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طريقة مقترحة لخطط تصميم متغير لعينات بناء على مؤشر القدرة العملية

محمد أسلم، **شين وي وو، ***محمد عزام و*تشي-هيوك يونيو**
*دائرة الإحصاءات العامة، كلية العلوم، جامعة الملك عبد العزيز، جدة 21551، المملكة العربية السعودية
**قسم الإدارة الصناعية، جامعة تايوان الوطنية للعلوم والتكنولوجيا، تايبيه، تايوان
***قسم للإحصاء وعلوم الحاسب، جامعة العلوم البيطرية والحيوانية لاهور، باكستان
****قسم الهندسة الصناعية والإدارة، POSTECH، بوهانغ 37673، جمهورية كوريا

الخلاصة

تقدم هذه المقالة خطة أخذ عينات قبول شددت المعتاد -شددت باستخدام مؤشر القدرة العملية CPK. يتم الحصول على معالم الخطة المقترحة من خلال حل مشكلة التحسين مثل توافق مخاطر المنتج ومخاطر المستهلك في وقت واحد لعدة مجموعات محددة من مستوى جودة مقبول، والحد من مستوى الجودة، على التوالي. يتم جدولة ثوابت الخطة ومناقشتها للاستخدام الصناعي عندما تكون الكسور غير مطابقة لحدود المواصفات الدنيا أو العليا سواء كانت متماثلة أو غير متماثلة. ونحن نتحقق من فعالية واستخدام الخطة المقترحة على مثال فعلي. تتم مقارنة كفاءة الخطة المقترحة مع نشر وإعادة تقديم خطط أخذ العينات بنفس قيم AQL و LQL. وتعطي الخطة المقترحة قيمة ASN أصغر من خطة أخذ عينة واحدة وخطة أخذ العينات بنفس قيم AQL و LQL.

A proposed procedure for variable sampling plans design based on process capability index

Muhammad Aslam*, Chien-Wei Wu**, Muhammad Azam*** and Chi-Hyuck Jun****

**Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah 21551, Saudi Arabia*

***Department of Industrial Engineering and Engineering Management, National Tsing Hua University, Hsinchu, Taiwan*

****Department of Statistics and Computer Science, University of Veterinary & Animal Sciences Lahore, Pakistan*

*****Department of Industrial and Management Engineering, POSTECH, Pohang 37673, Republic of Korea*

**Corresponding author: aslam_ravian@hotmail.com*

ABSTRACT

The article presents the tightened-normal-tightened acceptance sampling plan, using the process capability index C_{pk}. The parameters of the proposed plan are obtained by solving an optimization problem, such that the producer's risk and the consumer's risk are satisfied simultaneously for several specified combinations of acceptable quality level and limiting quality level, respectively. The plan parameters are tabulated and discussed for industrial use, when the fractions non-conforming beyond the lower and the upper specification limits are symmetric or asymmetric. We validate the effectiveness and use of the proposed plan with an actual example. The efficiency of the proposed plan is compared with published and resubmitted sampling plans with the same AQL and LQL values. The proposed plan provides smaller ASN than the single sampling plan and the resubmitted sampling plan under the same values of AQL and LQL.

Keywords: Normal distribution; normal inspection; producer and consumer risks; process capability index; tightened inspection.

INTRODUCTION

There are a number of methods in industry that are widely used to enhance the quality of the product and to protect producers and consumers from bad quality. Some methods may not be appropriate and not always possible, when the purpose is to inspect products from a producer or parts supplied to the producer. 100% inspection is often impractical, as it increases the cost of the inspection. In such cases, acceptance sampling is a solution that not only provides the protection to producers and consumers, but also determines the minimum sample size for the inspection purpose.

Acceptance sampling is a major area of quality design and control that, in some situations, provides means for efficient and effective inspection of the product.

There are many acceptance sampling schemes that have been used in certain situations to provide the optimal sample size for the inspection of the product. For example, variable sampling is used, when the acceptance criteria is based on the measurement process. Attribute sampling is used, when the product is classified into good or bad. Variable sampling plans are more costly, but provide smaller sample size as compared to attribute acceptance sampling plans, and this reduction can offset the increased cost (Montgomery, 2009). Calvin (1997) provided the tightened-normal-tightened (TNT) scheme for attribute acceptance sampling plans. He developed the acceptance sampling plan utilizing two single sampling plans having different sample size with zero acceptance number and using the switching rule, which is characterized by three plan parameters namely; the sample size for the tightened inspection (n_T), the sample size for the normal inspection (n_N) and the acceptance number (c). Recently, there have been some studies on improving the TNT sampling plans for attributes and variables quality characteristics. These will be described in Section 2.

On the other hand, the process capability indices (PCIs) have been widely used in industry to manufacture products according to the given specification limits. Several PCIs, including C_p, C_{pk} and C_{pm}, have been widely used in many industrial applications to provide numerical measures on whether the process is capable of reproducing items meeting the quality requirements. By exploring the literature, we found that none or little work has been done on TNT schemes using PCIs. The TNT schemes available in the literature do not consider the process capability, but just help engineers either to accept or to reject the lot of the product more efficiently. The development of a TNT scheme using the PCIs will help the industrial engineers to inspect the product according to the specification limits as well as to make decision on lots more effectively.

LITERATURE REVIEW

Soundararajan & Vijayaraghavan (1992) developed tables and procedures for the selection of TNT-(n_T,n_N;c) scheme for various entry parameters. Vijayaraghavan & Soundararajan (1996) proposed another type of TNT sampling scheme using the equal sample size and two acceptance numbers, designated as TNT-(n;c_1,c_2) scheme. Muthuraj & Senthilkumar (2006) proposed the variable sampling plan using the TNT scheme. The plan given by them is characterized by three parameters (n_σ, k_T, k_N) where n_σ is the sample size, k_T and k_N are two acceptance numbers under the tightened and normal inspection, respectively. Senthilkumar & Muthuraj (2010) developed the variable sampling scheme based on three plan parameters (n_T, n_N, k). Aslam *et al.* (2010) designed a TNT scheme for Weibull distributions. More details about various types of acceptance sampling plans can be seen in Aslam *et al*. (2015a); Yen *et al*. (2015); Aslam *et al.* (2015b); Aslam *et al.* (2015c) and Aslam *et al*. (2015d).

Kotz & Johnson (2002) and Wu *et al.* (2012) discussed the advantages and applications of PCIs for quality assurance in different fields. In the literature, many authors have developed variable single acceptance sampling plans based on several PCIs. Pearn & Wu (2006a) proposed the variable sampling plan for low fraction defective and Pearn & Wu (2006b) presented parts per million fraction defective plans using the process loss consideration. Pearn & Wu (2006) provided an acceptance sampling plan for effective decision making. Wu & Pearn (2008) proposed the variable plans using C_{pmk}. Yen & Chang (2009) presented variable sampling plans using the process loss function, which is closely related to PCIs. Negrin *et al.* (2009, 2011) proposed and discussed the applications of variable sampling plans using the PCIs. They developed the new PCIs based on models assuming the normal distribution. Recently, Wu (2012) designed an efficient variable inspection plan using the Taguchi process index. Later, Wu *et al*. (2012) proposed the variable sampling plans using the resubmitted sampling scheme by considering C_{pk} and deriving the exact sampling distribution. Aslam *et al.* (2013) proposed a variable sampling plan for resubmitted lots based on C_{pk} under a normal distribution.

Research work on TNT variable sampling plans using the process capability index C_{pk} is very little or non-excitant. Therefore, this paper attempts to develop the variable TNT acceptance sampling plan using the process capability index C_{pk} for a normally distributed process with specified upper and lower specification limits.

DESIGNING TNT VARIABLE PLANS BASED ON C_{PK} INDEX

Suppose that the quality characteristic of interest has two-sided specification limits, USL and LSL, and that it follows a normal distribution with unknown mean μ and unknown standard deviation σ. The process capability index C_p was the first one introduced by Kane (1986), which is defined as

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

Kane (1986) further developed the index C_{pk} that takes into account the magnitude of process variation and process location. It is defined as

$$\mathrm{C_{pk}} = \min\left\{\frac{\mathrm{USL} - \mu}{3\sigma}, \frac{\mu - \mathrm{LSL}}{3\sigma}\right\} \tag{2}$$

Here, the process mean μ and the standard deviation σ are usually unknown in practice, so the estimator of the index C_{pk} can be used by substituting the sample mean $\bar{X} = \sum_{i=1}^{n} X_i/n$ and the sample standard deviation $S = \sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2/(n-1)}$ for μ and σ. That is, the estimator of C_{pk} can be obtained by

$$\hat{C}_{pk} = \min\left\{\frac{\text{USL}-\bar{X}}{3S}, \frac{\bar{X}-\text{LSL}}{3S}\right\} \tag{3}$$

More details about sampling plan based on PCIs can be seen in Aslam *et al.* (2014), Liu *et al.* (2014), Govindaraju & Kissling (2014) and Yen *et al.* (2015).

Based on the idea of TNT acceptance sampling, we propose the following TNT variable sampling plan with two different sample sizes and the common acceptance number based on the C_{pk} :

Step-1 (Tightened inspection): Start with the tightened inspection by taking a random sample of size n_T. Then, compute the $\hat{C}_{pk}$ based on the measured quality characteristics from the sample. Accept the lot if $\hat{C}_{pk} \geq k_a$ and reject, otherwise.

Step-2: Switch to the normal inspection if *t* lots in a row are accepted.

Step-3 (Normal inspection): Under the normal inspection take a random sample of size $n_N(< n_T)$. Calculate $\hat{C}_{pk}$ based on the collected sample. Accept the lot if $\hat{C}_{pk} \geq k_a$ and reject, otherwise.

Step-4: Switch to the tightened inspection if an additional lot is rejected in the next *s* lots after a rejection.

The proposed plan is characterized by five parameters namely, n_T: the sample size at the tightened inspection, n_N: the sample size at the normal inspection, k_a: the acceptance number, s: the lot acceptance criterion at the tightened inspection for switching to the normal inspection, and t: the lot acceptance criterion at the normal inspection for switching to the tightened inspection.

The probability of lot acceptance based on the C_{pk} index from the sample size n can be derived as follows:

$$P_a = P\{\hat{C}_{pk} \geq k_a\} = P\left\{\frac{USL-\bar{X}}{3S} \geq k_a, \frac{\bar{X}-LSL}{3S} \geq k_a\right\}. \qquad 4)$$

Let us define p_U and p_L as the fraction non-conforming outside LSL and USL, respectively. That is,

$p_U = P\{X > USL\}$ and $p_L = P\{X < LSL\}$.

Then,

$$\frac{USL-\mu}{\sigma}=z_{p_U} \text{ and } \frac{LSL-\mu}{\sigma}=-z_{p_L}.$$

where z_p is the standard normal value corresponding to the fraction non-conforming p. After some simplifications, Equation (4) is reduced to

$$P_a=\Phi\left((z_{p_U}-3k_a)\sqrt{\frac{n}{1+9k_a^2/2}}\right)-\Phi\left(-(z_{p_L}-3k_a)\sqrt{\frac{n}{1+9k_a^2/2}}\right) \quad (5)$$

The operating characteristics (OC) function of the proposed TNT acceptance sampling plan is obtained as the same form in Calvin (1977) and given by below:

$$P_a(p)=\frac{P_T(1-P_N^s)(1-P_T^t)(1-P_N)+P_NP_T^t(1-P_T)(2-P_N^s)}{(1-P_N^s)(1-P_T^t)(1-P_N)+P_T^t(1-P_T)(2-P_N^s)}, \quad (6)$$

where p is the fraction non-conforming in a lot, and P_T is the proportion of lots to be accepted under the tightened inspection scheme (n_T, k_a) and P_N is the proportion of lots to be accepted under the normal inspection scheme (n_N, k_a). Thus, based on Equation (5) these two proportions can be expressed as

$$P_T=\Phi\left(\left(z_{p_U}-3k_a\right)\sqrt{\frac{n_T}{1+9k_a^2/2}}\right)-\Phi\left(-\left(z_{p_L}-3k_a\right)\sqrt{\frac{n_T}{1+9k_a^2/2}}\right), \quad (7)$$

$$P_N=\Phi\left(\left(z_{p_U}-3k_a\right)\sqrt{\frac{n_N}{1+9k_a^2/2}}\right)-\Phi\left(-\left(z_{p_L}-3k_a\right)\sqrt{\frac{n_N}{1+9k_a^2/2}}\right). \quad (8)$$

The average sample number (ASN) of the TNT sampling plan is given by Calvin (1977) as

$$ASN(p)=\frac{n_T(1-P_N^s)(1-P_T^t)(1-P_N)+n_NP_T^t(1-P_T)(2-P_N^s)}{(1-P_N^s)(1-P_T^t)(1-P_N)+P_T^t(1-P_T)(2-P_N^s)}. \quad (9)$$

Symmetric case

The probability of rejecting a good lot is called the producer's risk, say α and the probability of accepting a bad lot is called the consumer's risk β. The producer desires that the probability of lot acceptance should be larger than $1-\alpha$ at the acceptable quality level (AQL) and the consumer wants that it should be smaller than β at the limiting quality level (LQL). In this situation, the plan parameters should be determined such that the producer's and the consumer's requirements can be satisfied at the same time. The probabilities p_U and p_L may not be same, so we will consider two cases of the symmetric and the asymmetric cases.

For the symmetric case when $p_U = p_L = p/2$, Equations (7)-(8) are reduced to Equations (10)-(11)

$$P_T = 2\Phi\left(\left(z_{p/2} - 3k_a\right)\sqrt{\frac{n_T}{1+9k_a^2/2}}\right) - 1, \tag{10}$$

$$P_N = 2\Phi\left(\left(z_{p/2} - 3k_a\right)\sqrt{\frac{n_N}{1+9k_a^2/2}}\right) - 1. \tag{11}$$

Let p_1 denote AQL and p_2 denote LQL. Then, the plan parameters $(n_T, n_N; k_a, s, t)$ for the proposed TNT sampling scheme are determined so that the following two inequalities are satisfied.

$$P_a(p_1) = \frac{P_{T1}(1-P_{N1}^s)(1-P_{T1}^t)(1-P_{N1})+P_{N1}P_{T1}^t(1-P_{T1})(2-P_{N1}^s)}{(1-P_{N1}^s)(1-P_{T1}^t)(1-P_{N1})+P_{T1}^t(1-P_{T1})(2-P_{N1}^s)} \geq 1-\alpha, \tag{12}$$

$$P_a(p_2) = \frac{P_{T2}(1-P_{N2}^s)(1-P_{T2}^t)(1-P_{N2})+P_{N2}P_{T2}^t(1-P_{T2})(2-P_{N2}^s)}{(1-P_{N2}^s)(1-P_{T2}^t)(1-P_{N2})+P_{T2}^t(1-P_{T2})(2-P_{N2}^s)} \leq \beta. \tag{13}$$

where

$$P_{T1} = 2\Phi\left(\left(z_{p_1/2} - 3k_a\right)\sqrt{\frac{n_T}{1+9k_a^2/2}}\right) - 1,$$

$$P_{T2} = 2\Phi\left(\left(z_{p_2/2} - 3k_a\right)\sqrt{\frac{n_T}{1+9k_a^2/2}}\right) - 1,$$

$$P_{N1} = 2\Phi\left(\left(z_{p_1/2} - 3k_a\right)\sqrt{\frac{n_N}{1+9k_a^2/2}}\right) - 1,$$

$$P_{N2} = 2\Phi\left(\left(z_{p_2/2} - 3k_a\right)\sqrt{\frac{n_N}{1+9k_a^2/2}}\right) - 1,$$

where P_{T1} is the lot acceptance probability under the tightened inspection at AQL=p_1 and P_{T2} is the lot acceptance probability under the tightened inspection at LQL=p_2. Similarly, P_{N1} is the lot acceptance probability under the normal inspection at AQL=p_1 and P_{N2} is the lot acceptance probability under the normal inspection at LQL=p_2.

However, using Equations 12 and 13, we may find many combinations of the plan parameters, which satisfy the above conditions. So, we solve the plan parameters for the proposed TNT sampling scheme by minimizing the ASN at LQL. The complete optimization model with constraints is

$$\text{Minimize } ASN(p_2) \tag{14}$$

Subject to

$$P_a(p_1) \geq 1-\alpha \text{ ,}$$

$$P_a(p_2) \leq \beta \text{ ,}$$

$$2 \leq n_N < n_T \text{ ,}$$

$$s \leq t \text{ .}$$

Plan parameters in the symmetric case are determined for several combinations of the AQL and LQL and placed in Table 1. The ASN at LQL for selected values of the plan parameters are also placed in the same table.

Table 1. Proposed plans for symmetric case

p_1	p_2	n_T	n_N	t	s	k_a	ASN
0.001	0.003	183	179	16	16	0.9838	183
	0.004	117	106	5	7	0.9538	117
	0.006	67	63	5	8	0.9051	67
	0.008	51	43	2	18	0.8833	51
	0.010	35	33	9	18	0.8463	35
	0.015	19	18	9	15	0.7953	19
	0.020	18	16	3	19	0.7648	18
0.0025	0.005	383	361	8	13	0.9313	383
	0.010	82	81	16	17	0.8492	82
	0.015	48	47	2	13	0.8037	48
	0.020	33	30	3	19	0.7681	33
	0.025	25	23	7	14	0.7340	25
	0.030	23	21	9	18	0.7182	23
	0.050	13	12	1	20	0.6515	13
0.005	0.010	304	280	18	19	0.8555	304
	0.015	117	116	9	9	0.8099	117
	0.020	67	64	9	18	0.7742	67
	0.030	30	28	12	20	0.7141	30
	0.040	25	23	7	14	0.6794	25
	0.050	22	16	1	7	0.6463	22

	0.100	12	5	16	20	0.5412	12
0.01	0.020	219	205	9	20	0.7709	219
	0.030	82	75	4	15	0.7206	82
	0.040	45	43	1	13	0.6752	45
	0.050	32	30	10	18	0.6409	32
	0.100	13	11	14	18	0.5420	13
	0.150	8	6	11	15	0.4668	8
	0.200	5	4	5	8	0.4215	5
0.03	0.060	127	123	4	9	0.6208	127
	0.090	48	35	9	18	0.5559	48
	0.120	24	20	11	16	0.5063	24
	0.150	20	16	2	5	0.4670	20
	0.300	13	6	1	12	0.3360	13
0.05	0.100	85	82	7	18	0.5419	85
	0.150	37	26	6	6	0.4728	37
	0.200	17	15	3	15	0.4140	17
	0.250	12	11	10	12	0.3797	12
	0.500	8	6	13	13	0.2235	8

From the results in Table 1, we note the following points:

1. We could not find the plan parameters satisfying the conditions for p_1=0.001 and p_2=0.002.
2. For the same value of p_1, as the p_2 increases from 0.002 to 0.02, the plan parameters including ASN, t, n_N and n_T decrease.
3. This type of trend is also true for other values of AQL and LQL.

Asymmetric case

We also consider the asymmetric case where $p_U \neq p_L$ to derive the proposed plan. Particularly, we consider the case of $p_L = p/4$, and $p_U = 3p/4$ and the case of $p_L = p/3$, $p_U = 2p/3$. For the first case, for example, p_{1L}=0.025, p_{1U}=0.075, p_{2L} =0.01 and p_{2U}=0.03 if p_1=0.01 and p_2=0.04. So, the plan parameters should be determined so that the following two inequalities should be satisfied for the given constraints.

$$P_a(p_1) = \frac{P_{T1}(1-P_{N1}^s)(1-P_{T1}^t)(1-P_{N1})+P_{N1}P_{T1}^t(1-P_{T1})(2-P_{N1}^s)}{(1-P_{N1}^s)(1-P_{T1}^t)(1-P_{N1})+P_{T1}^t(1-P_{T1})(2-P_{N1}^s)} \geq 1-\alpha, \quad (15)$$

$$P_a(p_2) = \frac{P_{T2}(1-P_{N2}^s)(1-P_{T2}^t)(1-P_{N2})+P_{N2}P_{T2}^t(1-P_{T2})(2-P_{N2}^s)}{(1-P_{N2}^s)(1-P_{T2}^t)(1-P_{N2})+P_{T2}^t(1-P_{T2})(2-P_{N2}^s)} \leq \beta, \quad (16)$$

where for the case of $p_L = p/4$, and $p_U = 3p/4$

$$P_{T1} = \Phi\left((z_{3p_1/4} - 3k_a)\sqrt{\frac{n_T}{1+9k_a^2/2}}\right) - \Phi\left(-(z_{p_1/4} - 3k_a)\sqrt{\frac{n_T}{1+9k_a^2/2}}\right),$$

$$P_{T2} = \Phi\left((z_{3p_2/4} - 3k_a)\sqrt{\frac{n_T}{1+9k_a^2/2}}\right) - \Phi\left(-(z_{p_2/4} - 3k_a)\sqrt{\frac{n_T}{1+9k_a^2/2}}\right),$$

$$P_{N1} = \Phi\left((z_{3p_1/4} - 3k_a)\sqrt{\frac{n_N}{1+9k_a^2/2}}\right) - \Phi\left(-(z_{p_1/4} - 3k_a)\sqrt{\frac{n_N}{1+9k_a^2/2}}\right),$$

$$P_{N2} = \Phi\left((z_{3p_2/4} - 3k_a)\sqrt{\frac{n_N}{1+9k_a^2/2}}\right) - \Phi\left(-(z_{p_2/4} - 3k_a)\sqrt{\frac{n_N}{1+9k_a^2/2}}\right).$$

Similarly, the plan parameters for the proposed TNT sampling scheme for asymmetric case are determined by minimizing the ASN at LQL. The plan parameters for the asymmetric case under the same combinations of p_1 and p_2 are showed in Tables 2-3. Table 2 is constructed when $p_L = p/4$, $p_U = 3p/4$ and Table 3 is constructed when $p_L = p/3$, $p_U = 2p/3$.

Table 2. Proposed plans for asymmetric case of $p_L = p/4, p_U = 3p/4$

p_1	p_2	n_T	n_N	t	s	k_a	ASN
0.001	0.003	378	362	5	15	0.9913	378
	0.004	185	168	15	20	0.9612	185
	0.006	81	76	12	16	0.9180	81
	0.008	55	48	5	14	0.8868	55
	0.010	40	39	5	12	0.8615	40
	0.015	34	29	5	6	0.8162	34
	0.020	21	17	6	9	0.7734	21
0.0025	0.010	135	131	13	13	0.8634	135
	0.015	58	54	5	12	0.8115	58
	0.020	40	38	1	3	0.7872	39.99
	0.025	35	32	9	14	0.7582	35
	0.030	30	26	6	16	0.7368	30
	0.050	13	12	6	6	0.6489	13

0.005	0.015	205	200	9	10	0.8127	205
	0.020	114	112	4	5	0.7804	114
	0.030	43	39	1	2	0.7272	42.95
	0.040	28	23	11	11	0.6865	28
	0.050	27	20	1	10	0.6612	27
	0.100	16	14	6	16	0.5477	16
0.01	0.020	481	463	10	10	0.7617	481
	0.030	153	151	2	4	0.7262	153
	0.040	70	65	7	16	0.6859	70
	0.050	51	46	5	15	0.6579	51
	0.100	18	14	8	20	0.5605	18
	0.150	11	9	14	14	0.4844	11
	0.200	6	5	8	19	0.4272	6
0.03	0.060	260	232	5	19	0.6098	260
	0.090	76	73	9	13	0.5636	76
	0.120	38	30	2	18	0.5198	38
	0.150	27	22	12	16	0.4901	27
	0.300	8	6	1	3	0.3564	7.99
0.05	0.100	170	168	11	19	0.5299	170
	0.150	49	48	3	5	0.4769	49
	0.200	32	26	2	7	0.4296	32
	0.250	20	15	3	13	0.3859	20
	0.500	9	8	11	15	0.2294	9

Table 3. Proposed plan for asymmetric case of $p_L = p/3, p_U = 2p/3$

p_1	p_2	n_T	n_N	t	s	k_a	ASN
0.001	0.003	257	255	2	13	0.9888	257
	0.004	146	139	6	6	0.9586	146
	0.006	75	65	4	19	0.9161	75
	0.008	52	44	13	14	0.8786	52
	0.010	38	36	1	16	0.8601	38
	0.015	24	23	7	12	0.8052	24
	0.020	16	15	7	20	0.7724	16
0.0025	0.010	111	109	10	13	0.8602	111
	0.015	50	44	8	10	0.8051	50
	0.020	42	32	5	12	0.7712	42
	0.025	24	21	10	14	0.7391	24

	0.030	22	21	6	14	0.7158	22
	0.050	16	11	2	13	0.6424	16
0.005	0.015	139	132	5	14	0.8079	139
	0.020	74	71	19	20	0.7740	74
	0.030	51	46	16	17	0.7234	51
	0.040	30	22	6	13	0.6854	30
	0.050	22	16	14	18	0.6550	22
	0.100	11	8	1	12	0.5462	11
0.01	0.020	411	407	2	18	0.7726	411
	0.030	103	102	10	16	0.7228	103
	0.040	62	50	1	14	0.6839	62
	0.050	44	43	10	12	0.6478	44
	0.100	13	12	2	12	0.5548	13
	0.150	7	6	18	20	0.4790	7
	0.200	6	4	19	20	0.4156	6
0.03	0.060	242	229	7	9	0.6231	242
	0.090	65	50	7	16	0.5620	65
	0.120	35	31	6	6	0.5182	35
	0.150	25	24	2	17	0.4848	25
	0.300	8	7	6	13	0.3395	8
0.05	0.100	180	175	4	16	0.5431	180
	0.150	43	37	8	13	0.4787	43
	0.200	35	16	2	11	0.4220	35
	0.250	14	13	12	13	0.3827	14
	0.500	11	9	9	19	0.2213	11

From these tables, we observe the similar trends of the plan parameters with the symmetric case. The ASN for the proposed TNT plan when $p_L = p/4$, $p_U = 3p/4$ is larger than $p_L = p/3$, $p_U = 2p/3$ for all the combinations of p_1 and p_2.

APPLICATION OF THE PLAN

Suppose that a manufacturer of steel coils want to inspect the tensile strength (TS, in kg/cm^2) of a coil based on the proposed sampling plan. The lower and the upper specification limits are known as 45 and 78, respectively. The AQL is known as 0.005 at the producer's risk of 5%, while the LQL is known as 0.040 at the consumer's risk of 10% ($p_1 = 0.005$, $p_2 = 0.040$). It is also known that the current production process is asymmetric, so the fraction defective beyond the *USL* accounts for 75%.

That is, $p_U = 3p/4$ and $p_L = p/4$. For this case, the plan parameters from Table 2 are obtained by $n_T = 28, n_N = 23,\ s = 11, t = 11$ and $k_a = 0.6865$.

Thus, based on the operating procedure for the proposed TNT sampling plan, we start with the tightened inspection, i.e. take a random sample of size $n_T = 28$ and the acceptance number is $k_a = 0.6865$. Now, the collected data of size 28 taken from the lot randomly are showed in Table 4.

Table 4.The data of tensile strengths (in kg/cm^2) from a lot (28 measurements).

55.0	61.0	68.9	54.9	59.6	57.3	53.1
71.4	65.0	63.7	72.6	59.1	51.5	61.6
69.3	67.8	72.8	54.8	64.0	62.2	64.6
56.8	53.2	67.8	51.2	64.4	60.1	62.3

These observed measurements are also shown to be fairly close to the normal distribution according to the Anderson-Darling normality test. Figure 1 displays the normal probability plot of the collected data for testing the normality assumption. Figure 2 displays the histogram of the collected data with the lower and the upper specification limits.

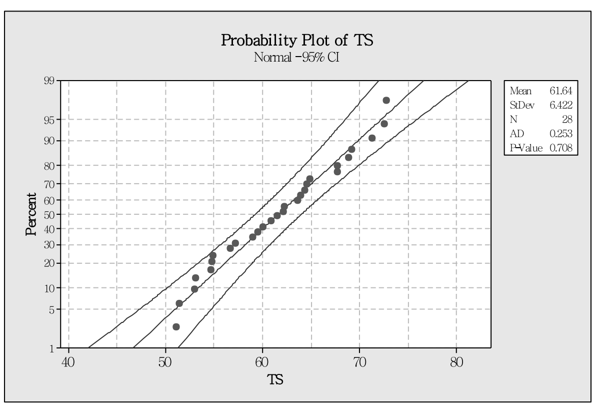

Fig 1. The normal probability plot of the collected data

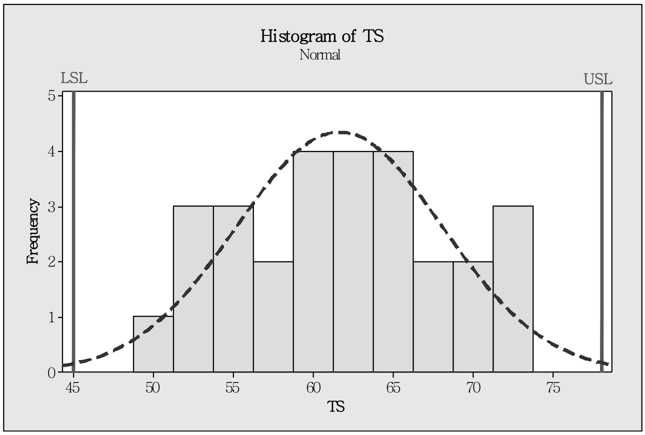

Fig 2. The histogram of the collected data with specification limits.

Further, the sample mean and the sample standard deviation from the collected samples are calculated as $\overline{x} = 61.6429$, $s = 6.4215$. So, the estimated C_{pk} index is $\hat{C}_{pk} = 0.8491$. Thus, in this case, the manufacturer should accept the lot since $\hat{C}_{pk} = 0.8491$ is larger than the acceptance number $k_a = 0.6865$. If the following additional 10 lots in a row are all accepted then the sampling inspection can be switched to the normal inspection.

Comparison of the proposed plan with existing plans

In this section, we will compare the efficiency of the proposed plan in terms of the sample size (average sample number, ASN) with existing sampling plans using C_{pk}. We will only consider the symmetric case for the proposed plan and the existing sampling plans and compare the plan under the same values of AQL and LQL. Table 5 shows the ASN for the proposed plan and compares with the resubmitted sampling plan using C_{pk} by Aslam *et al.* (2013) and the single sampling plan using C_{pk}.

Table 5. Sample size comparison of proposed plan with existing plans

p_1	p_2	Proposed Plan	Resubmitted Plan	Single Plan
0.001	0.003	183	374.06	351
	0.004	117	216.86	166
	0.006	67	113.87	74
	0.008	51	78.88	47
	0.010	35	59.68	34
	0.015	19	37.98	21

	0.020	18	27.10	15
0.0025	0.005	383	750.74	822
	0.010	82	152.14	118
	0.015	48	78.60	53
	0.020	33	51.60	32
	0.025	25	40.67	24
	0.030	23	32.53	18
	0.050	13	18.97	10
0.005	0.010	304	567.65	623
	0.015	117	195.21	186
	0.020	67	111.19	87
	0.030	30	56.95	37
	0.040	25	38.09	23
	0.050	22	27.12	17
	0.100	12	10.84	7
0.01	0.020	219	407.29	449
	0.030	82	138.38	132
	0.040	45	75.93	61
	0.050	32	51.52	37
	0.100	13	18.97	11
	0.150	8	10.85	6
	0.200	5	7.60	4
0.03	0.060	127	214.18	240
	0.090	48	67.81	68
	0.120	24	35.24	31
	0.150	20	24.41	18
	0.300	13	8.13	5
0.05	0.100	85	149.13	167
	0.150	37	46.08	46
	0.200	17	24.39	20
	0.250	12	13.55	12
	0.500	8	5.42	3

From Table 5, it is very clear that the proposed plan provides the smaller ASN than the single sampling plan and the resubmitted sampling plan under the same values of AQL and LQL. Therefore, the proposed plan performs better than the two existing sampling plans in terms of the sample size required.

CONCLUDING REMARKS AND AREAS FOR FUTURE RESEARCH

In this paper, a variable TNT sampling plan based on the process capability index C_{pk} is developed for a normally distributed process with two-sided specification limits. Plan parameters are given and discussed for both symmetric and asymmetric cases, which will be helpful for practitioners to implement the TNT sampling plan. From the comparison of the proposed plan with the existing sampling plans using capability index C_{pk}, we found that the proposed plan is more efficient in terms of the average sample number as compared to the single and the resubmitted sampling plans. The proposed plan results in reduced cost and time of inspection for the decision on the submitted lots. In the proposed plan, we assumed that the quality characteristic follows the normal distribution. For future research, it would be suggested to develop a TNT variable sampling scheme for non-normal distributions. Designing the proposed plan using a cost model is also an interesting area for the future research.

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