Defect-free optimization of a polycentric prosthetic knee design using an imperialist competition-inspired optimization method

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ABSTRACT

Lower limb disability is one of the major problems facing human lives. To restore the missing functionality and aesthetic features of an amputee's locomotion, finding the optimal design of a human knee prosthesis is crucial. This paper focuses on the design optimization of a four bar knee mechanism capable of reproducing the complex flexion/extension knee joint motion in the sagittal plane with variable instantaneous centre of rotation positions. Thus, an optimization approach with appropriate constraints is formulated to consider the degree of compatibility between the instantaneous centre of rotation trajectories of the human reference knee joint and the four bar knee mechanism. To solve this highly nonlinear optimization problem, an algorithm based on a multiobjective modified imperialist competitive method, where all the constraints are managed with a penalty method, is proposed. The results obtained by the multiobjective modified imperialist competitive method show that it is possible to obtain a small tracking error. The obtained results prove the effectiveness of the proposed optimization approach for the optimal synthesis of a four bar knee mechanism compared with that of other literature techniques.

Keywords: Four Bar Knee Joint; Defect-Free Optimization; Tracking Error; Imperialist Competition Method.

INTRODUCTION

For human beings, walking is an important everyday task. There are people with walking disabilities due to certain neuromuscular diseases, stroke, trauma, ageing, etc., which cause many inconveniences in their daily lives. The most reliable way to restore the ability to walk is usually through the use of polycentric prostheses. Polycentric prosthetics usually contain FBKM, which is functionally flexible, topologically simple and easy to fabricate (Mohamed et al. 2017). A polycentric prosthetic knee should mimic human knee motion. In other words, the instantaneous centre of rotation (ICR) of the four bar knee mechanism (FBKM) should be very close to the ICR of the reference knee joint (Khan et al. 2015). Therefore, the dimensional optimization of the FBKM to match the reference knee joint movement is a challenge. Several determinist optimization methods have been suggested (Krishnanand and Ghose 2009, Zhang, Zhou and Ye 2000). However, it has been shown that determinist methods cannot converge to the global optimum, especially for constrained nonlinear problems (Hosseini and Al Khaled 2014). To overcome these issues, stochastic methods can be used to optimize the FBKM. Genetic algorithms (GAs) have been used to mechanism optimal synthesis (Khorshidi et al. 2011). Moreover, particle swarm optimization (PSO) has been used in mechanism synthesis (Eqra, Abiri and Vatankhah 2018, Syed et al. 2021). Differential evolution (DE) and simulated annealing (SA) have been used in the design optimization of four bar linkages with and without clearances (CHEN and ZHU 2009, Lin 2010, Naresh 2019). The imperialist competitive algorithm (ICA) has also been used to find the optimal design planar mechanisms (Bilel et al. 2018). Several researchers have focused on studying the drawbacks of the above algorithms and developed other improved versions. For example, NSGAII, MOPSO, and MGBICA are improved versions of the GA, PSO and ICA, respectively. In this context, MOMICA has recently been proposed as an efficient stochastic method that offers the best balance between convergence speed and solution diversity (Mohamed, Bilel and Alsagri 2020).

Nevertheless, MOMICA has not been used in prosthetic knee design because it is a quite recently developed tool. In the optimization of FBKM, most researchers have not paid close attention to constraint formulations, despite their important role in finding defect-free designs (Bapat and Sujatha 2017, Bertomeu et al. 2007, Ghaemi et al. 2012, Kittisares et al. 2020, Muñoz-César et al. 2013). In reality, branch, Grashof and order defects often occur during FBKM kinematic synthesis. If all possible orientations cannot be realized without dismantling the mechanism, an FBKM has a branch defect (Zhou and Cheung 2004). Grashof defects arise when the FBKM linkage needs to be fully rotated frequently, while order defects occur when the FBKM linkage rotation is not in the desired order (Bilel et al. 2017). With the exception of Grashof defects, which have been considered by a few researchers, the abovementioned defects are avoided in the literature (Aoustin and Hamon 2013, Sancibrian et al. 2016).

In this paper, adequate constraints for Grashof, branch and order defect eliminations have been clearly established in the optimization problem of the FBKM. The objective function of this optimization problem is to minimize the error between the reference human knee ICR and the FBKM ICR. For the first time, the design optimization of a prosthetic knee is conducted through MOMICA, which was developed quite recently. The findings indicate the efficacy of the proposed optimization strategy compared with that of other literature approaches.

BIOMECHANICS OF A HUMAN KNEE JOINT

In the human knee joint, the femur articulates with the tibia. The knee's primary role is to allow joint rotation while achieving adsorption and load transmission (Xu et al. 2016). The motion of the knee is generally modelled as a four-bar mechanism ABCD, as seen in Figure 1 (Dathe et al. 2016). The ICR is positioned at the point at which the anterior link AB and the posterior link DC cross. For different knee flexion angles, the ICR motion is described by a trajectory.

Figure 2 illustrates some familiar prosthesis knee designs. To create a moving ICR, a prosthesis knee includes four bar linkage, where their interior and posterior links intersect in their ICR.

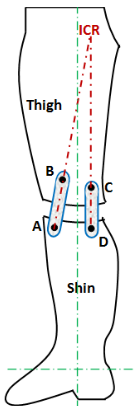


Figure 1 Four bar linkage as an equivalent of a human knee joint

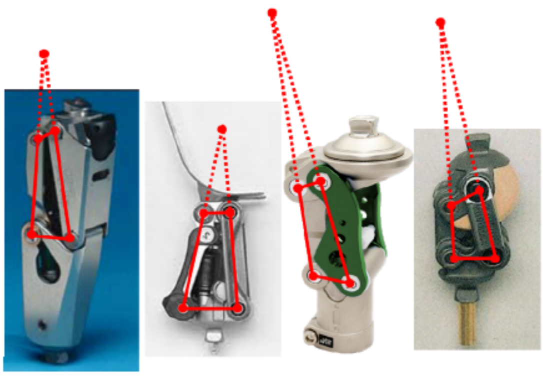


Figure 2 ICR of FBKM used in polycentric prosthesis (Etoundi, Vaidyanathan and Burgess 2012)

Ideally, the ICR points should mimic the human knee ICR trajectory. Therefore, the FBKM configuration requires optimization to follow a desired path.

DETERMINATION OF ICR COORDINATES OF THE FBKM

Figure 3 illustrates the adopted notations of the FBKM. The frame linkage (link 1) is positioned at the shank of the prosthesis. The coupler (link 3) is situated in the socket. Links 2 and 4 are positioned at both trimmings of links 1, A and D. The position of C is given by:

$$X_C = X_D + r_4 \cos\theta_4 = X_A + r_2 \cos\theta_2 + r_3 \cos\theta_3 \quad (1)$$

$$Y_C = Y_D + r_4 \sin\theta_4 = Y_A + r_2 \sin\theta_2 + r_3 \sin\theta_3 \quad (2)$$

Equations (1) and (2) can be presented with respect to θ_2 as:

$$r_2 \cos\theta_2 = r_4 \cos\theta_4 + C_1 \quad (3)$$

$$r_2 \sin\theta_2 = r_4 \sin\theta_4 + C_2 \quad (4)$$

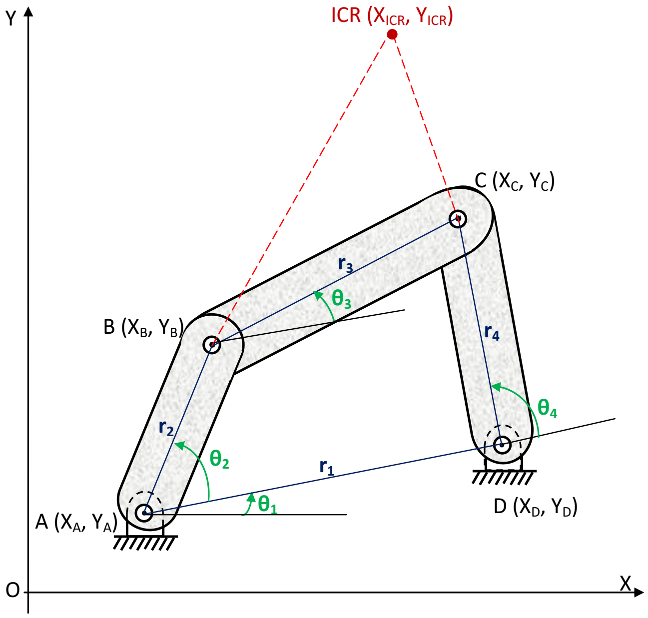


Figure 3 The FBKM configuration and adopted notations

where

$$C_1 = X_D - X_A - r_3 \cos\theta_3 \quad (5)$$

$$C_2 = Y_D - Y_A - r_3 \sin\theta_3 \quad (6)$$

By squaring and adding Equations (3) and (4), we obtain:

$$r_2^2 = r_4^2 + C_1^2 + 2C_1 r_4 \cos\theta_4 + 2C_2 r_4 \sin\theta_4 + C_2^2 \qquad (7)$$

By rearranging Equation (7), we can obtain the following equation of motion:

$$B\cos\theta_4 + A\sin\theta_4 = C \qquad (8)$$

$$A = 2C_2 r_4 \qquad (9)$$

$$B = 2C_1 r_4 \qquad (10)$$

$$C = r_2^2 - r_4^2 - C_1^2 - C_2^2 \qquad (11)$$

Equation (8) is a θ_4 function. The following substitutions can make this explicit.

$$\sin\theta_4 = \frac{2\tan\left(\frac{\theta_4}{2}\right)}{1+\tan^2\left(\frac{\theta_4}{2}\right)} \qquad (12)$$

$$\cos\theta_4 = \frac{1-\tan^2\left(\frac{\theta_4}{2}\right)}{1+\tan^2\left(\frac{\theta_4}{2}\right)} \qquad (13)$$

By incorporating Equations (12) and (13) in (8), the latter is reduced in $\left(\frac{\theta_4}{2}\right)$ quadratic form. Its solution is given by the following equation (Muñoz-César et al. 2013):

$$\theta_4 = 2\tan^{-1}\frac{A \pm \sqrt{A^2 + B^2 - C^2}}{B + C} \qquad (14)$$

As given in Equation (14), θ_4 presents two solutions according to the mechanism configuration.

The ICR of the FBKM can be calculated from the coordinates of the centres of rotation of links 2 and 4 and their angles (Muñoz-César et al. 2013):

$$X_{ICR} = \frac{Y_D - Y_A - X_D\tan\theta_4 - X_A\tan\theta_2}{\tan\theta_2 - \tan\theta_4} \qquad (15)$$

$$Y_{ICR} = Y_D + \tan\theta_4\left(X_{ICR} - X_D\right) \qquad (16)$$

Equations (15) and (16) give the coordinates of any point on the ICR trajectory.

OPTIMIZATION PROCEDURE

The optimization problem

The purpose of the optimization problem is to find an FBKM design that mimics as closely as possible human knee movement (based on a human knee ICR) while meeting the defined constraints. Thus, the objective function can be presented as a tracking error function (*TE*).

TE is defined by the sum of the Euclidean distance between the ICR trajectory $\left(X_{ICR}, Y_{ICR}\right)$ of the FBKM and the reference human knee ICR trajectory $\left(X_{R/ICR}, Y_{R/ICR}\right)$:

$$TE(\mathbf{x}) = \sum_{i=1}^{N} \sqrt{\left(X_{R/ICR}^{i} - X_{ICR}^{i}\right)^{2} + \left(Y_{R/ICR}^{i} - Y_{ICR}^{i}\right)^{2}} \qquad (17)$$

N is the number of reference human knee ICR trajectory points.

$\mathbf{x}$ represents the vector of unknown design parameters, which is given by:

$$\mathbf{x} = \left[r_1, r_2, r_3, r_4, X_A, Y_A, \theta_1, \theta_2^1, \theta_2^2, \theta_2^3 ....., \theta_2^N\right] \qquad (18)$$

The Grashof defect can be eliminated by satisfying the following Grashof rule.

$$h_1(\mathbf{x}) = r_1 + r_2 - r_3 - r_4 < 0 \;\; if\,(r_2 < r_3 < r_4 < r_1) \qquad (19)$$

The branch defect can be eliminated by considering the following constraint.

$$h_2(\mathbf{x}) = \theta_3^i - \theta_4^i < 0 \;\; i = 1,...., N \qquad (20)$$

Order defects occur when the FBKM linkage rotation is not in the desired order (clockwise or counterclockwise). To eliminate this defect, the following constraint is needed.

$$h_3(\mathbf{x}) = \theta_2^i - \theta_2^{i+1} < 0 \;\; i = 1,...., N \qquad (21)$$

The optimization problem regroups the objective function and the following constraints:

$$\begin{aligned}
&\text{Minimize: } TE(\mathbf{x}) = \sum_{i=1}^{N} \sqrt{\left(X_{R/ICR}^{i} - X_{ICR}^{i}\right)^{2} + \left(Y_{R/ICR}^{i} - Y_{ICR}^{i}\right)^{2}} \\
&\text{Constraints: } h_1(\mathbf{x}) = r_1 + r_2 - r_3 - r_4 < 0 \;\; if\,(r_2 < r_3 < r_4 < r_1) \\
&\qquad h_2(\mathbf{x}) = \theta_3^i - \theta_4^i < 0 \;\; i = 1,...., N \\
&\qquad h_3(\mathbf{x}) = \theta_2^i - \theta_2^{i+1} < 0 \;\; i = 1,...., N \\
&\qquad L_k \le x_k \le U_k \qquad k = 1,...., n
\end{aligned} \qquad (22)$$

where the index *k* represents the number of design variables. L_k and U_k define the lower and upper limits of the design variables search spaces, respectively.

Optimization method

The MOMICA method will be used to solve the optimization problem. MOMICA has recently been proposed as an efficient stochastic method that offers the best balance between convergence speed and solution diversity (Mohamed et al. 2020). Figure 4 illustrates the different steps of

The MOMICA method. First, MOMICA starts a random generation of the initial population

formed by many countries. The evaluation of different objective functions corresponding to each

country yields the determination of the countries' cost. Based on this cost, some powerful countries (with elevated costs) are considered "imperialist", and the other countries are referred to as "colonies". Each imperialist with some colonies forms an empire. Thus, several initial empires are formed. Second, the assimilation step begins where colonies move towards their imperialists, as illustrated in Figure 5-a (Mohamed et al. 2020).

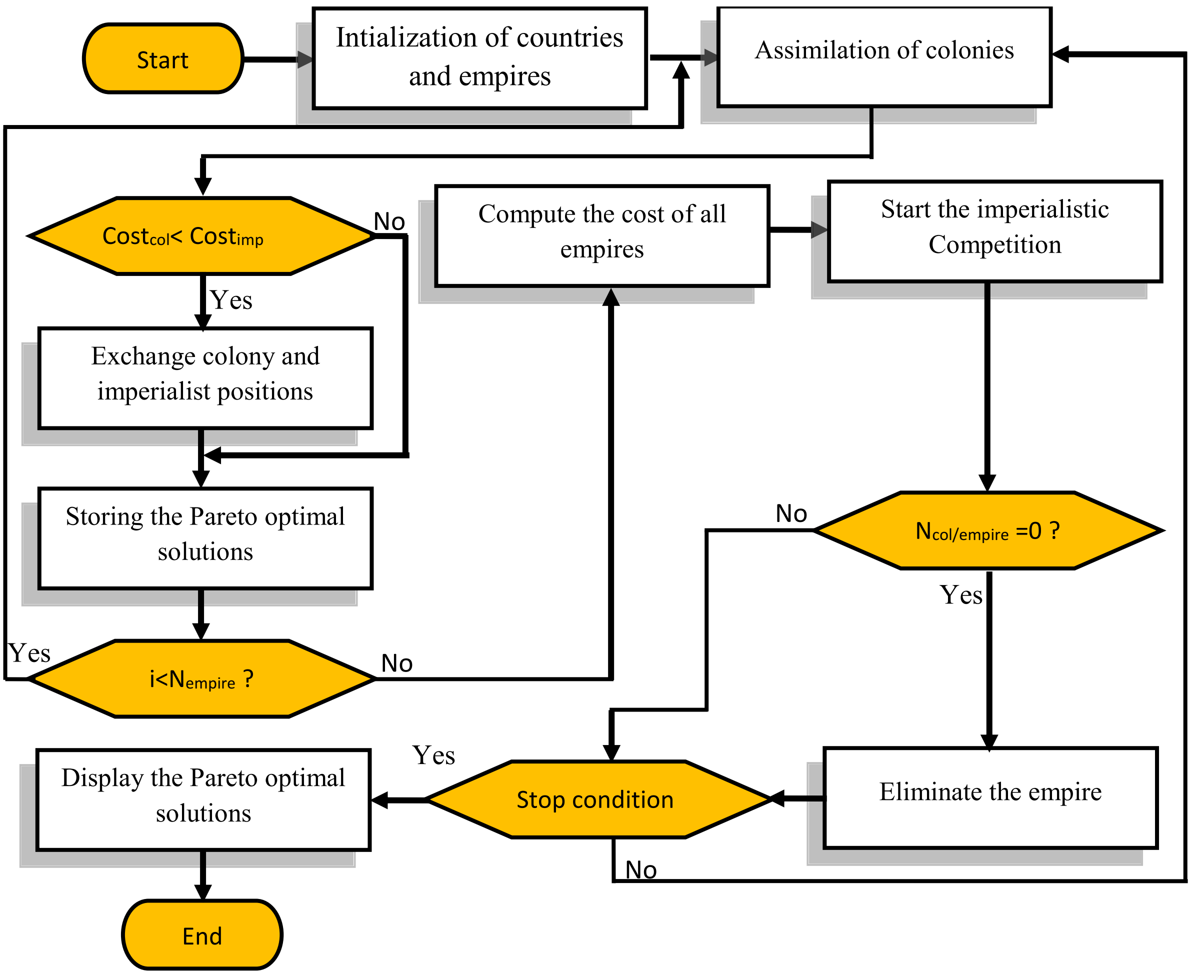
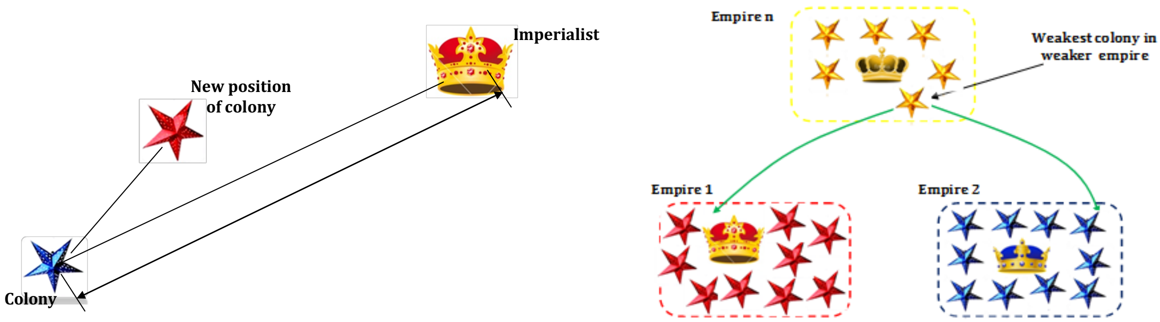


Figure 4 Different steps of MOMICA

After the assimilation phase, the costs of new colonies are recalculated (based on the objective function values) and compared with the cost of the imperialist. If any colony cost is less than the cost of its imperialist, then the imperialist should be swapped with that colony. Third, imperialist competition occurs, as presented in Figure 5-b. In fact, dominant empires acquire colonies of weaker empires (Mohamed et al. 2020). After several iterations, weaker empires will lose all their colonies and fall. This process repeats until only one empire remains.



a) b)

Figure 5 Empire improvements; a) Colonies movement b) Competition stage

RESULTS OF OPTIMIZATION

The MOMICA method is applied for the optimization of the defect-free FBKM for flexion/extension motion of the human knee. The reference human knee ICR trajectory is given in Table 1. The ranges of the different FBKM design parameters are illustrated in Table 2.

Table 1 Reference knee ICR trajectory (Sancibrian et al. 2016, Singh, Chaudhary and Singh 2017)

Points	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$X_{R/ICR}$	-50	-16	10	14	6	0	-5	-10	-15	-20	-25	-30	-35	-40
$Y_{R/ICR}$	200	150	100	60	18	0	-12	-19	-21.5	-22	-21	-18	-11	0

Table 2 Variable search domains (Sancibrian et al. 2016, Singh et al. 2017)

x	r_1(mm)	r_2(mm)	r_3(mm)	r_4(mm)	X_A(mm)	Y_A(mm)	θ_1(°)	θ_2^i(°)
L_k	20	40	20	40	-1500	-1500	0	0
U_k	90	170	90	170	1500	1500	360	360

To evaluate the MOMICA efficacy, the same optimization problem is solved by using well-known naturally inspired methods (GA and MOPSO). The evolution of *TE* as a function of the number of iterations for the best FBKM design is presented in Figure 6. One can note from Figure 6 that the lower value of *TE* is given by MOMICA compared with that of GA and MOPSO. In fact, GA reaches an optimum value of 12 mm in 150 iterations, MOPSO reaches 22 mm in 200 iterations and MOMICA obtains 6.7 mm in 178 iterations. Thus, MOMICA is capable of improving the *TE* results of GA and MOPSO by approximately 44% and 69%, respectively.

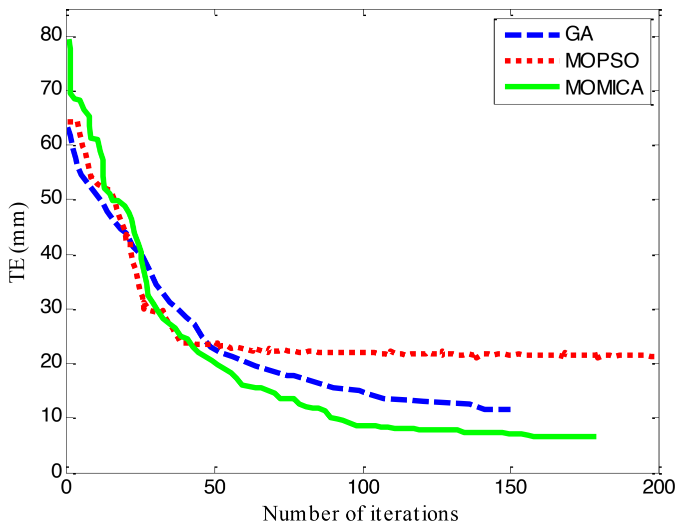


Figure 6 Convergence of *TE* with GA, MOPSO and MOMICA

The design parameters of the optimal FBKM are detailed in Table 3.

Table 3 Optimum FBKM obtained by GA, MOPSO and MOMICA

x	r_1 (mm)	r_2 (mm)	r_3 (mm)	r_4 (mm)	X_A (mm)	Y_A (mm)	θ_1 (°)	*TE* (mm)
GA	88.43	52.13	65.16	79.77	-101.86	-313.22	28.56	12
MOPSO	88.76	54.54	65.09	81.21	-219.33	-278.98	42.66	22
MOMICA	79.92	55.05	65.72	74.54	-373.44	-354.88	17.39	**6.7**

Figure 7 demonstrates the trajectories generated by the GA, MOPSO and MOMICA. The reference human knee ICR trajectory is also presented in the same figure.

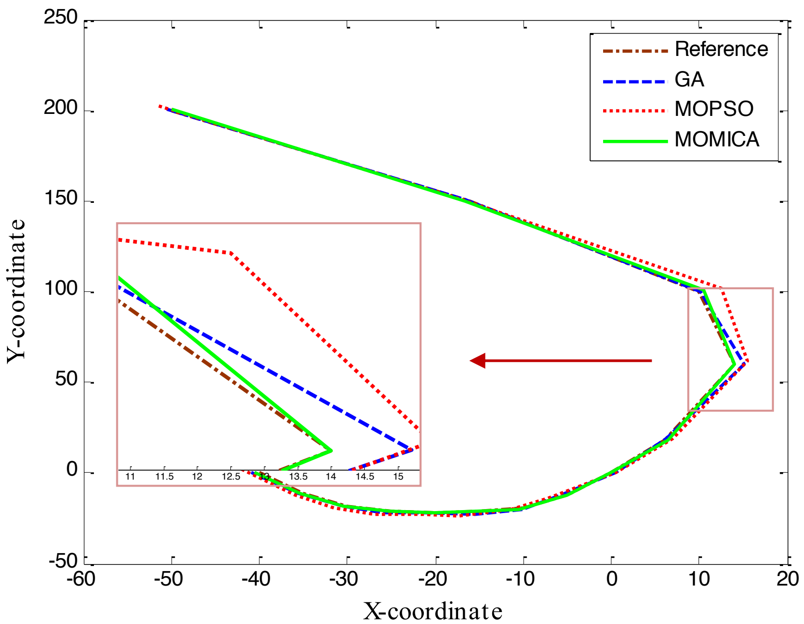


Figure 7 Different ICR paths (Reference, GA, MOPSO, and MOMICA)

It is shown that the MOMICA trajectory is the closest to the reference ICR path compared with that of GA and MOPSO. The errors between each reference point and the ICR trajectories are presented in Figure 8. It is noted that MOMICA dominates GA and MOPSO at all reference points, except at point 3. This proves the efficacy of MOMICA in FBKM design optimization.

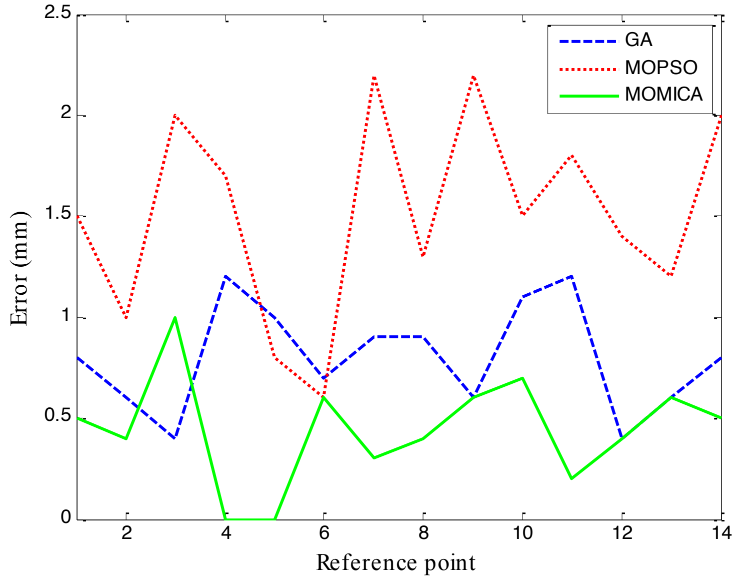


Figure 8 Evolution of errors as a function of reference points

Table 4 presents the minimum, average and maximum errors to the reference human knee ICR path obtained by different optimization methods. One can note that the MOMICA results in terms of the minimum, average and maximum errors are lower than those given by GA, MOPSO, TLBO and NSGAII. This result proves the effectiveness of the MOMICA method, especially in FBKM optimization. In fact, in MOMICA, an attraction and repulsion concept is implemented in the assimilation phase to improve the performance of the algorithm to reach the global optimal position and ameliorate the diversity rates of the algorithm (Mohamed et al. 2020)

Table 4 Optimum FBKM obtained by GA, MOPSO and MOMICA

Error	GA	MOPSO	MOMICA	TLBO (Singh et al. 2017)	NSGA II (Bertomeu et al. 2007)
Minimum	0.43	0.62	**0.001**	0.86	0.63
Average	0.81	1.57	**0.41**	3.2	1.27
Maximum	1.28	2.21	**1.05**	9.21	1.91

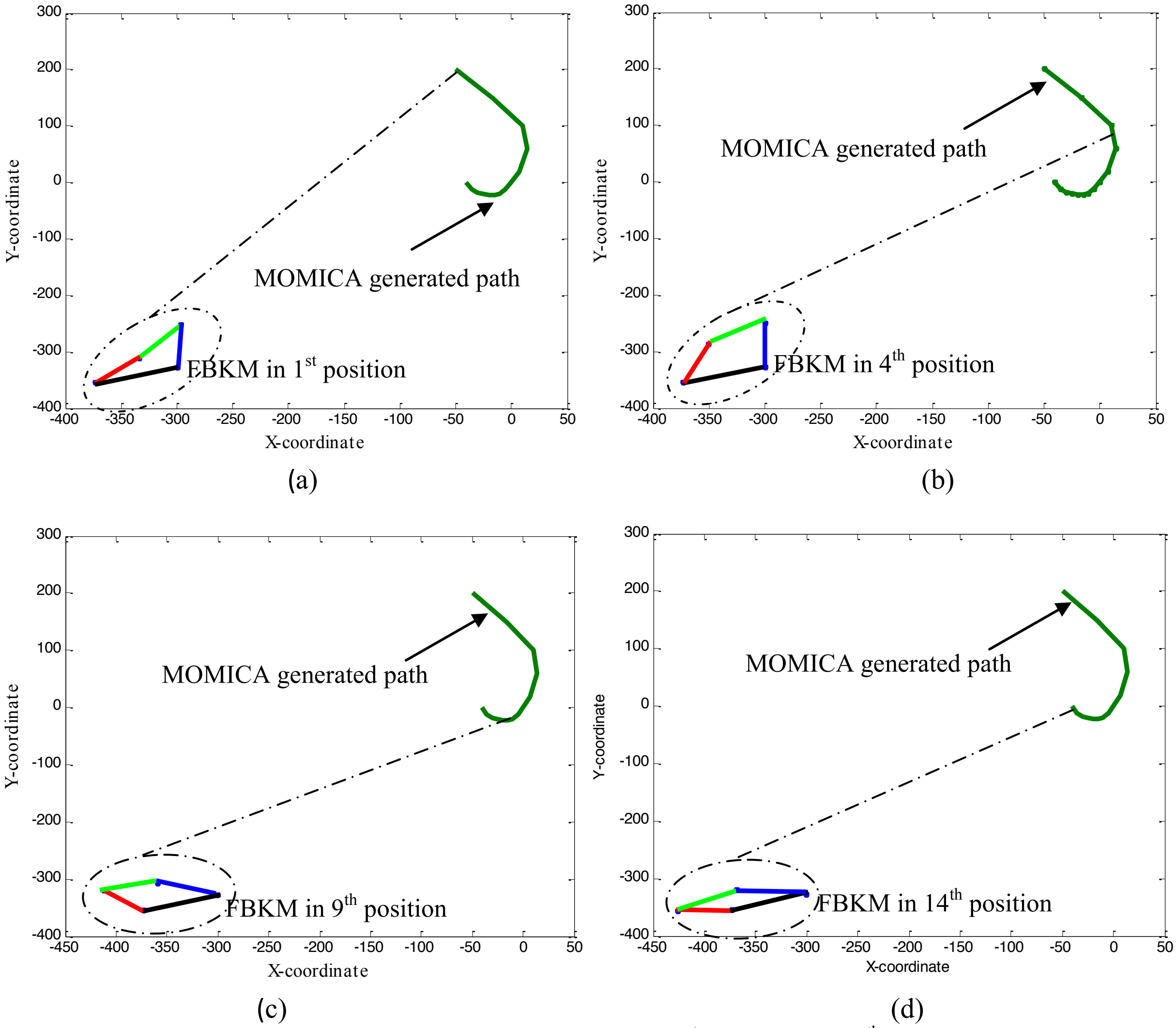


Figure 9 FBKM motion simulation: a) 1st position b) 4th position c) 9th position d) 14th position

To verify that the developed FBKM is a defect-free system, it is necessary to check the Grashof, branch and order constraints adopted in the optimization strategy. Figure 9 presents four sequential positions of the FBKM corresponding to four randomly selected values of θ_2. This confirms that the system travels sequentially without any stationary configuration, which proves that the Grashof defect is avoided. Figure 10-a represents the θ_2 evolution as a function of the 14 sequential reference points. One can note that by passing from one reference point to the successive one, the value of θ_2 increases. Therefore, the link 2 rotation is in the counterclockwise order. Hence, the order defect is eliminated. Figure 10-b demonstrates the evolution of

$\theta_3 - \theta_4$ values as a function of the 14 reference points. For all the reference points, θ_3 is always inferior to θ_4. Therefore, the branch defect is avoided.

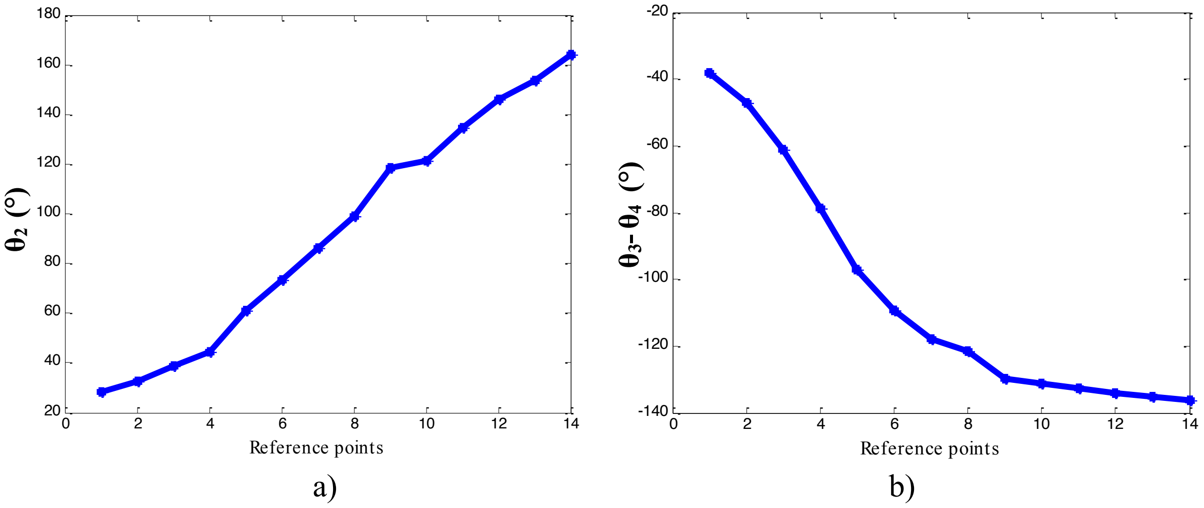


Figure 10 Angle evolution as a function of reference points: a) θ_2 evolution b) $\theta_3 - \theta_4$ evolution

The presented results turn out to be a valuable tool for designers to choose the defect-free optimal FBKM. In fact, the proposed optimization approach could help designers in the choice of the most adequate design parameters yielding the FBKM with the minimum tracking error.

CONCLUSION

This paper presents a new design of a four bar knee mechanism by adopting the appropriate constraints that avoid Grashof, branch and order defects. An optimization strategy based on the multiobjective modified imperialist competitive algorithm (MOMICA) is developed to solve this nonlinear problem. For comparison reasons, GA and MOPSO are also used to solve the optimization problem. It is shown that MOMICA can offer an FBKM tracking error improved by approximately 44% and 69% compared with those given by GA and MOPSO, respectively. In addition, it is found that the proposed MOMICA mechanisms satisfy all the constraints used for defect-free optimization without any stationary configuration. Thus, the proposed optimization strategy using MOMICA proves to be an effective tool for the optimal synthesis of the FBKM.

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