Journal of Engg. Research Vol.10 No. (2A) pp. 210-229 DOI: 10.36909/jer.15209

A mathematical programming approach for a vessel scheduling-transportation problem with multiple sources and destinations and normal daily demand distributions

Fatmah Almathkour *,*, Salem Al-Yakoob** and Dina Alghoul***

** Statistics & Operations Research Department, Kuwait University, Kuwait.*
*** Mathematics Department, Kuwait University, Kuwait.*
**** Corresponding Author: fatmah.almazkoor@ku.edu.kw*

Submitted ***:*** 14/08/2021
Revised ***:*** 19/01/2022
Accepted ***:*** 02/02/2022

ABSTRACT

The principal focus of this research effort is to investigate a maritime stochastic transportation problem. In this case, crude oil shipments are to be transported from multiple sources to multiple destinations. The demand at the destinations is normally distributed, and the violation of certain specified lower and upper daily storage limits can lead to various types of penalties. We aim to utilize (stochastic) mathematical programming approaches to minimize the overall cost of such fleet operation by optimizing the vessel schedules and maintaining acceptable daily storage levels considering the stochastic demand structures. This research effort signifies that the nature of daily demand distributions in the multiple sources-destinations scheduling-inventory scenario significantly impacts the overall fleet schedules and the total expected cost. Therefore, it is crucial to grasp essential stochastic aspects of the daily demands to avoid potential misrepresentation of the operational costs. The robustness of the adopted approach is illustrated by presenting computational results that are based on a wide range of test problems. Moreover, our computational study also examines the impact of variations in demand and the probability of meeting demands on the cost structures.

Keywords: Chance constraints; Integer nonlinear programming; Inventory/shortages; Stochastic vessel scheduling; Transportation.

1. AN OVERVIEW AND CONTRIBUTION

Transportation by sea remains the major mean of transportation in the world-Jean-Paul Rodrigue (2020). Crude oil and its derivatives are the major transported products worldwide, making for about 5.9% of the world value. For example, in 2019, the overall dollar value of transported crude oil was about $1.004 trillion. The bulk of crude oil was shipped from the Gulf Countries, totaling for about $241.7 billion or 24.1% of the worldwide crude oil exports, International Trade Centre (2020). It is imperative to use efficient methods to handle overall vessel operation because of the complexity, uncertainty, and combinational natures of such operation. The average per barrel crude oil price in 2020 and 2021 is, respectively, about $42 and $65. This price dropped below $20 in early 2020, while the recent price (September 2021) is about $69 https://en.wikipedia.org/wiki/COVID-19_pandemic. The continuous changes in the global oil market signify the importance of using practical quantitative transportation approaches to decide on a suitable fleet mix, generate vessel schedules, and better understand demand trends.

This proposed research work investigates a maritime transportation-inventory problem that may encounter, for example, oil companies in the Gulf countries such as KPC (Kuwait Petroleum Corporation) of Kuwait or Aramco of Saudi Arabia. In this case, a company aim to transport crude oil from various source points to multiple destinations according to some contracts that specify demand and penalty structures. Typically, the transporting fleet is composed of a heterogeneous fleet of self-owned and chartered ships of various types to satisfy demand requirements at each destination. Hence, the deliveries of shipments from source points to the destinations are restricted by constraints such as the rates of consumption at each destination, penalties imposed by customers when violating certain customers (destinations) specified lower and upper storage limits, the available vessel transportation routes, and other requirements agreed upon between customers and the company. The principal goal of the transporting company is to minimize the expected overall cost of such vessel operation. This cost is composed of the overall cost of operation of vessels, the expected total penalties, as well as costs of vessel chartering. This objective can be realized by optimizing the vessel schedules while satisfying the stochastic demands at the destinations with acceptable reliability levels.

This research work extends the efforts in Soroush and Al-Yakoob (2018), where a similar vessel scheduling problem based on a single source-destination operation is considered. In this case, the authors consider normally distributed daily demands at the destinations. This article aims to extend findings established in the previous work to a multiple sources-destinations scenario.

Maritime transportation of crude oil and related products is a complex and costly operation. Therefore, efficient routing and scheduling of oil tankers become necessary to reduce operational and penalty expenses. Uncertainties in short- and long-term oil demands directly impact the pertinent fleet operational expenses, shortage and excess inventory penalties, chartering costs. We aim to employ a mathematical modeling approach to simultaneously optimize the fleet schedules and maintain desirable daily storage levels to meet the stochastic demand requirements at the destinations with acceptable reliability levels. Different methods for solving such problems with uncertainties such as chance-constrained programming, stochastic programming, robust optimization, simulation-based optimization and simulation-based heuristics, and fuzzy programming. Although all of these methods can provide good enough solutions, our modeling approach with advances in software and computing power is still a very solid and robust option. In this paper, chance-constrained programming is used to formulate exact mixed-integer nonlinear programming models to obtain good results.

The remainder of this article is organized as follows. Section 2 discusses some of the recent literature pertinent to the studied problem. Section 3 first provides basic modeling constructs such as assumptions, notation, and penalties and then introduces a stochastic vessel scheduling model (SVSM) for the investigated problem. Normal daily demands are incorporated into Model SVSM, and the variational impact of demands on the expected penalty expenses based on the normal demand distributions is examined in Section 4. Section 5 presents computational results and sensitivity analyses for the developed models based on a set of test problems. Finally, a summary and conclusions related to the proposed work are given in Section 6.

2. LITERTURE REVIEW

Literature pertinent to the specific maritime stochastic transportation inventory scenario investigated in the current research effort can be deemed as a stochastic case of the inventory routing problem (IRP) with multiple sources and multiple destinations. The Inventory Routing Problem (IRP) is an extension of the Vehicle Routing Problem (VRP) as it considers a combination of both inventory management and routing decisions into a single problem. In general, the objective of the VRP mainly focuses on minimizing the total cost. Çam et al. (2020) formulated a linear programming model for the VRP. The authors proposed a new type of objective function and new constraints. The objective function tends to minimize the total idle time as in job shop problem rather than minimizing the total cost in most of VRP. In order to meet this objective, the vehicle is suggested to work more in order to minimize the total waiting time. Their constraints allow subtours and multi visits in different time which will reduce the number of vehicle used and increase their efficiency. The authors used the exact method (linear programming) to solve the problem.

In this section, we focus on the literature related to the stochastic maritime transportation-inventory problems; which is relatively limited when compared to the deterministic scenarios. Uncertainties in maritime transportation affect various aspects such as demands, loading/unloading and shipping times, and freight rates, etc. Most of the pertinent studies tackle these uncertainties by utilizing approaches such as chance constrained programming, stochastic programing, robust optimization, simulation-based optimization, and simulation-based heuristics.

Al-Yakoob and Sherali (2018) investigated different scenarios and solution heuristics for a single-source and single-destination deterministic maritime scheduling transportation-inventory problem. In particular, Soroush and Al-Yakoob (2018) studied a single source-destination stochastic maritime scheduling transportation-inventory problem with normally distributed daily demands. The authors presented a stochastic optimization model using chance-constrained programming to obtain the exact mixed-integer nonlinear program with a convex objective function and linear constraints. Later, the work in Soroush and Al-Yakoob (2018) was extended by Soroush et al. (2020) by studying gamma, exponential, and uniform daily demand distributions. Zhao Y. et al. (2018) considered stochastic time and demand for a stochastic intermodal service network design problem and developed a two-stage chance constrained programming model. Sun Y. et al. (2018) formulated a fuzzy chance-constrained mixed-integer nonlinear stochastic program with uncertainty in travel time and capacity to optimize the CO_2 emissions in the container routing problem.

Agra et al. (2018) studied a single product maritime IRP with a heterogeneous fleet of vessels, multiple production and consumption ports with limited storage capacity, constant production and consumption rates, and uncertain sailing times. The authors applied an adaptable robust optimization approach to deal with sailing time uncertainties and presented a decomposition scheme along with an iterative local search heuristic to report some computational results based on a set of real instances. G. Diz et al. (2019) proposed a robust optimization method for a maritime IRP with uncertain total time that vessels spend at ports. The goal was to determine the vessel routings while keeping the inventory levels at ports within some limits.

Sanaz S. et al. (2020) introduced a maritime inventory-routing problem for liquefied natural gas under uncertainty of travel time. The purpose of the study is to examine and compare the shipping costs of split and nonsplit delivery. A metaheuristic method is applied, and the effectiveness of the results is compared with a commercial solver. The authors concluded that split delivery is not recommended in maritime transportation with uncertain nature. Msakni et al. (2018) proposed a mixed-integer program for a maritime inventory transportation problem of liquefied natural gas deliveries and used a column generation approach to solve this problem. Zhang et al. (2017) studied a maritime emergency resource allocation problem under dynamic demands and used a robust optimization model as a general deterministic model. Siswanto et al. (2019) considered a maritime inventory routing problem with multiple time windows. The authors proposed a multiheuristic based genetic algorithm. Friske et al. (2022) proposed a metaheuristic method for solving a maritime inventory routing problem over two discrete-time formulations.

Stochastic programming is a framework for optimization problems with uncertainty. Konur et al. (2017) focused on an integrated stochastic inventory control with order splitting problem when the demands have normal, gamma and Poisson distributions. The authors in Konur et al. (2017) also presented a bi-objective mixed-integer nonlinear program to generate the Pareto fronts for the problem. Rahimi et al. (2017) studied an IRP with multiobjectives and solved it using a fuzzy distribution approach. Roldán et al. (2017) examined the literature on IRP with stochastic demands for a single depot as well as multiple depots. Lima et al. (2018) developed a multistage stochastic programming approach to solve the refined distribution problem of products. In this case, the stochastic model relies on a time series analysis and a tree analysis to effectively deal with uncertainty in oil prices and demands. Rodrigues et al. (2019) studied the uncertainties related to the travel times of MIRP using a stochastic programming model as one of five different general models of the problems. Rafie-Majd et al. (2018) used a Lagrangian relaxation algorithm to solve an integrated inventory-location-routing problem with normally distributed demands. Markov et al. (2018) introduced a unified framework for rich vehicle and inventory routing problems with complex physical and temporal constraints.

In our study, we focused on the stochastic maritime transportation-inventory problems with uncertainty in demand that is normally distributed. In this paper, a stochastic optimization model is presented using chance-constrained programming to obtain the exact mixed-integer nonlinear program with a convex objective function and linear constraints. To the best of our knowledge, this is the first research effort to optimally solve and compare the solutions to a stochastic maritime transportation-inventory problem in which demands have normal probability distribution, with multiple sources and destinations, and four different penalty types are imposed for violating pre-specified lower and upper bounds on the destination's storage levels.

3. PROBLEM FORMULATION

As mentioned earlier, this research effort extends the studies in Soroush and Al-Yakoob (2018) and Soroush et al. (2020). Therefore, to ease presentation, we will commence in Section 3.1 by restating necessary assumptions and notation drawn from Soroush and Al-Yakoob (2018) and Soroush et al. (2020). Section 3.2 discusses various penalty types associated with storage levels that includes type I and type II associated with shortages and type III and type IV associated with excesses similar to those presented in Soroush and Al-Yakoob (2018) and Soroush et al (2020). New assumptions, notation, and constructs will be also introduced as necessary in the remainder of the paper. Section 3.3 then presents expected penalties and chance constraints, which are employed in Section 3.4 to formulate a stochastic vessel model (SVSM) for the proposed problem.

3.1 Preliminary Constructs

The studied stochastic scenario is investigated based on the following assumptions:

- Days are units of the time horizon.
- US dollar is the currency of all costs and all imposed penalties.
- Barrels are the quantities measure of the product.
- A time horizon is specified based on a given contract where the product is transported under specific consumption rates from multiple sources to multiple destinations.
- Same type vessels have identical properties such as size, capacity, speed, etc.
- Same type vessels have equal loading/unloading and roundtrip voyage times.
- A vessel voyage duration is at least two days.
- Vessel loading/unloading and roundtrip voyage times are known a priori.
- A vessel is fully loaded when leaving a source, fully unloaded at the designated destinations, then this vessel returns to a source. (Partially loaded and unloaded scenarios are investigated in a future research).
- Based on a vessel availability, a vessel is potentially chartered for the entire duration of the time horizon or for a subset of the time horizon. Further notations are introduced next.

Table 1. Indices and parameters of the problem.

Vessel related sets, indices, and parameters	
$\text{LEG}_{h,t,n,s_1,d,s_2}$	Leg for vessel n of type $t \in T$ that leaves source $s_1 \in S$ toward destination $d \in D$ on day $h \in H$, and then returns to source $s_2 \in S$.
$T^{Lo}_{t,s_1,d}$	Sum of loading time of a vessel of type $t \in T$ at source $s_1 \in S$ plus voyage time to destination $d \in D$.
T^{Un}_{t,d,s_2}	Sum of unloading time of a vessel of type $t \in T$ at destination $d \in D$ plus voyage time from d back to source $s_2 \in S$.
T_{t,s_1,d,s_2}	Total loading/unloading and roundtrip voyage time for a vessel of type $t \in T$ from source $s_1 \in S$ to destination $d \in D$, and then returning from destination d to source $s_2 \in S$, *i.e.*, $T_{t,s_1,d,s_2} = T^{Lo}_{t,s_1,d} + T^{Un}_{t,d,s_2}$.
$UT_{t,n}$	Maximum number of days that a vessel n, $n = 1,\ldots,TN_t$, of type $t \in T$, can be sent during the time horizon (this time restriction is mainly required for maintenance purposes).
$OC_{t,n}$	Daily operational cost of vessel n, $n = 1,\ldots,TN_t$, of type $t \in T$.
C_{t,n,s_1,d,s_2}	Total operational cost of vessel n, $n = 1,\ldots,TN_t$, of type $t \in T$, from the source $s_1 \in S$ to destination $d \in D$, and then returning back to source $s_2 \in S$, *i.e.*, $C_{t,n,s_1,d,s_2} = T_{t,s_1,d,s_2}(OC_{t,n})$.

3.2 Penalty Types

Penalties are calculated on daily bases, being determined based on the destinations' storage levels. These penalties are extended from those presented in Soroush and Al-Yakoob (2018) to take into consideration multiple destinations. We define the following four penalties.

Type I Penalty:

$$\boldsymbol{P_{1,h,d} = \alpha^m_d\left(SL^m_d - S_{h,d}\right)}, \textbf{ if } \boldsymbol{S_{h,d} \in SL^m_d - A^m_d, SL^m_d}), \tag{3.1}$$

Type II Penalty:

$$\boldsymbol{P_{2,h,d} = \alpha^m_d A^m_d + \beta^m_d\left(SL^m_d - A^m_d - S_{h,d}\right), \beta^m_d > \alpha^m_d}, \textbf{ if } \boldsymbol{S_{h,d} \in (LB_d, SL^m_d - A^m_d)}, \tag{3.2}$$

Type III Penalty:

$$\boldsymbol{P_{3,h,d} = \alpha^M_d\left(S_{h,d} - SL^M_d\right)}, \textbf{ if } \boldsymbol{S_{h,d} \in SL^M_d, SL^M_d + A^M_d}, \tag{3.3}$$

Type IV Penalty:

$$\boldsymbol{P_{4,h,d} = \alpha_d^M A_d^M + \beta_d^M\left[S_{h,d} - \left(SL_d^M + A_d^M\right)\right], \beta_d^M > \alpha_d^M, \text{ if } S_{h,d} \in \left(SL_d^M + A_d^M, UB_d\right).} \quad (3.4)$$

3.3 Expected Penalıtıes And Chance Constraınts

Note that $S_{h,d}, h \in H, d \in D,$ are random variables with probability density functions (pdfs) $f_{h,d}(t)$, since all daily demands are random variables.

Therefore, the expected daily penalties of Types I-IV, respectively, given by $E\left[P_{i,h,d}\right]\ for\ i = 1,\ldots,4$, (referring to (3.1)–(3.4)), are defined as follows:

$$E\left[P_{1,h,d}\right] = \alpha_d^m \int_{SL_d^m - A_d^m}^{SL_d^m} (SL_d^m - t)\, f_{h,d}(t)dt, h \in H, d \in D, \quad (3.5)$$

$$E\left[P_{2,h,d}\right] = \int_{LB_d}^{SL_d^m - A_d^m} [\alpha_d^m A_d^m + \beta_d^m (SL_d^m - A_d^m - t)]\, f_{h,d}(t)dt, h \in H, d \in D, \quad (3.6)$$

$$E\left[P_{3,h,d}\right] = \alpha_d^M \int_{SL_d^M}^{SL_d^M + A_d^M} (t - SL_d^M)\, f_{h,d}(t)dt, h \in H, d \in D, \quad (3.7)$$

And

$$E\left[P_{4,h,d}\right] = \int_{SL_d^M + A_d^M}^{UB_d} \left[\alpha_d^M A_d^M + \beta_d^M [t - (SL_d^M + A_d^M)]\right] f_{h,d}(t)dt, h \in H, d \in D. \quad (3.8)$$

Therefore, the overall expected penalty on day $h \in H$ and destination $d \in D$ using (3.5)–(3.8), is given by $\sum_{i=1}^{4} E\left[P_{i,h,d}\right]$.

Figure 1 shows a schematic network structure of some routes in the operation along with the storage levels and the penalty types.

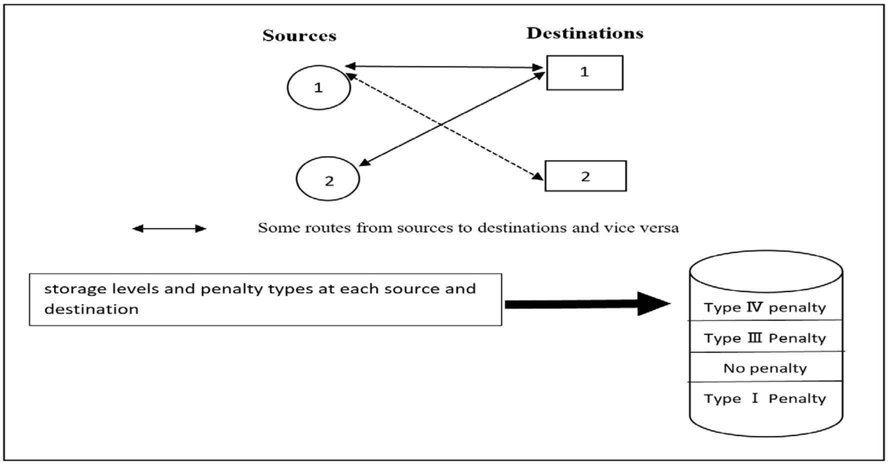

Figure 1. Illustration of network structure, storage levels and penalty types of the problem.

3.4 Model SVSM

Next, we extend model SVSM which is formulated in Soroush and Al-Yakoob (2018). Next, we set binary decision variables. Let

$$X_{h,t,n,s_1,d,s_2} = \begin{cases} 1 & \text{if LEG}h,t,n,s_1,d,s_2 \text{ is selected,} \\ 0 & \text{otherwise.} \end{cases}$$

Another set of binary variables is defined to ensure that a vessel is dispatched from sour s on day h only if it is unused on this day.

Let

$$Y_{h,t,n,s} = \begin{cases} 1 & \text{if vessel } n \text{ of type } t \text{ is available at source } s \text{ on day } h, \\ 0 & \text{otherwise.} \end{cases}$$

Finally, the chartering decision variables are defined as follows:

$$V_{t,n} = \begin{cases} 1 & \text{if vessel } n \in \{O_t + 1, \ldots, O_t + CH_t\} \text{ of type } t \text{ is selected for chartering during the time horizon,} \\ 0 & \text{otherwise,} \end{cases}$$

Note that $Y_{h,t,n,s_1} = 0$ leads to $X_{h,t,n,s_1,d,s_2} = 0 \forall (d, s_2)$.

Let φ_X and φ_Y respectively denote the set of *X*- and *Y*-variables that are initialized to zero values. Our vessel scheduling stochastic model is then formulated as follows. (Note that all coming indices are supposed to take exact values.)

SVSM:

$$\textbf{Minimize} \sum_h \sum_t \sum_n \sum_{s_1} \sum_d \sum_{s_2} C_{t,n,s_1,d,s_2} X_{h,t,n,s_1,d,s_2}$$

$$+ \sum_h \sum_{i=1}^{4} \sum_d E[p_{i,h,d}] + \sum_t \sum_{n=O_t+1}^{TN_t} \$_{t,n} V_{t,n},$$

Subject to

$$S_{h,d} = w_d + \sum_t \sum_n \sum_{s_1} \sum_{s_2} \sum_{h: h+T^{Lo}_{t,s_1,d,s_2} \in \{1,\ldots,h\}} \Omega_t X_{h,t,n,s_1,d,s_2} - TC_{h,d}, \quad (3.9)$$

$\forall h$ **and** $d \in \{1, \ldots, |D|\}$,

$$prob(SL_d^m \le S_{h,d} \le SL_d^M) \ge (1 - r_{h,d}), \forall h \text{ and } d \in \{1, \ldots, |D|\}, \quad (3.10)$$

$$\boldsymbol{Y_{h,t,n,s}} = \boldsymbol{Y_{h-1,t,n,s}} - \sum_d \sum_{s_2} \boldsymbol{X_{h-1,t,n,s,d,s_2}} + \sum_{s_1} \sum_d \sum_{\substack{h: \\ h+T_{t,s_1,d,s}=h}} \boldsymbol{X_{h,t,n,s_1,d,s}}, \tag{3.11}$$

$\forall h \geq 2, t, n, s,$

$$\sum_d \sum_{s_2} \boldsymbol{X_{h,t,n,s,d,s_2}} \leq \boldsymbol{Y_{h,t,n,s}}, \forall \boldsymbol{h, t, n, s}, \tag{3.12}$$

$$\sum_s \boldsymbol{Y_{h,t,n,s}} \leq \boldsymbol{V_{t,n}}, \ \forall \ \boldsymbol{h, t, n} \in \{\boldsymbol{O_t + 1, \ldots, O_t + CH_t}\}, \tag{3.13}$$

$$\sum_t \sum_n \sum_d \sum_{s_1} \boldsymbol{\Omega_t X_{h,t,n,s,d,s_1}} \leq \boldsymbol{Q_s}, \forall \boldsymbol{h} \textbf{ and } \boldsymbol{s} \in \{\boldsymbol{1, \ldots, |S|}\}, \tag{3.14}$$

$$\sum_h \sum_{s_1} \sum_d \sum_{s_2} X_{h,t,n,s_1,d,s_2} \boldsymbol{T_{t,s_1,d,s_2}} \leq \boldsymbol{UT_{t,n}}, \ \forall \ \boldsymbol{t, n}, \tag{3.15}$$

$X_{h,t,n,s_1,d,s_2} \in \{0,1\}$ for $X_{h,t,n,s_1,d,s_2} \notin \phi_X$, $\forall h, t, n, s_1, d, s_2$, and (3.16)

$Y_{h,t,n,s} \in [0,1]$, for $Y_{h,t,n,s} \notin \phi_Y$, $\forall h, t, n, s$, and

$V_{t,n} \in [0,1]$, $\forall \ t, n = O_t + 1, .., O_t + CH_t$,

$\boldsymbol{S_{h,d}} \geq \boldsymbol{0}$, $\forall \boldsymbol{h}$ **and** $\boldsymbol{d} \in \{\boldsymbol{1, \ldots, |D|}\}$.

4. MODEL SVSM WITH NORMAL DAILY DEMAND

Initially, we further study the chance constraint given by (3.10) daily demands that are normally distributed. Then, we state four theorems to set the foundation for deriving closed-form expressions for the exact expected total daily penalties of Types I-IV defined in (3.5-3.8). Our proposed stochastic optimization model with normal demands (SVSM-N) is then presented.

For a given $h \in H$ and $d \in D$, the daily demands $R_{h_1,d}, h_1 = 1, \ldots, h$ are independent normal random variables with means $\hat{\mu}_{h_1,d}$ and variances $\hat{\sigma}^2_{h_1,d}$ *i.e.*, $R_{h_1,d} \sim N(\hat{\mu}_{h_1,d}, \hat{\sigma}^2_{h_1,d})$. The cumulative daily demands $TC_{h,d} = \sum_{h_1=1}^{h} R_{h_1,d}$ are also normally distributed, *i.e.*, $TC_{h,d} \sim N(\mu_{h,d}, \sigma^2_{h,d})$, where the mean and variance of $TC_{h,d}$ are defined by $\mu_{h,d} = \sum_{h_1=1}^{h} \hat{\mu}_{h_1,d}$ and $\sigma^2_{h,d} = \sum_{h_1=1}^{h} \hat{\sigma}^2_{h_1,d}$, respectively. Also, the storage level $S_{h,d}$ given in (3.9) is a normal random variable with mean $M_{h,d}$ and variance $\sigma^2_{h,d}$. Hence, $M_{h,d}$ is defined as follows:

$$M_{h,d} = w_d + \sum_{t \in T} \sum_{n=1}^{TN_t} \sum_{s_1 \in S} \sum_{s_2 \in S} \sum_{h_1 \in \{1, \ldots, h - T^{Lo}_{t,s_1,d,s_2}\}} \Omega_t X_{h_1,t,n,s_1,d,s_2} - \mu_{h,d}, \forall h \in H, d \in D. \tag{4.1}$$

Then, we deduce the following constraint:

$$M_{h,d} \geq 3.5\sigma_{h,d}, h \in H, d \in D. \tag{4.2}$$

The chance constraint (3.10) can be equivalently written as

$$Prob\left[\frac{SL_d^m - M_{h,d}}{\sigma_{h,d}} \leq N_{S_{h,d}} \leq \frac{SL_d^M - M_{h,d}}{\sigma_{h,d}}\right] \geq \left(1 - r_{h,d}\right), h \in H, d \in D. \tag{4.3}$$

thus, the chance constraint (4.3) can be stated as follows:

$$M_{h,d} \leq SL_d^M - \varepsilon_{h,d}^* \sigma_{h,d}, h \in H, d \in D. \tag{4.4}$$

Hence,

$$Prob\left[N_{S_{h,d}} \leq \frac{SL_d^m - M_{h,d}}{\sigma_{h,d}}\right] \leq \tau_{h,d}, h \in H, d \in D, \tag{4.5}$$

and

$$Prob\left[N_{S_{h,d}} \geq \frac{SL_d^M - M_{h,d}}{\sigma_{h,d}}\right] \leq \varsigma_{h,d}, h \in H, d \in D, \tag{4.6}$$

where $\tau_{h,d}$ and $\varsigma_{h,d}$ are specific known constants defined as $0 < \tau_{h,d} < 1$ and $0 < \varsigma_{h,d} < 1$, and $\tau_{h,d} + \varsigma_{h,d} = r_{h,d}$, in order to guarantee a maximum probabilities that $S_{h,d} \in [SL_d^m, SL_d^M], h \in H, d \in D$. Now, for $h \in H, d \in D$, let $\tau_{h,d}^*$ be as $Prob\left[N_{S_{h,d}} \leq \tau_{h,d}^*\right] = \int_{-\infty}^{\tau_{h,d}^*} \phi(t)dt = \tau_{h,d}$. This implies that $\frac{(SL_d^m - M_{h,d})}{\sigma_{h,d} \leq} \tau_{h,d}^*, h \in H, d \in D$, and thus, the chance constraint (4.5) can be stated as follows:

$$M_{h,d} \geq SL_d^m - \tau_{h,d}^* \sigma_{h,d}, h \in H, d \in D. \tag{4.7}$$

Also, $h \in H, d \in D$, let $\varsigma_{h,d}$ be such that $Prob\left[N_{S_{h,d}} \geq \varsigma_{h,d}^*\right] = \int_{\varsigma_{h,d}^*}^{\infty} \phi(t)dt = \varsigma_{h,d}$. This implies that $\frac{\varsigma_{h,d}^* \leq (SL_d^M - M_{h,d})}{\sigma_{h,d}}$; thus, the chance constraint (4.6) can be stated as follows:

$$M_{h,d} \leq SL_d^M - \varsigma_{h,d}^* \sigma_{h,d}, h \in H, d \in D. \tag{4.8}$$

Based on Theorems 1–4 that are presented and proved in Soroush and Al-Yakoob (2018), the exact expected total daily penalties of Types I-IV $E\left[P_{h,d}\right] = \sum_{i=1}^{4} E\left[P_{i,h,d}\right]$, can be written in the following closed-form expression:

$$E\left[P_{h,d}\right] = \sigma_{h,d} \begin{bmatrix} \alpha_d^m\left[\phi\left(z_{1,h,d}\right) + z_{1,h,d}\Phi\left(z_{1,h,d}\right)\right] + (\beta_d^m - \alpha_d^m)\left[\phi\left(z_{2,h,d}\right) + z_{2,h,d}\Phi\left(z_{2,h,d}\right)\right] \\ +\alpha_d^M\left[\phi\left(z_{3,h,d}\right) + z_{3,h,d}\Phi\left(z_{3,h,d}\right)\right] \\ +(\beta_d^M - \alpha_d^M)\left[\phi\left(z_{4,h,d}\right) + z_{4,h,d}\Phi\left(z_{4,h,d}\right)\right] \\ -\beta_d^M z_{4,h,d} \end{bmatrix} \tag{4.9}$$

Where

$$z_{1,h,d} = \frac{(SL_d^m - M_{h,d})}{\sigma_{h,d}} \tag{4.10}$$

$$z_{2,h,d} = \frac{(SL_d^m - A_d^m - M_{h,d})}{\sigma_{h,d}} \tag{4.11}$$

$$z_{3,h,d} = \frac{(SL_d^M - M_{h,d})}{\sigma_{h,d}} \tag{4.12}$$

$$z_{4,h,d} = \frac{(SL_d^M + A_d^M - M_{h,d})}{\sigma_{h,d}} \tag{4.13}$$

$$+\alpha_d^M A_d^M$$

and $M_{h,d}$ is given by (4.1). Next, we state our mixed-integer nonlinear program SVSM-N.

SVSM-N:

Minimize $\sum_h \sum_t \sum_n \sum_{s_1} \sum_d \sum_{s_2} C_{t,n,s_1,d,s_2} \boldsymbol{X_{h,t,n,s_1,d,s_2}}$

$+ \sum_{\boldsymbol{h}} \sum_{\boldsymbol{i=1}}^{\boldsymbol{4}} \sum_{\boldsymbol{d}} \boldsymbol{E}\left[\boldsymbol{p_{i,h,d}}\right] + \sum_t \sum_{n=O_t+1}^{TN_t} \$_{\boldsymbol{t,n}} \boldsymbol{V_{t,n}}$,

where $E\left[P_{h,d}\right]$ is given by (4.9), $\mu_{h,d} = \sum_{h_1=1}^{h} \hat{\mu}_{h_1,d}$ and $\sigma_{h,d} = \sqrt{\sum_{h_1=1}^{h} \hat{\sigma}_{h_1,d}^2}$, $\forall h \in H, d \in D$.

subject to (3.11)–(3.16), (4.1), (4.2), (4.7), and (4.8).

The objective function of model SVSM-N minimizes the sum of the overall vessel operational expenses, total expected penalty cost, and total chartering costs. Constraints (3.11)–(3.16) impose certain restrictions on the operation of vessels. Constraint (4.1) defines the mean daily storage level $M_{h,d}$. Constraint (4.2) guarantees the nonnegativity of the daily storage level. Constraints (4.7) and (4.8) express the chance constraint (3.10) and ensure that the mean storage level lies in $[SL_d^m, SL_d^M]$, for $h \in H$, $d \in D$.

5. COMPUTATIONAL RESULTS AND SENSITIVITY ANALYSES

This section presents some computational experiments related to solving SVSM-N and its respective deterministic model DVSM. Also, we conduct sensitivity analyses in order to assess that if the optimal expected total cost is affected by the changes in the probabilities and variations of daily demands.

5.1. Generatıon of Test Problems

We consider 12 test problems, denoted by P_1–P_{12}, based on which, for each test problem, we randomly generate 50 instances. We present the same example and inputs used in Soroush and Al-Yakoob (2018) Tables 2, 3, and 4 given subsequently provide extra data to all test problems.

Table 2. Basic data and assumptions.

Sources $\lvert S\rvert = 2$:	$S = \{1,2\}$
Destinations$\lvert D\rvert = 2$:	$D = \{1,2\}$
Vessel types $\lvert T\rvert = 3$:	$T = \{1,2,3\}$
$UT_{t,n} = UT_t$, for $t \in T$ and $n = 1,\dots,NT_t$	The maximum vessel usage in days within a time horizon is the same for all vessels of the same type, say denoted by UT_t.

Table 3. Intervals for randomly generating the common input data for the test problems.
P1 –P12

Total loading/unloading and roundtrip voyage times for vessels of types 1, 2, and 3, *i.e.*, for $(T_{1,s_1,d,s_2}, T_{2,s_1,d,s_2}, T_{3,s_1,d,s_2})$, where it is assumed that $T^{Lo}_{t,s_1,d,s_2} = T^{Un}_{t,s_1,d,s_2}, s_1, s_2 = 1,2 \ \ d = 1,2$:	([4, 8], [10, 14], [16, 20])
Total loading/unloading and roundtrip voyage times for vessels of types 1, 2, and 3, *i.e.*, for $(T_{1,s_1,d,s_2}, T_{2,s_1,d,s_2}, T_{3,s_1,d,s_2})$, where it is assumed that $T^{Lo}_{t,s_1,d,s_2} = T^{Un}_{t,s_1,d,s_2}, s_1, s_2 = 1,2, \qquad d = 1,2$:	([4, 8], [10, 14], [16, 20])

Table 4. Intervals for randomly generating specific input data for test problems P_1 –P_{12} with normal demands.

Normal demands	
$(\hat{\mu}_{h,d}, \hat{\sigma}_{h,d}), h \in H, d = 1,2$	([150, 200], [35, 40])
$w_d, d = 1,2$	[5000, 6000]
$SL_d^m, d = 1,2$	[1750, 2750]
$SL_d^M, d = 1,2$	[5500, 6500]
$A_d^m, d = 1,2$	[900, 1100]
$A_d^M, d = 1,2$	[900, 1100]

For the Normal distributed daily demands, the means and standard deviations $(\hat{\mu}_{h,d}, \hat{\sigma}_{h,d})$ are, respectively, randomly generated from ([150, 200], [35,40]), where $Prob(R_{h,d} \geq 0) \approx 1, \forall h \in H, d = 1,2$ (as indicated earlier in chapter 4. Models SVSM-N are solved based on the randomly generated 750 instances, fifty instances of each of the problems P_1 – P_{12}. The 750 instances of DVSM relative to SVSM-N were generated by specifying the deterministic daily demands equal to $\hat{\mu}_{h_1,d}, h_1 \in H, d = 1,2$, ($\hat{\sigma}_{h_1,d}$ is ignored).

5.2 Computatıonal Experıments

To conduct our computational experimentations, we have used a HP Laptop 14-cf0xxx computer with Intel(R) Core(TM) i5-8250U CPU, 1.80 GHz, 8 GB of RAM, 64-bit OS. We solved DVSM using the CPLEX optimization package with its enhanced branch-and-cut methods for solving mixed-integer programs, whereas, to solve models SVSM-N, we applied Particle Swam Optimization using MATLAB R2020a for solving mixed-integer nonlinear programs. The termination limit was set at 16,500,000 iterations. We also set the convergence criteria at "Global Optimality." Obtained results indicated that all the generated instances of SVSM-N and DVSM were optimally solved within the set limits. We define the following terms that will be used in our computational and sensitivity analyses in the remainder of this chapter.

1. *v**(DVSM): Optimal objective function value of model DVSM, which represents the total cost (overall operational cost, penalty cost, and chartering cost) obtained via solving the deterministic model DVSM.
2. *v**(SVSM-I): Optimal objective function value of model SVSM-I, which represents the expected total cost (overall operational cost, expected total penalty cost, and chartering cost) obtained via solving the stochastic model SVSM-I, where I=N for the normal demand.
3. Overcost-I (%) = 100[*v**(SVSM-I) – *v**(DVSM)] / *v**(DVSM): Percentage over cost of *v*∗(SVSM-I) relative to *v*∗(DVSM), where I=N for the normal case.
4. Min-I: Least Overcost-I (%) obtained when solving Model SVSM-I based on instances of a given test problem, where I=N for the normal case.

5. Max-I: Largest Overcost-I (%) obtained when solving Model SVSM-I based on instances of a given test problem, where I=N for the normal case.
6. Med-I: Median Overcost-I (%) obtained when solving Model SVSM-I based on instances of a given test problem, where I=N for the normal case.
7. Ave-I: Average of Overcost-I (%) obtained when solving Model SVSM-I based on instances of a given test problem, where I=N for the normal case.
8. Op-cost-I: Average of operational costs of vessels obtained when solving Model SVSM-I based on instances of a given test problem, where I=N for the normal case.
9. Exp-total-pen-I: Average of expected total penalties obtained when solving model SVSM-I based on instances of a given test problem, where I=N for the normal case.
10. Chart-cost-I: Average of chartering costs obtained when solving model SVSM-I based on instances of a given test problem, where I=N for the normal case.
11. RT: Average of run times (in seconds) for solving model DVSM. based on instances of a given test problem,
12. RT-I: Average of run times (in seconds) for solving stochastic model SVSM-I, based on instances of a given test problem, where I=N for model SVSM-N.

Table 5. Summary statistics of Overcost-N(%), the breakdowns (in percent) of the expected total cost, and the run times RT-N and RT.

	Over-cost-N (%)				Exp. total cost (%)			Run times	
Problem	Min-N	Max-N	Med-N	Avg-N	Op-cost-N	Exp-total-pen-N [Types I..., IV]	Chart-cost-N	RT-N	RT
P 1	2.66	39.19	18.98	19.67	78.53	21.47 [10.43, 0.0, 11.04, 0.0]	0.0	1.217	0.099
P 2	2.76	40.3	17.93	18.91	77.95	22.05 [11.02, 0.0, 11.03, 0.0]	0.0	1.460	0.102
P 3	2.55	43.61	18.73	20.63	56.12	25.89 [11.33, 3.06, 11.5, 0.0]	17.99	1.851	0.272
Average	**2.66**	**41.03**	**18.55**	**19.74**	**70.87**	**23.14 [10.93, 1.02, 11.19, 0.0]**	**6.00**	**1.510**	**0.158**
P 4	6.99	66.36	27.81	29.96	76.14	23.86 [12.16, 0.0, 11.7, 0.0]	0.0	9.011	0.492
P 5	7.71	62.86	28.13	28.88	75.02	24.98 [12.22, 0.0, 12.76, 0.0]	0.0	10.162	0.577
P 6	7.53	67.98	29.27	30.3	50.39	27.99 [12.06, 3.18, 12.75, 0.0]	21.62	10.898	0.657
Average	**7.41**	**65.73**	**28.4**	**29.71**	**67.18**	**25.61 [12.15, 1.06, 12.40, 0.0]**	**7.20**	**10.024**	**0.575**
P 7	10.81	94.63	44.09	46.97	71.38	28.62 [14.5, 0.0, 14.12, 0.0]	0.0	32.092	1.512
P 8	11.98	99.97	47.3	48.89	69.66	30.34 [15.25, 0.0, 15.09, 0.0]	0.0	36.694	1.764
P 9	11.64	106.6	48.75	49.96	45.24	31.81 [14.22, 3.46, 14.13, 0.0]	32.95	37.749	1.801
Average	**11.48**	**100.4**	**46.71**	**48.61**	**62.10**	**30.26 [14.66, 1.15, 14.45, 0.0]**	**10.97**	**35.512**	**1.692**
P 10	17.7	99.9	59.85	65.11	68.05	31.95 [16.88, 0.0, 15.07, 0.0]	0.0	60.011	2.539
P 11	19.1	110.2	67.31	68.34	67.22	32.78 [16.73, 0.0, 16.05, 0.0]	0.0	62.631	2.674
P 12	17.6	107.3	69.87	69.97	29.88	34.11 [15.05, 4.02, 15.04, 0.0]	36.01	65.745	2.936
Average	**18.13**	**105.8**	**65.68**	**67.81**	**55.05**	**32.95 [16.22, 1.34, 15.39, 0.0]**	**12.00**	**62.796**	**2.783**

Based on the results of Table 5, the following observations under the summary statistics of the Overcost-N (%), expected total cost components obtained via solving SVSM-N, and the average run times are presented by using one-tailed or two tailed Wilcoxon rank-sum tests.

1. The over cost given by Overcost-N (%) is noticeably larger than zero (p -value < 0.0001, one-tailed test) due to meeting the demands from the no penalty interval for the storage levels with the prescribed reliabilities.

2. The overcosts of problem instances having similar time horizon for a stochastic model are (in most cases) not significantly different (*p* -value > 0.2284, two-tailed test); hence, the configurations of the self-owned and chartered vessels (in most cases) does not lead to significant impact on the over cost.
3. Increasing the days of the time horizon consistently leads to an increase in Overcost-N (%) (*p* -value < 0.0001, one-tailed test).
4. The expected total cost obtained via solving a stochastic model is significantly higher than that of its deterministic counterpart (*p*-value < 0.00001, one-tailed test). As an example, with regard to the expected total cost components obtained via solving SVSM-N, the expected Type I penalty percentage is significantly higher than the expected Type II penalty percentage using one-tailed test with p -value < 0.0001; and the expected Type III penalty percentage is significantly higher than that of Type IV (p -value < 0.0001, one-tailed test). The expected Type I and Type III penalties percentages are significantly the same using two-tailed test with p -value > 0.2557, since their relevant symbols are generated randomly from same intervals.
5. It was observed that, at any time horizon, chartering vessels significantly increase the following:
   (i) The cost percentage using one-tailed test with p -value < 0.0001.
   (ii) The expected type 2 penalty percentage, in order to prevent any stock-outs of the product using one-tailed test with p -value < 0.0001.
   (iii) The chartering vessels cost percentage using one- tailed test with p -value < 0.0001.
6. We found that any increase in the number of days in horizon impacts the following:
   (i) The vessel operational percentage cost will significantly decrease (p -value < 0.0001, one-tailed test),
   (ii) The expected total penalty percentage will significantly increase (p -value < 0.0001, one-tailed test),
   (iii) The chartering cost percentage will not be affected (p -value > 0.1884, one-tailed test).
7. For the run time of the operation, we noticed the following:
   (i) The run solution time for a stochastic model is more than that of its deterministic counterpart instance (*p*-value < 0.00001, one-tailed test).
   (ii) The run times for solving various instances with the same time horizon of a stochastic model are not significantly different (*p*-value > 0.2532, two-tailed test). It's also occur for their corresponding deterministic instances.
   (iii) An increase in the days of time horizon constantly leads to an increase in the run solution time for a stochastic model (*p*-value < 0.0001, one-tailed test). This also applies to the run time of its deterministic counterpart.

The above findings substantiate the proposition that the nature of the daily demand probability distribution significantly impacts the expected total cost as well as its components. Therefore, discarding demand uncertainties is likely to underestimate the overall operational costs, penalty expenses and chartering costs, leading to unrealistic solutions. The same findings were also deduced in Soroush and Al-Yakoob (2018) and Soroush et al. (2020) for the single source and single destination scenario.

We have also examined the similarity of the optimal schedules obtained from SVSM-N and its DVSM. A measure was introduced to determine the percentage of similarity between an optimal schedule obtained via one of the stochastic models (SVSM-N) and its DVSM counterpart. This percentage is defined as 100 [no. of similar voyages (of all vessels of all types) in the optimal schedules obtained from a stochastic model and its DVSM counterpart] / [max. of the no. of that voyages]. As example, suppose that 0%, 25%, 50%, and 100% similarities, respectively, show that our voyages are entirely different, 25% similar, 50% similar, and completely the same.

Table 6 presents the input data and normal demand for a specific instance. Table 7 provides the optimal schedules for SVSM-N, and their DVSM counterparts as well as the percentages of similarity and overcost between the optimal schedules. The percentage of similarity of the optimal schedules for SVSM-N and its DVSM is 8.33% (the two schedules share only one identical vessel voyage, i.e., vessel 1 of type 1 leaving on day 2 and returning on day 6).

Table 6. Extra Input data for a problem instance with normal demand.

(a) specific input data	
	Normal demand with $(\hat{\mu}_{h,d}, \hat{\sigma}_{h,d})$=(150, 40), $h \in H, d = 1,2$
$w_d, d = 1,2$	4000
$SL_d^m, d = 1,2$	2250
$SL_d^M, d = 1,2$	6000
$A_d^m, d = 1,2$	1000
$A_d^M, d = 1,2$	1000

Table 7. Optimal schedules of SVSM-N, and DVSM for the problem instance of Table 6 and the percentages of their overcosts and schedule similarities.

(a) SVSM-N versus its DVSM							
SVSM-N optimal schedule				DVSM optimal schedule			
Vessel no.	Vessel type	Leav. day	Ret. day	Vessel no.	Vessel type	Leav. day	Ret. day
1	1	1	5	1	1	2	6
1	1	2	6	3	1	5	9
2	1	8	14	1	1	7	11
3	1	10	16	2	1	9	13
2	1	13	17	3	1	11	15
4	1	14	18	4	1	13	17
1	1	15	19	2	1	15	19
3	1	17	21	3	1	20	24
1	1	20	24	2	1	21	25
4	1	21	25	1	1	22	26
3	1	22	26				
2	1	23	27				
v ∗(SVSM-N) = $359,580				v ∗(DVSM) = $299,760			
Overcost-N (%) = 19.96% and, schedule similarity percentage = 8.33%							

Table 8. Summary statistics on the similarities' percentages of the SVSM-N, and DVSM optimal schedules for $\mathbf{P_1} - \mathbf{P_{12}}$

(a) SVSM-N versus its DVSM			
Similarity (%)			
Test problem (1)	Min (2)	Max (3)	Avg (4)
P_1	5.25	18.99	11.86
P_2	5.44	19.76	11.97
P_3	6.02	19.98	12.19
Average	**5.57**	**19.58**	**12.01**
P_4	4.12	15.88	9.88
P_5	4.23	16.05	9.97
P_6	4.56	16.74	10.08
Average	**4.30**	**16.22**	**9.98**
P_7	3.06	11.05	5.64
P_8	3.68	11.58	5.85
P_9	3.79	12.07	6.01
Average	**3.51**	**11.57**	**5.83**
P_{10}	1.09	4.63	1.87
P_{11}	1.25	5.28	2.37
P_{12}	1.62	5.71	2.76
Average	**1.32**	**5.21**	**2.33**

We also have computed the percentages of similarity between the optimal schedules for SVSM-N and its respective DVSM associated with the various instances of each of the test problems $P_1 - P_{12}$. Table 8 displays the minimum, average, and maximum of such percentages for every problem. The results indicate the following:

1. The percentages of similarities are small (with an average of at most 12.19% for SVSM-N and its DVSM.
2. The percentages of similarity among SVSM-N and its corresponding DVSM are different ((p-value < 0.00001, two-tailed Wilcoxon rank-sum test).
3. The percentages of similarity for the instances of the same number of days in horizon of each problem are significantly the same using two-tailed Wilcoxon rank-sum test with p-value > 0.3327.
4. The percentages of similarity are noticeably decreased according to the increase of the number of days in the time horizon using one-tailed Wilcoxon rank-sum test with p-value < 0.00001.

For more details related to the computational results and analysis, please refer to the

Dina E. A. (2021). A Vessel Scheduling and Inventory Problem with Normal and Gamma Demand Distributions – Multiple Sources and Destinations
[Master's Thesis, Kuwait University] (http://dx.doi.org/10.13140/RG.2.2.18801.40806)

5.3 Sensıtıvıty Analyses

This section investigates the effects when $(1 - r_{h,d}), h \in H, d \in D$, (*i.e.*, service levels) takes various values on the Overcost-N(%) relative to its DVSM of the test problems $P_1 - P_{12}$. From Table 9, we can notice that the increase in $(1 - r_{h,d})$values leads to the increase in the Overcost-N(%) relative to its DVSM (p -value < 0.0001, one-tailed test). Then, higher probabilities in order to meet demand consistently lead to higher overcosts.

Table 9. Overcost-N(%) for the test problems $\mathbf{P_1} - \mathbf{P_{12}}$ when $(\mathbf{1} - \boldsymbol{r_{h,d}})$ takes various values.

(a) SVSM-N versus its DVSM				
$(1 - r_{h,d})$				
Test problem	0.80	0.85	0.90	0.95
P_1	14.67	18.04	19.56	21.19
P_2	14.04	19.12	19.87	21.71
P_3	15.51	19.68	21.02	22.47
Average	**14.74**	**18.95**	**20.15**	**21.79**
P_4	22.51	26.72	29.04	31.36
P_5	22.87	25.32	29.57	31.24
P_6	23.03	28.08	30.89	32.66
Average	**22.80**	**26.71**	**29.83**	**31.75**
P_7	37.92	44.02	46.87	50.01
P_8	38.79	45.69	47.42	50.24
P_9	39.58	46.83	48.05	51.07
Average	**38.76**	**45.51**	**47.45**	**50.44**
P_{10}	55.28	63.07	65.82	66.15
P_{11}	56.79	64.26	66.02	67.23
P_{12}	57.08	64.03	66.97	68.04
Average	**56.38**	**63.79**	**66.27**	**67.14**

In addition, in Table 10, we sought the effects when the standard deviations $\hat{\sigma}_{h_1,d}$ of normal daily demands are randomly generated from various intervals based on the test problems $P_1 - P_{12}$. Table 10 displays the results for in which the standard deviations of daily demands, $\hat{\sigma}_{h_1,d}$, $h_1 \in H, d \in D$ are randomly sampled from various intervals, while the expected demands $\hat{\mu}_{h_1,d}$ are generated from the same interval [150, 200] for SVSM-N versus its DVSM. The results of Table 10 indicate the following:

1. A decrease in demand variations reduces the overcost (p-value < 0.0001, one-tailed test).
2. When the demand variations reduce to zero, the overcosts will also decrease to zero; that is, v * (SVSM-N) reduced to its respective v * (DVSM) values.

Table 10. Overcost-N(%) for the test problems $\mathbf{P_1} - \mathbf{P_{12}}$ when the standard deviations $\hat{\boldsymbol{\sigma}}_{\boldsymbol{h_1},\boldsymbol{d}}$ of normal daily demands is randomly generated from various intervals.

(a) SVSM-N versus its DVSM				
	$\hat{\sigma}_{h_1,d}$			
Test problem	[5,10]	[15,20]	[25,30]	[35,40]
P_1	6.12	9.54	14.87	17.99
P_2	6.04	9.76	15.96	18.91
P_3	7.41	10.68	17.52	19.48
Average	**6.52**	**9.99**	**16.12**	**18.79**
P_4	8.50	16.42	24.13	28.06
P_5	10.57	17.38	26.02	30.39
P_6	9.88	19.25	27.19	31.90
Average	**9.65**	**17.68**	**25.78**	**30.12**
P_7	12.02	35.81	38.81	45.45
P_8	13.75	37.87	39.52	46.24
P_9	14.28	39.66	41.95	47.08
Average	**13.35**	**37.78**	**40.09**	**46.26**
P_{10}	20.04	44.45	52.81	64.45
P_{11}	22.75	47.54	54.02	66.24
P_{12}	24.28	49.89	56.95	68.08
Average	**22.36**	**47.29**	**54.59**	**66.26**

6. CONCLUDING REMARKS

The specific vessel scheduling transportation problem considered in this research effort incorporates multiple sources, and multiple destinations with normal demand distribution. We have formulated exact mixed-integer non-linear programming models SVSM-N for the normal demand scenario using chance-constrained programming.

The following findings highlight our computational experimentation:

1. The optimal fleet schedules for SVSM-N are significantly different from those obtained from their corresponding deterministic models.
2. An increase in the days of time horizon consistently widens the gap between the optimal objective values for SVSM-N and their respective deterministic counterparts.
3. Any increase in the probabilities for meeting the demands triggers an increase in the optimal objective values for SVSM-N.
4. Any reduction in the variations in daily demands and the CV of demands diminishes the optimal objective values for SVSM-N.
5. The optimal objective values for SVSM-N are significantly higher than those of their deterministic counterparts.

Thus, this research effort signifies that the nature of demand distribution in the multiple sources and multiple destinations scheduling-inventory scenario significantly impacts the overall fleet schedules and the total expected cost. Therefore, it is crucial to grasp essential stochastic aspects of the daily demands to avoid potential misrepresentation of the operational costs. The limitation of the proposed method is essentially the difficulty to solve large-scale mix-integer programming models. However, with the advances in optimization solvers and computer power, we can nowadays solve large-scale mix-integer programming models in a relatively reasonable time.

An extension of our modeling approach is to generalize problem scenarios with other penalty structures. Another extension is to investigate new stochastic demand scenarios with multiple sources and destinations. Future work should also consider probabilistic aspects related to the vessel loading/unloading, travel times, and partially loaded/unloaded vessels.

ACKNOWLEDGMENT

This research effort extends the work in Soroush and Al-Yakoob (2018) and Soroush et al. (2020). Prof. Hossein Soroush sadly passed away on February 27, 2020. Prof. Soroush was a pivotal contributor in Soroush and Al-Yakoob (2018) and Soroush et al. (2020) as well as the initial conceptualization and computational experimentation of this work. Therefore, we not only acknowledge his contribution in this article, but we also dedicated this work to him.

REFERENCES

Agra, A., Christiansen, M., Hvattum, L. M., and Rodrigues, F. (2018). Robust optimization for a maritime inventory routing problem. *Transportation Science*, *52*(3), 509-525.

Al-Yakoob, S. M., & Sherali, H. D. (2018). A Mathematical Modelling and Optimization Approach for a Maritime Facility Location Transshipment Problem. *Informatica*, *29*(4), 609-632.

Dina E. A. (2021). A Vessel Scheduling and Inventory Problem with Normal and Gamma Demand Distributions – Multiple Sources and Destinations. *Master's Thesis, Kuwait University*, http://dx.doi.org/10.13140/RG.2.2.18801.40806

Gustavo Souto dos Santos Diz, Silvio Hamacher, &Fabricio Oliveira (2019). A robust optimization model for the maritime inventory routing problem. *Flexible Services and Manufacturing Journal*. , *31*(3), 675-701.

International Trade Centre, *Trade Map*. Accessed on June 9, 2020. ISBN 978-0-367-36463-2.

Konur, D., Campbell, J. F., & Monfared, S. A. (2017). Economic and environmental considerations in a stochastic inventory control model with order splitting under different delivery schedules among suppliers. *Omega*, *71*, 46-65.

Lima, C., Relvas, S., and Barbosa-Póvoa, A. (2018). Stochastic programming approach for the optimal tactical planning of the downstream oil supply chain. *Computers & Chemical Engineering*, *108*, 314-336.

Marcelo W. Friske , Luciana S. Buriol , Eduardo Camponogara (2022). A relax-and-fix and fix-and-optimize algorithm for a Maritime Inventory Routing Problem. *Computers & Operations Research*. Volume 137, January 2022, 105520.

Markov, I., Bierlaire, M., Cordeau, J. F., Maknoon, Y., & Varone, S. (2018). A unified framework for rich routing problems with stochastic demands. *Transportation Research Part B: Methodological*, *114*, 213-240.

Nurhadi Siswanto , Stefanus Eko Wiratno , Ahmad Rusdiansyah and Ruhul Sarker (2019).

Maritime inventory routing problem with multiple time windows. *Journal of Industrial & Management Optimization*, 2019, 15 (3) : 1185-1211. doi: 10.3934/jimo.2018091.

Ömer Nuri Çam and Hayrettin Kemal Sezen (2020) .The Formulation of a Linear Programming Model for the Vehicle Routing Problem in Order to Minimize Idle Time. *Decision Making: Applications in Management and Engineering,* Vol. 3, Issue 1, 2020, pp. 22-29.

Rafie-Majd, Z., Pasandideh, S. H. R., & Naderi, B. (2018). Modelling and solving the integrated inventory-location-routing problem in a multi-period and multi-perishable product supply chain with uncertainty: Lagrangian relaxation algorithm. *Computers & chemical engineering*, *109*, 9-22.

Rahimi, M., Baboli, A., & Rekik, Y. (2017). Multi-objective inventory routing problem: A stochastic model to consider profit, service level and green criteria. *Transportation Research Part E: Logistics and Transportation Review*, *101*, 59-83.

Rodrigues, F., Agra, A., Christiansen, M., Hvattum, L. M., and Requejo, C. (2019). Comparing techniques for modelling uncertainty in a maritime inventory routing problem. *European Journal of Operational Research*, *277*(3), 831-845.

Roldán RF, Basagoiti R, and Coelho LC (2016). Robustness of inventory replenishment and customer selection policies for the dynamic and stochastic inventory-routing problem. *Computers and Operations Research* 74: 14-20.

Sanaz Sheikhtajian ,Ali Nazemi , and Majid Feshari (2020). Marine Inventory-Routing Problem for Liquefied Natural Gas under Travel Time Uncertainty. *International Journal of Supply and Operations Management (IJSOM).* Volume 7, Issue 1, pp. 93-111.

Soroush, H. M., Al-Yakoob, S. M., and Alqallaf, F. A., (2020). A Stochastic Maritime Transportatio-Inventory Problem with Gamma, Exponential and Uniform Demand Distrubutions. *European Journal of Industrial Engineering*, 14(6): 813-850.

Soroush, H. M., and Al-Yakoob, S. M., (2018). A maritime scheduling transportation-inventory problem with normally distributed demands and fully loaded/unloaded vessels. *Applied Mathematical Modeling*, 53: 540-566.

Yan Sun, Martin Hrušovský, Chen Zhang, and Maoxiang Lang (2018). A Time-Dependent Fuzzy Programming Approach for Green Multimodal Routing Problem with Rail Service Capacity Uncertainty and Road Traffic Congestion. *Complexity,* Volume 2018, Article. ID 8645793, https://doi.org/10.1155/2018/8645793

Yi Zhao ,Qingwan Xue, Zhichao Cao, and Xi Zhang (2018). A Two-Stage Chance Constrained Approach with Application to Stochastic Intermodal Service Network Design Problems. *Journal of Advanced Transportation,* Volume 2018, Article ID 6051029, 18 pages https://doi.org/10.1155/2018/6051029.

Zhang C, Nemhauser G, Sokol J, Cheon MS, Keha A (2018). Flexible solutions to maritime inventory routing problems with delivery time windows. *Computers and Operations Research,* 89:153-162.