As shown in (24) and (25), since the mean and standard deviation expressions of *OC* have been obtained, CDF's sensitivities of *OC* to UPFC, as shown in (7), could be reformed as follows:

$$\begin{aligned}\gamma_{oc} &= \frac{\partial F_{oc}}{\partial \mu_{oc}}\frac{\partial \mu_{oc}}{\partial X_U} + \frac{\partial F_{oc}}{\partial \sigma_{oc}}\frac{\partial \sigma_{oc}}{\partial X_U} = \frac{\partial F_{oc}}{\partial \mu_{oc}}\sum_{i=1}^{S}\sum_{k}^{2}\omega_{i,k}\frac{\partial OC(i,k)}{\partial X_U} \\ &+ \frac{1}{2}\frac{\partial F_{oc}}{\partial \sigma_{oc}}(E[OC^2]-\mu_{oc}^2)^{-1/2}\sum_{i=1}^{S}\sum_{k}^{2}2\omega_{i,k}OC(i,k)\frac{\partial OC(i,k)}{\partial X_U} \\ &- \mu_{oc}\frac{\partial F_{oc}}{\partial \sigma_{oc}}(E[OC^2]-\mu_{oc}^2)^{-1/2}\sum_{i=1}^{S}\sum_{k}^{2}\omega_{i,k}\frac{\partial OC(i,k)}{\partial X_U}\end{aligned} \tag{26}$$

To calculate CDF's sensitivities of *OC* to UPFC, all components but $\partial OC/\partial X_U$ could be calculated by using PEM to solve POPF problem. How to compute the partial differential function $\partial OC/\partial X_U$ will be discussed in the next section.

(4) SENSITIVITIES of OC to UPFC ANALYSIS (move)

we deduce the expression of $\partial OC/\partial X_U$, which could be obtained by using interior point method to solve OPF problem.

Now, let us construct the Lagrangian for the OPF problem, in which the random variable $\tilde{x}_s$ has been transferred to certain ones ($\bar{x}_s$) by PEM.

$$L(X_C,X_U,\bar{X}_S,\kappa,\nu) = f_{\cos t}(X_C,X_U,\bar{X}_S) + \sum_{m=1}^{M}\kappa_m h_m(X_C,X_U,\bar{X}_S) + \sum_{n=1}^{N}\nu_n g_n(X_C,X_U,\bar{X}_S) \tag{27}$$

where κ_m and ν_n are the Lagrange multipliers for the equality and inequality constraints, respectively. Therefore, if an inequality constraint is binding, we could treat it as an equality constraint. Otherwise, ignore it. Then, we rewrite (27) as

$$L(X_C,X_U,\bar{X}_S,\kappa) = f_{\cos t}(X_C,X_U,\bar{X}_S) + \sum_{m\in A}\kappa_m h_m(X_C,X_U,\bar{X}_S) \tag{28}$$

where A is the set of active constraints. We consider the case where the UPFC is installed between buses *i* and *k*. The derivation $\partial OC/\partial X_U$ is simply the amounts by which the operation cost could be changed by allowing a small change of the UPFC control variables. Then, obtain the $\partial OC/\partial X_U$ by assuming that there is a UPFC, which is not put into operation. Thus, three additional constraints are added into the original OPF problem.

$$T_{ik} = T, \phi_{ik} = \phi, \rho_{ik} = \rho \tag{29}$$

Then, using KKT condition for OPF problem with the constraints, we can obtain

$$\frac{\partial f_{\cos t}}{\partial X_U} = -\sum_{m\in A}\kappa_m^*\frac{\partial h_m}{X_U} \tag{30}$$

where κ_m^* is the Lagrange multipliers of equality constraints in the optimal situation.

Note that (30) is easy to compute. Then, combining (26) and (30), we could get CDF's sensitivities of *OC* to UPFC finally. Thus, we can obtain CDF's sensitivities of *OC* to UPFC control variables for each possible transmission line by solving only the base case POPF.

RESULTS AND ANALYSIS

The proposed CDF's sensitivity method was tested on IEEE-5 and IEEE 14 systems to validate its effectiveness. These systems have 7 and 15 lines, respectively. Fig. 4 shows the five-bus system. The line data of the five-bus system are given in Ref. 6.

(1) IEEE-5 bus system

The system consists of two generators and buses 1 and 2. The generation cost function, measured in \$/h, is defined as follows:

$$f_{cost}(P_{Gi}) = a_i P_{Gi}^2 + b_i P_{Gi} + c_i \tag{31}$$

where P_{Gi} is the generator's real power generation, measured in MW. a_i, b_i, and c_i, cost parameters for generator i, are summarized in Table 1. The load data and statistic characteristic are given in Table 2.

Table 1. Generator Data.

generator	ai	bi	ci	Pmax/MW	Pmin/MW	Qmax/MVar	Qmin/MVar
1	0.00082	11	692.32	250	45	150	-100
2	0.000776	12	692.32	150	15	50	-40

Table 2. Load data and statistic characteristic.

Bus	Pload/MW	Qload/MVar	Variance of Pload	Variance of Qload
1	0	0	--	--
2	21.7	12.7	2.31	1.12
3	94.2	19	43.4	2.32
4	47.8	3.9	11.22	1.01
5	7.6	1.6	1.44	0.67

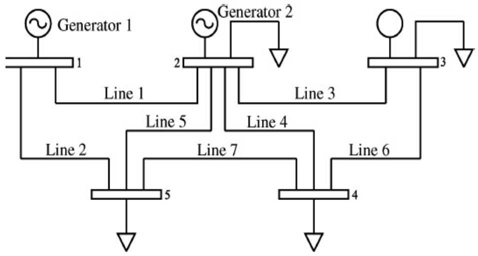

Fig. 4. Diagram of five-bus system.

To analyze the impact of the UPFC installed in different lines to CDF of *OC*, we calculate CDF's sensitivities of *OC* to control variables of UPFC. The sensitivities of UPFC in all possible line are shown in Table 3. By analyzing the results, CDF's ranges of UPFC control variables are 0.00069~0.05477, 0.00747~0.22716, and 1.07×10^{6}~5.99×10^{6}, respectively. Obviously, phase shifting angle has minimal impact on CDF of *OC* totally. From the view of UPFC located in different branches, when UPFC is installed in Line 1 (0.00587, 0.11569, 3.15×10^{6}), Line 2 (0.07926, 0.32861, 3.15×10^6), or Line 5 (0.02887, 0.0939, 4.59×10^5), the probability distribution of *OC* in high value region is lower than other lines. Among these three candidates, Line 2 is the most suitable line for UPFC installed.

Table 3. Sensitivities of UPFC control variables.

UPFC location	Sensitivities to T_{ij} of UPFC	Sensitivities to ϕ_{ij} of UPFC	Sensitivities to ρ_{ij} of UPFC
Line 1	0.00587	0. 11569	3.15×10^{6}
Line 2	0.07926	0.32861	3.15×10^{6}
Line 3	0.00147	0.01068	4.59×10^{6}
Line 4	0.00156	0.01112	4.59×10^{5}
Line 5	0.02887	0.09390	4.59×10^{5}
Line 6	0.00099	0.01067	3.60×10^{6}
Line 7	0.00228	0.021790	1.54×10^{4}

To illustrate the effectiveness of stochastic sensitivity to optimal location of UPFC, CDF of *OC* is calculated in the situation of all branches with equal capacity of UPFC (T_{ij}, φ_{ij}, ρ_{ij} are set to 0.98p.u., 0.004p.u.,0.3p.u. partly). The corresponding results are shown in Table 4. Before UPFC is installed, the operation cost of the system is 3282.46$. With UPFC being installed in different lines, the operation cost will be lower than 3282.46, and the larger reduction margin is 22.46%, 8.04%, and 6.3%, when UPFC is located in Line 2, Line 5, and Line 1 accordingly.

Table 4. Operation cost of CDF when UPFC is located in different lines.

UPFC location	Operation Cost		CDF when OC is mean value without UPFC	CDF's Reduction margin with UPFC
	Mean /$	standard deviation		
Without UPFC	3282.46	63.48	0.5	--
Line 1	3277.4	63.42	0.5315	6.30%
Line 2	3264.8	61.79	0.6123	22.46%
Line 3	3282.1	63.47	0.5021	0.43%
Line 4	3282.1	63.47	0.5022	0.44%
Line 5	3276.1	63.39	0.5402	8.04%
Line 6	3282.2	63.48	0.5017	0.34%
Line 7	3281.5	63.46	0.5058	1.16%

Compared to the *OC* without UPFC, PDF of *OC* with UPFC in Line 1, Line 2, and Line 5 is shown in Fig. 5. All possible installed methods could move PDF of *OC* towards lower value region, so as to reduce CDF of *OC* in higher value area.

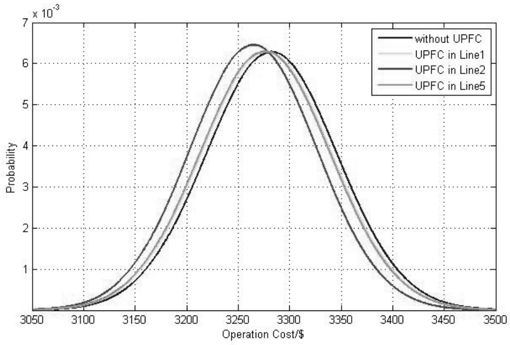

Fig. 5. Operation cost of PDF when UPFC is located in different lines.

(2) IEEE-14 bus system

There are two generators and 15 lines in IEEE-14 bus system. Generator data are the same as IEEE-5 bus system. Based on the sensitivity of *OC*'s CDF calculation, the result is obtained and shown in Table 5. It could be seen that line 2 is the most acceptable position for UPFC, because the CDF's sensitivity of UPFC's control variables is 0.07206, 0.25671, and 8.01×10^6.

Table 5. Sensitivities of UPFC control variables.

UPFC location	Stochastic sensitivity of UPFC's variables		
	T_{ij}	φ_{ij}	ρ_{ij}
Line 1	0.00625	0. 09511	8.01×10^6
Line 2	0.07206	0.25671	8.01×10^6
Line 3	0.00407	0.01144	1.02×10^5
Line 4	0.00881	0.01276	1.02×10^5
Line 5	0.00604	0.07089	1.02×10^5
Line 6	0.00332	0.01146	7.71×10^6
Line 7	0.00211	0.01328	6.17×10^6
Line 8	0.00958	0.00728	8.60×10^5
Line 9	0.00367	0.00240	8.60×10^5
Line 10	0.00370	0.00606	8.60×10^4
Line 11	0.00946	0.00728	2.22×10^4
Line 12	0.00012	0.00366	2.22×10^4
Line 13	0.00956	0.00728	3.42×10^4
Line 14	0.00364	0.00240	2.29×105
Line 15	0.00013	0.00366	1.18×104

In order to explain the principle that *OC* is affected by UPFC, real powers' PDF of generators is got when UPFC is installed in line 2, and control variables are set to 0.98p.u., 0.003 p.u., 0.3 p.u. As shown in Fig. 6 and Fig. 7, the mean real powers of generators are 171.88MW and 88.87 MW without UPFC, while the mean active powers change to 212.72MW and 48.69MW due to UPFC in line 2. Because the generation marginal cost is lower, it is profitable to obtain more real power from generator 1, as long as its adjusted marginal cost stays lower, and no operational limits are reached. Therefore, for the optimal location of the UPFC in this system, it is profitable to reduce the total generation cost rather than the transmission line loss since two generators have different generation costs. After UPFC is installed, generator 1 output would increase, while generator 2 output decreases. As a result, the operation cost of the power system could be lower than that of the original situation.

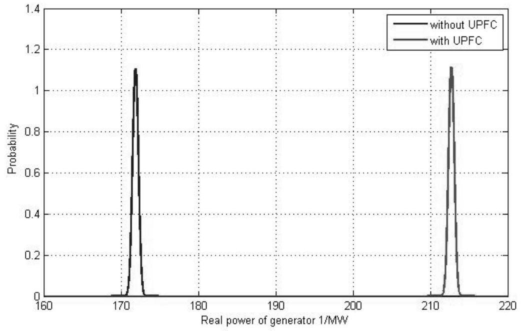

Fig. 6. Comparison of PDF of Generator 1 active power.

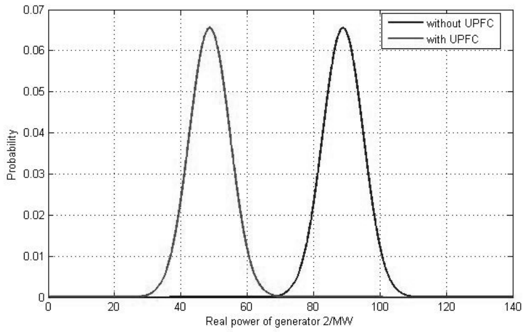

Fig. 7. Comparison of PDF of Generator 2 active power.

CONCLUSION

We have proposed a kind of stochastic method to optimize UPFC's installing location. Different from the existing algorithms, the proposed method considers randomness in power system, such as load's uncertainty. As a result, the operation cost would be a stochastic variable, not a certain value any more. To evaluate the impact of UPFC at different transmission lines to CDF of *OC*, CDF' sensitivity is introduced in this paper. POPF and PEM models are also used to calculate the sensitivities. Finally, based on CDF' sensitivity, the optimal UPFC location would be decided by comparison. IEEE-5 and IEEE-14 test systems are employed to illustrate the validity and efficiency of the proposed method. In future research, we will take the voltage regulation ability of UPFC into consideration and optimize UPFC's installation by dealing with active power imbalance and lack of voltage stability.

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