Table 5. The natural frequencies corresponding to different mass ratios and mass locations for two masses, where $N = 100$.

α_1	α_2	η_1	η_2	ω_1	**Özk.01**	ω_2	**Özk.01**	ω_3	**Özk.01**	ω_4	**Özk.01**
1	1	0.1	0.3	6.118	6.118	27.536	26.506	55.338	55.412	98.966	99.097
			0.7	6.183	6.183	22.588	22.598	60.165	60.226	124.852	125.021
		0.5	0.3	4.730	4.785	25.116	19.802	60.830	45.252	141.073	95.238
			0.7	4.730	4.730	25.116	25.128	60.830	60.883	141.073	141.289
1	10	0.1	0.3	2.509	2.509	26.066	26.075	50.993	51.069	94.388	94.505
			0.7	2.516	2.516	20.052	20.060	58.763	58.824	124.117	124.285
		0.5	0.7	2.387	2.387	17.916	17.925	59.518	59.569	136.776	136.993
10	1	0.1	0.3	4.513	4.514	18.548	18.563	38.536	38.578	96.578	96.694
			0.7	4.671	4.671	12.423	12.429	50.941	50.992	121.270	121.432
		0.5	0.7	2.078	2.078	22.025	22.036	54.599	54.647	140.648	140.866
10	10	0.1	0.3	2.356	2.357	16.238	16.257	29.949	29.975	92.758	92.863
			0.7	2.412	2.413	8.845	8.850	48.883	48.883	120.858	121.018
		0.5	0.7	1.677	1.677	9.806	9.812	53.472	53.516	136.317	136.535

Table 6. The natural frequencies corresponding to different mass ratios and mass locations for three masses, where $N = 100$.

α_1	α_2	α_3	η_1	η_2	η_3	ω_1	**Özk.01**	ω_2	**Özk.01**	ω_3	**Özk.01**	ω_4	**Özk.01**
1	1	1	0.1	0.4	0.8	5.130	5.130	18.908	18.915	40.627	40.668	101.805	101.949
1	1	10	0.1	0.4	0.8	3.011	3.011	11.726	11.731	39.407	39.445	98.570	98.713
1	10	1	0.1	0.4	0.8	2.182	2.182	17.179	17.186	37.318	37.356	99.182	99.323
10	1	1	0.1	0.4	0.8	4.141	4.142	13.012	13.021	25.934	25.958	99.303	99.439
10	10	10	0.1	0.4	0.8	1.864	1.864	6.672	6.675	14.144	14.161	93.644	93.774
1	1	1	0.2	0.5	0.7	4.411	4.411	18.193	18.201	39.151	39.189	137.782	137.980
1	1	10	0.2	0.5	0.7	2.350	2.350	13.463	13.469	34.966	35.001	134.570	134.770
1	10	1	0.2	0.5	0.7	2.048	2.048	18.178	18.185	29.348	29.378	137.760	137.958
10	1	1	0.2	0.5	0.7	2.857	2.857	10.767	10.771	35.346	35.379	137.078	137.274
10	10	10	0.2	0.5	0.7	1.540	1.540	6.381	6.383	13.564	13.578	134.055	134.252

As shown in Table 4, the numerical results obtained for the natural frequencies are extremely close to analytical results. Similar findings are presented in Table 5 for two concentrated masses and in Table 6 for three masses.

CONCLUSIONS

In this study, a general model is considered to analyze the dynamic behavior of structural elements that may have variable material, cross-section, or some other discontinuities. Unlike the classical approach, instead of writing a separate equation for each span containing discontinuity, a single equation with singularity function is discussed. This provides great convenience to us in the solution, as all discontinuities occurring in any structural element are modeled with a single equation. To demonstrate the accuracy of the present technique, the general solution procedure has been applied to two different problems, such as the multilinear elastic spring beam and beam-mass system. As a result of the comparisons made, it has been observed that the results obtained as a result of applying the classical approach and the present method are extremely close to each other.

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