M	0.051	0.060	0.064	0.081	0.110	0.120	0.145	0.151	0.170	0.180
$\tau=13$											
$\varphi=0,\theta=0$	T^2	0.063	0.068	0.092	0.101	0.130	0.147	0.163	0.190	0.201	0.231
	WL/M	0.063	0.070	0.102	0.108	0.137	0.156	0.173	0.200	0.209	0.286
$\varphi=0.9,\theta=0.1$	T^2	0.056	0.067	0.085	0.105	0.133	0.135	0.160	0.186	0.207	0.221
	WL/M	0.055	0.066	0.088	0.111	0.142	0.151	0.178	0.209	0.223	0.254
$\varphi=0.1,\theta=0.9$	T^2	0.056	0.067	0.082	0.103	0.123	0.134	0.164	0.189	0.209	0.227
	WL/M	0.056	0.068	0.090	0.120	0.133	0.148	0.174	0.192	0.217	0.239
$\tau=16$											
$\varphi=0,\theta=0$	T^2	0.067	0.074	0.099	0.124	0.155	0.198	0.228	0.270	0.309	0.333
	WL/M	0.066	0.071	0.105	0.125	0.164	0.210	0.237	0.290	0.312	0.345
$\varphi=0.9,\theta=0.1$	T^2	0.063	0.074	0.102	0.129	0.162	0.199	0.227	0.263	0.300	0.336
	WL/M	0.067	0.080	0.105	0.133	0.168	0.218	0.245	0.300	0.315	0.358
$\varphi=0.1,\theta=0.9$	T^2	0.061	0.079	0.100	0.124	0.161	0.198	0.236	0.260	0.307	0.341
	WL/M	0.060	0.085	0.103	0.130	0.165	0.225	0.248	0.310	0.322	0.364

CONCLUSIONS AND FUTURE RESEARCH

In this paper, Phase I monitoring of multivariate simple linear profiles in the case of within-profile autocorrelation was studied. First, a transformation method was utilized to eliminate the autocorrelation structure of consecutive response values within each profile. Then, two methods, Hotelling's T^2 and Wilks' lambda/M, were proposed based on the transformation applied to response values. A simulation study in terms of signal probability criterion was applied to evaluate and compare the power of the two methods to detect different step shifts in the regression parameters. The results of the simulation showed that the performance of both methods is satisfactory for Phase I monitoring of multivariate simple linear profiles when the response values within each profile follow autoregressive-moving average (ARMA(1,1)) model. The results of the simulation also indicate that the Wilks' lambda/M method outperforms the T^2 method under most step changes. For future research, we recommend taking into account other autocorrelation structures such as AR(1) and MA(1) to monitor multivariate simple linear profiles.

REFERENCES

Abdel-Salam, A. S. G., Birch, J. B. & Jensen, W. A. 2013. A semiparametric mixed model approach to phase I profile monitoring. Quality and Reliability Engineering International **29**(4): 555-569.

Amiri, A., Jensen, W. A. & Kazemzadeh, R. B. 2010. A case study on monitoring polynomial profiles in the automotive industry. Quality and Reliability Engineering International **26**(5): 509-520.

Amiri, A., Mehrjoo, M. & Pasek, Z. J. 2013. Modifying simple linear profiles monitoring schemes in phase II to detect decreasing step shifts and drifts. The International Journal of Advanced Manufacturing Technology **64**(9-12): 1323-1332.

Eyvazian, M., Noorossana, R., Saghaei, A. & Amiri, A. 2011. Phase II monitoring of multivariate multiple linear regression profiles. Quality and Reliability Engineering International **27**(3): 281-296.

Farahani, E., Noorossana, R. & Koosha, M. 2014. Profile monitoring in the presence of outliers. The International Journal of Advanced Manufacturing Technology **74**(1-4): 251-256.

Golnabi, S. & Houshmand A. A. 1999. Multivariate Shewhart X-bar chart. Internet Statistics, 4, a web-based journal: http://interstat.stat.vt.edu/interstat/index/Sep99.html.

Hotelling, H. 1947. Multivariate quality control in techniques of statistical analysis, edited by Eisenhart, Hastay and Walls. McGraw-Hill 111-184.

Jensen, W. A. & Birch, J. B. 2009. Profile monitoring via nonlinear mixed models. Journal of Quality Technology **41**(1): 18-34.

Keramatpour, M., Niaki, S. T. A. & Amiri, A. 2014. Phase-II monitoring of AR(1) autocorrelated polynomial profiles. Journal of Optimization in Industrial Engineering **7**(14): 53-59.

Keramatpour, M., Niaki, S. T. A., Khedmati, M. & Soleymanian, M. 2013. Monitoring and change point estimation of AR(1) autocorrelated polynomial profiles. International Journal of Engineering-Transactions C: Aspects **26**(9): 933-942.

Khedmati, M. & Niaki, S. T. A. 2015. Phase II monitoring of general linear profiles in the presence of between-profile autocorrelation. Quality and Reliability Engineering International **32**(2): 443-452.

Mahmoud, M. A. & Woodall, W. H. 2004. Phase I analysis of linear profiles with calibration applications. Technometrics **46**(4): 380-391.

Mason, R. L. & Young, J. C. 2002. Multivariate statistical process control with industrial applications, ASA-SIAM.

Murphy, B. J. 1987. Selecting out of control variables with the T2 multivariate quality control procedure. The Statistician **36**(5): 571-581.

Narvand, A., Soleimani, P. & Raissi, S. 2013. Phase II monitoring of auto-correlated linear profiles using linear mixed model. Journal of Industrial Engineering International **9**(1): 1-9.

Noorossana, R., Amiri, A. & Soleimani, P. 2008. On the monitoring of autocorrelated linear profiles. Communications in Statistics-Theory and Methods **37**(3): 425-442.

Noorossana, R., Eyvazian, M., Amiri, A. & Mahmoud, M. A. 2010b. Statistical monitoring of multivariate multiple linear regression profiles in phase I with calibration application. Quality and Reliability Engineering International **26**(3): 291-303.

Noorossana, R., Eyvazian, M. and Vaghefi, A. 2010a. Phase II monitoring of multivariate simple linear profiles. Computers & Industrial Engineering **58**(4): 563-570.

Noorossana, R., Fatemi, S. A. & Zerehsaz, Y. 2015. Phase II monitoring of simple linear profiles with random explanatory variables. The International Journal of Advanced Manufacturing Technology **76**(5-8): 779-787.

Noorossana, R., Saghaei, A. & Dorri, M. 2010c. Linear profile monitoring in the presence of non-normality and autocorrelation. International Journal of Industrial Engineering **21**(4): 221-230.

Saghaei, A., Mehrjoo, M. & Amiri, A. 2009. A CUSUM-based method for monitoring simple linear profiles. The International Journal of Advanced Manufacturing Technology **45**(11-12): 1252-1260.

Soleimani, P. & Hadizadeh, R. 2014. Monitoring simple linear profiles in the presence of GARCH and non-normality effects. Proceedings of Control, Decision and Information Technologies (CoDIT) Conference: 393-399.

Soleimani, P., Narvand, A. & Raissi, S. 2013a. Online monitoring of auto correlated linear profiles via mixed model. International Journal of Manufacturing Technology and Management **27**(4-6): 238-250.

Soleimani, P. & Noorossana, R. 2014. Monitoring multivariate simple linear profiles in the presence of between profile autocorrelation. Communications in Statistics-Theory and Methods **43**(3): 530-546.

Soleimani, P., Noorossana, R. & Amiri, A. 2009. Simple linear profiles monitoring in the presence of within profile autocorrelation. Computers & Industrial Engineering **57**(3): 1015-1021.

Soleimani, P., Noorossana, R. & Niaki, S. T. A. 2013b. Monitoring autocorrelated multivariate simple linear profiles. The International Journal of Advanced Manufacturing Technology **67**(5-8): 1857-1865.

Tracy, N. D., Young, J. C. & Mason, R. L. 1992. Multivariate quality control charts for individual observations. Journal of Quality Technology **24**(2): 88-95.

Vasilopoulos, A. V. & Stamboulis, A. P. 1978. Modification of control chart limits in the presence of data correlation. Journal of Quality Technology **10**(1): 20-30.

Woodall, W. H. 2007. Current research on profile monitoring. Production, **17**(3): 420-425.

Zhang, Y., He, Z., Zhang, C. & Woodall, W. H. 2014. Control charts for monitoring linear profiles with within-profile correlation using Gaussian process models. Quality and Reliability Engineering International **30**(4): 487-501.

Zhang, J., Li, Z. & Wang, Z., 2009. Control chart based on likelihood ratio for monitoring linear profiles. Computational statistics & data analysis, **53**(4): 1440-1448.

Zou, C., Tsung, F. & Wang, Z. 2012. Monitoring general linear profiles using multivariate exponentially weighted moving average schemes. Technometrics **49**(4): 395-408.

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