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State Feedback Repetitive Control: Robustness and Load Disturbance Conditions

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ABSTRACT

Robust and load disturbance conditions for state feedback Repetitive Control (RC) are investigated for linear time-invariant system. The Conditions found sets either an upper limit or lower limit weighting parameter depending on the case investigated in the design. The repetitive design investigated is a development of previously reported work, where the new design incorporates both past error feedforward and current error feedback rather than a current error feedback alone. The design idea is to include a pure delay model acting on the system input representing periodic disturbances. Isolating the delay model, finding the overall transfer function around the delay model, and using the small gain theorem, a stability condition is obtained that assures overall system stability and periodic disturbances accommodation. The conditions found, as the simulation results obtained show, had suppressed the uncertainty effect in the case where a weighting parameter is used compared to the case where the weighting parameter is omitted.

Keywords: Repetitive control; state feedback; uncertainty.

INTRODUCTION

In industry, it is required from a system to follow pre-defined task for ad-infinitum number of executions known as “trials” in a continuous manner without resetting between trials. Repetitive systems are those where the reference trajectory required to follow to a high precision is of a repetitive structure (Rogers et al., 2007). One well known controller to such systems is the Repetitive Controller (RC), where the persistence of this controller is to learn from previous experiences to enhance reference tracking and rejects periodic disturbances continuously (Hara et al., 1988). The start of the RC is in the reported work of (Inoue et al., 1981b), where the objective was to accommodate periodic disturbances in power supply control application. The work reported in Inoue et al. (1981a) was introduced to track periodic reference in a motion control application. Thus, it is clear that RC has a direct impact on industry applications especially those of rotary movement, such as in disc drives (Moon et al., 1998, Chen et al., 2006), electrical motors (Mattavelli et al., 2005), robotics (Kaneko et al., 1997, Consor et al., 1990), and other systems performing operations as such.

The human learning process is the main idea behind RC principle, where it uses previous trials to modify the controlling signal such that the overall system learns to follow a periodic reference trajectory with period T to a high precision. The research efforts adopted this idea and most of the produced work can be seen in the continuous time domain. This is a trend in research efforts due to repetitive system nature, and the use of the time instants t to form the forcing function for $t + T$ in the update law. A good source to RC principle and several designed updating laws can be seen in Longman (2010) and Wang et al. (2009). The internal model principle (IMP) was introduced by Francis and Wonham (1975), and since then it became the main principle to all of the RC designs. The IMP suggested modelling a periodic signal as an autonomous system inside a positive feedback loop. Then, the fundamental of the small gain theorem is used to design the control system such that the overall system is stable to achieve the required task; track/reject periodic signal without steady-state errors.

Iterative Learning control (ILC) is another controlling technique used to accommodate periodic disturbances and enhances the performance of repetitive systems. ILC controller start is referred to the work proposed by Arimoto et al. (1984) where the main task was to use the error signal of the current trial as a forcing function to update the control signal for the next trial. Here it can be thought that RC and ILC are the same, but they are not similar even though they use the same updating technique. The main difference is that the RC does not reset states between trials; system initial states for trial k are those of the final states of trial $k-1$ such that $x_{k+1}(0) = x_k(T-1)$, in the sense of a continuous periodic signal. In ILC, the system resets to the home position after each trial to start the next trial. For a list of controllers differences, the work in Wang et al. (2009) provides a table a comparison between RC and ILC, which clarifies those similarities and differences.

Longman (2000) suggested that the similarities in the general structure of the two methods allow diverting the design to lift the batch process description to be formed in a matrix representation.

Due to system nature, the control problem is stated in *2D* formulation, where time and trial indices are both considered in designing RC controllers. A uniform structure, which depends on the trial index alone known as a lifted form, can be considered in general instead of using time and trial indexes expression. This paper revisits the state feedback RC design, then both system uncertainty and load disturbance conditions are investigated to extend controller capability against system uncertainty and disturbance rejection to complete the design.

The design steps start with modelling the periodic signal first as an autonomous system containing a pure delay model along the forward path with positive feedback (Inoue et al., 1981b), then it uses the internal model principle by duplicating that delay model inside a feedback loop with the RC control, where the delay model affects the system output (DeRoover et al., 1997, DeRoover et al., 2000). The work presented in DeRoover et al. (1997) and DeRoover et al. (2000) showed that both RC and ILC are not similar, but they are related by duality and the solution found in one controller can be used to accommodate periodic disturbances in another under certain structure. This has been done for explicit use of current error feedback structure in state feedback and output injection. Freeman et al. (2013) designed a modified framework that incorporates both current error feedback and past error feedforward in the design instead of using the current error feedback explicitly. It also showed the advantage of using past error feedforward over current error feedback to accommodate periodic disturbances experimentally.

This paper, as mentioned earlier, revisits the RC design within the proposed framework given in Freeman et al. (2013), where it incorporates both past and current error signals, in state feedback structure “only”. This paper then extended the design presented in Freeman et al. (2013) by including new robust conditions based on the state feedback case using singular values, which forms the novelty and the major contribution of this paper. The idea of investigating robust boundaries with singular values had been studied earlier; one literature that uses singular values method to predict robust stability conditions for milling operation is Hajdu et al. (2017). The conditions found set either an upper limit or a lower limit to a weighting parameter to limit the associated uncertainty effect. The conditions are expressed in singular values because this design is made for linear systems, and the singular values selection gives a direct reflection on vectors weights. Conditions found are different than those in Freeman et al. (2008), where the latter discusses uncertainty condition for the design in the presence of current error feedback “only” in the frequency domain. Simulation results obtained show the reliability of the new design to perform well in the presence of modelling mismatch.

The following section discusses in brief the RC design in the general case under the proposed framework in Freeman et al. (2013). Design robustness against unmodelled dynamics of the proposed RC framework is presented afterwards. This is followed by Load disturbance limitations conditions. Simulation results are discussed. Finally, conclusion and future work are considered.

Controller Design Background

Start with a linear time-invariant system with m outputs, p inputs, and n states, having a discrete overall transfer function in the state space form given by $P(z) = C(zI_n - A)^{-1}B + D$. The matrices A, B, C, and D are of valid dimensions. Also assume the system output to be $y(z)$ and $u(z)$ is the system input, then the process output equation is given as $y(z) = P(z)u(z)$.

A lifted form describing the system can be considered, for a " single trial " with a finite time duration; N samples, the model of the system dynamics at a trial k can be written as

$$x_k(i+1) = Ax_k(i) + Bu_k(i), \quad x_k(0) = x_{k-1}(N-1)$$

$$y_k(i) = Cx_k(i) + Du_k(i) \tag{1}$$

where $0 \le i \le N-1$. From above, RC controller does not reset to the initial state after each trial. Now, introduce the input and output vectors as

$$u_k = [u_k(0), u_k(1), \cdots, u_k(N-1)]^T$$

$$y_k = [y_k(0), y_k(1), \cdots, y_k(N-1)]^T$$

The process dynamics for each trial can be written as

$$y_k = Pu_k \tag{2}$$

with the process matrix P defined as

$$P = \begin{bmatrix} D & 0 & 0 & \cdots & 0 \\ CB & D & 0 & \cdots & 0 \\ CAB & CB & D & \cdots & 0 \\ \vdots & \vdots & CB & \ddots & \vdots \\ CA^{N-2}B & CA^{N-3}B & CA^{N-4} & \cdots & D \end{bmatrix} \tag{3}$$

where the elements of P are the Markov parameters. Similarly, define the reference in discrete form, to hold the vector elements of

$$r = [r(0), r(1), \cdots, r(N-1)]^T$$

The block $\Phi(z)$ in the RC control problem structure given in Figure 1 is a diagonal transfer function matrix, which has an internal model representation along its diagonals. As DeRoover et al. (2000) pointed out, in RC case, there are m channels as the block operates in the output space with N state variables.

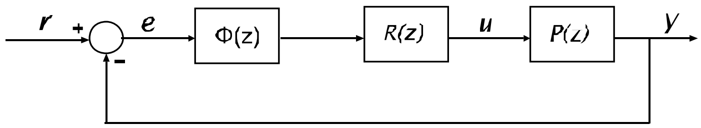

Figure 1. RC as a feedback problem (Freeman et al., 2013).

The autonomous system consisting of a positive feedback control loop with a pure time delay in the forward path with appropriate initial conditions representing a periodic disturbances can be modelled with a signal of length N in discrete-time as

$$x_w(t_{k+1}) = A_w x(t_k), x_w(t_0) = x_{w0}$$

$$w(t_k) = C_w x(t_k) \tag{4}$$

where the $N \times N$ matrix A_w is given by

$$A_w = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

and the $1 \times N$ row vector C_w as $C_w = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}$.

The control problem then can be defined as the need to find a robust controller $K(z)$ (where z denotes the discrete-time delay operator) for the robust periodic control problem that fulfills the following:

Given a $m \times l$ transfer-function matrix $P(z)$ with an input vector consists of the plant input and a disturbance input; $u = u_p + u_w$, the output signal as defined in (2) and a reference signal $r(t_k) = r(t_{k+N}), t_k = 0, \Delta T, 2\Delta T, \cdots$, with N sampling time. It is required to design $K(z)$ such that 1) the overall closed loop system is asymptotically stable. 2) The tracking error, $e_k = r - y_k$, tends to zero along the trial domain. 3) The conditions above are robust.

The solution considered in DeRoover et al. (1997) and DeRoover et al. (2000) uses the internal model principle, as well as the small gain theorem to set stability conditions to design the feedback gain and the observer gain using Linear Quadratic Regulator (LQR), where the periodic disturbances act on the system output. Freeman et al. (2013) considered a more general case as it incorporates both current error feedback and past error feedforward in the designed framework instead of the current error feedback alone.

This paper reintroduces the RC design scheme in state feedback reported in Freeman et al. (2013) with different stability conditions depending on the error case considered: current error or past error feedforward. The RC design (Freeman et al., 2013) is explained in brief next and the stability conditions, which form the novelty of this paper for each case, are introduced later.

RC Controller Design via State Feedback

Consider the system in (4) for a single channel; also introduce the following $N \times 1$ vectors:

$$B_w = \begin{bmatrix} 0 & \cdots & 0 & 0 & 1 \end{bmatrix}^T$$

and

$$D_w = \begin{cases} 0 & \text{for past error feedforward case} \\ 1 & \text{for current error feedback case} \end{cases}$$

For a multi-input multi-output (MIMO) case define the diagonal matrix A_r consisting of A_w along its diagonal,

$$A_r = \text{diag}\{A_w\}$$

B_r, C_r and D_r are also defined the same as well, where each diagonal block is repeated m times (acting on the system output). Thus, considering the periodic problem proposed in Figure 1, the transfer function of the delay model, $\Phi(z)$, is given as

$$C_r(zI_{N_m} - A_r)^{-1}B_r + D_r = \begin{cases} (z^N I_m - I_m)^{-1} & \text{if } \mathrm{D_w} = 0 \\ (I_m - z^{-N} I_m)^{-1} & \text{if } \mathrm{D_w} = 1 \end{cases}$$

The overall idea of the design considered that uses the state feedback in Freeman et al. (2008; 2013) is to combine both the plant and the internal model in one structure as

$$\begin{bmatrix} x_r(i+1) \\ x(i+1) \end{bmatrix} = \begin{bmatrix} A_r & -B_rC \\ 0 & A \end{bmatrix} \begin{bmatrix} x_r(i) \\ x(i) \end{bmatrix} + \begin{bmatrix} -B_rD \\ B \end{bmatrix} u_k(i) + \begin{bmatrix} B_r \\ 0 \end{bmatrix} r(i) \quad (5)$$

Stabilising this system guarantees periodic disturbances accommodation since the output of the combined system is the plant output and its input is the control input signal, where x_r is the internal model system state. Manipulating the combined system and choosing the control input of the combined system to be

$$u(i) = -K_r \begin{bmatrix} \hat{x}_r(i) \\ \hat{x}(i) \end{bmatrix} \quad (6)$$

adding an observer to estimate the combined system states, this in turn will end up with the overall system of the form (Freeman et al., 2013)

$$\begin{bmatrix} \hat{x}_r(i+1) \\ \hat{x}(i+1) \end{bmatrix} = \begin{bmatrix} A_r & -B_rC \\ 0 & A \end{bmatrix} \begin{bmatrix} \hat{x}_r(i) \\ \hat{x}(i) \end{bmatrix} - \begin{bmatrix} -B_rD \\ B \end{bmatrix} K_r \begin{bmatrix} \hat{x}_r(i) \\ \hat{x}(i) \end{bmatrix}$$
$$+ L_r \left(v(i) - \left(\begin{bmatrix} C_r & -D_rC \end{bmatrix} + D_rDK_r \right) \begin{bmatrix} \hat{x}_r(i) \\ \hat{x}(i) \end{bmatrix} \right) \quad (7)$$

As the internal model principle suggests, the design at this stage requires isolating the delay $z^{-N}I_m$ and finding the overall transfer function, $H(z)$, that links its output by its input.

Stability condition following the small gain theorem then would be

$$\|H(z)\| < 1 \quad (8)$$

The overall transfer function around the delay operator, $H(z)$, definition differs depending on the error case considered for either past error feedforward or current error feedback, for **Past error feedforward case**, $H(z)$ is defined as

$$H(z) = (G(z) + P(z))G(z)^{-1} \quad (9)$$

while in the **Current error feedback case**

$$H(z) = G(z)(G(z) + P(z))^{-1} \quad (10)$$

$G(z)$ in both cases is governed by the following:

$$G(z) = \begin{bmatrix} C_r & -D_rC \end{bmatrix} \left(zI - \begin{bmatrix} A_r & -B_rC \\ 0 & A \end{bmatrix} + \begin{bmatrix} B_rD \\ -B \end{bmatrix} K_r \right)^{-1} \begin{bmatrix} -B_rD \\ B \end{bmatrix} - D_rD \quad (11)$$

Solving the Linear Quadratic Regulator (LQR) via the Riccati equation as Freeman et al. (2013) suggests is the key to find the state feedback solution, K, such that the model to consider is the difference between the combined system and the estimator structure to minimize the defined cost function.

Robust Conditions for RC Design in State Feedback

Modelled systems do not hold the exact model description, or they suffer from the non-linearities at high frequencies. It is required to complete the design to investigate the design robustness and set conditions for system stability under such model mismatch. In this section the robust stability conditions are set for the RC control in Freeman et al. (2013) in past error feedforward and current error feedback. This is based on the stability condition assigned in (8). The previous reported works did not discuss this subject in Singular Values concept, which forms the main novelty of the work presented in this paper since the system considered is linear time-invariant. In Freeman et al. (2008), the algorithm robustness was discussed in the frequency domain for the case of current error feedback only. We start with the stability condition given in (8) and considering the following cases:

• Current error feedback in state feedback design.

The starting point is the stability condition given in (8), where the induced norm is required to be less than 1 to guarantee system stability. Consider the system uncertainty to be (Δ) acting on the system in operation. Define $[P = P_o + P_o\Delta W]$, where P_o, Δ, W are the nominal plant, plant uncertainty, and the uncertainty weight, respectively. Each of the defined variables is assumed stable, causal, and linear time invariant for simplicity, in combination with the more conservative definition of $H(z)$ given in (10) in terms of singular values as (Ringwood, 1995)

$$\overline{\sigma}\left(\frac{G}{G - P_o - P_o\Delta W}\right) < 1 \qquad (12)$$

where $\overline{\sigma}(.)$ represents the maximum singular value and $\underline{\sigma}(.)$ is the minimum singular value. The above again can be manipulated to give

$$\overline{\sigma}(\Delta) > \frac{\sigma(G) - \sigma(G) - \sigma(P_o)}{\sigma(W)\sigma(P_o)} \qquad (13)$$

Since the uncertainty assumed to be stable ($\overline{\sigma}(\Delta) < 1$), then equation (13) can be written as

$$\frac{\sigma(G) - \sigma(G) - \sigma(P_o)}{\sigma(W)\sigma(P_o)} < 1 \qquad (14)$$

Equation (14) will give the proper condition for the weighting factor (W) such that the left hand side is minimized, which gives the choice of the numerator to be selected as

$$1 > \overline{\sigma}(W) > \frac{\overline{\sigma}(G) - \underline{\sigma}(G) - \underline{\sigma}(P_o)}{\overline{\sigma}(P_o)} \qquad (15)$$

The above defines the weighting parameter such that its upper limit is less than 1 and its lower limit is the right hand side of (15). Thus it sets a lower limit to the weighting parameter.

Note that $\overline{\sigma}(G) \neq \underline{\sigma}(G)$ unless G is scalar multiplied by the identity, which is not true in our design.

• Past error feedforward in state feedback design.

The starting point again is the stability condition given in (8), which requires that the induced norm to be less than 1 to guarantee system stability. As pointed earlier, a more conservative restriction is to consider the singular values; instead, the stability condition then is written as

$$\overline{\sigma}(H(z)) < 1$$

Verifying this condition in the maximum case assures reference tracking and periodic disturbances accommodation. Following the same assumptions, define $[P = P_o + P_o\Delta W]$, where P_o, Δ, W are the nominal plant, plant uncertainty, and the uncertainty weight, respectively. Each of the defined variables is assumed to be stable, causal, and linear time invariant for simplicity. In combination with the definition of $H(z)$ given in (9), we can write the following derivation:

$$\overline{\sigma}(G + P_o + P_o\Delta W) < \overline{\sigma}(G)$$

with singular value properties, it can be written as

$$\underline{\sigma}(G+P_o)+\overline{\sigma}(P_o\Delta W)<\overline{\sigma}(G)$$

where this option is the only choice that can guarantee keeping the left hand side to its maximum and not to exceed the right hand side. Now, taking the uncertainty part in one side and the other parts to the right side yields

$$\overline{\sigma}(P_o\Delta W)<\overline{\sigma}(G)-\underline{\sigma}(G)-\underline{\sigma}(P_o) \quad (16)$$

maximizing the left hand side will give the possible variation in system dynamics, right hand side, and sets the upper bound for the system not to have unwanted performance through the operation. This can be found if the right hand side was of the form $\overline{\sigma}(G)-\underline{\sigma}(G)-\overline{\sigma}(P)$. Extending the previous property sets the uncertainty weight that gives an upper bound and permit the design to perform better in the case of model mismatch

$$\sigma(\Delta)<\frac{\overline{\sigma}(G)-\underline{\sigma}(G)-\underline{\sigma}(P_o)}{\sigma(W)\overline{\sigma}(P_o)} \quad (17)$$

maximizing the left hand side of equation (17) such that the right hand side is kept minimum can be seen as solving the following:

$$\overline{\sigma}(\Delta)<\frac{min}{max}=\frac{\overline{\sigma}(G)-\underline{\sigma}(G)-\underline{\sigma}(P_o)}{\sigma(W)\overline{\sigma}(P_o)} \quad (18)$$

To suppress the uncertainty effect to a higher level, further investigation toward the weight (W) is taken into account and can be expressed, with the fact that $\overline{\sigma}(\Delta)<1$, in the following:

$$1<\frac{\overline{\sigma}(G)-\underline{\sigma}(G)-\underline{\sigma}(P_o)}{\sigma(W)\overline{\sigma}(P_o)}$$

$$\overline{\sigma}(W)<\frac{\overline{\sigma}(G)-\underline{\sigma}(G)-\underline{\sigma}(P_o)}{\overline{\sigma}(P_o)}<1 \quad (19)$$

Condition (19) now sets the upper limit to the weighting factor such that uncertainty is extended and the performance is enhanced.

Condition (15) is the same as that in (19) to a limit where in (15) it sets the lower limit to the weight selection while (19) sets the upper limit to the uncertainty weight, which has a wider and better range than that of (15).

The next section discusses load disturbances case and sets disturbance conditions to complete the design given earlier in Freeman et al. (2013), which is the second part of the novelty introduced within this work.

Load Disturbances Conditions in RC State Feedback Design

Define for now the system described in (1) in single-input single-output case in terms of load and measurement disturbances, $d_k(t)$ and $n_k(t)$, respectively, as

$$\Psi_k(t+\delta)=P(q)u_k(t)+d_k(t), \quad (20)$$

$$y_k(t)=\Psi_k(t)+n_k(t),\ t=0,1,\ldots,n-1. \quad (21)$$

where the load disturbance $d_k(t)$ can be described as a non-periodic disturbance acting on the system input and tends not to make the error goes to zero as long as it affects the operation. The subscript k represents the iteration index and q is the forward shift operator. The time delay operator, δ, is inserted in the output equation. Without loss of generality the process matrix $P(q)$ is assumed not to have a delay. At the start of the operation, the state of the system starts from a home position and continues to move on within the required path as the operation continues. Thus it is assumed that the number of samples in one operation/iteration is $N+\delta$.

Now, if a control action takes place at time $t=q$, the system will respond when $t=q+\delta$. Thus, it is trivial to control the output $\Psi_k(t)$ in repetitive control lifted form at times $\delta \le t \le \delta + N - 1$, using the input $u_k(t)$ at times $0 \le t \le N-1$. The reference $r(t+N)=r(t)$ is then defined over the period $\delta \le t \le \delta + N - 1$, and the control problem would be as to let $\Psi_k(t)$ to follow $r(t)$ as close as possible, where $r(t)$ is periodic.

Within the same context, the description given in (20) and the control input signal $u_k(t)$ defined earlier, the output $\Psi_k(t)$ for trial k can be defined as

$$\Psi_k = [\Psi_k(\delta), \Psi_k(\delta+1), \cdots, \Psi_k(\delta+N-1)]^T \tag{22}$$

The load disturbance vector d_k is analogous to $u_k(t)$ and the measurement disturbance n_k, the measured output vector y_k and the reference vector r are defined analogous to (22). Required assumptions are made about d_k and n_k where (i) they are zero mean. (ii) They are uncorrelated with each other. (iii) They are uncorrelated between iterations (Johannes et al., 2014). To examine the load disturbance limitation such that it does not effect system performance and tends to stabilize the response, let us start with the stability condition described in (9) for the state feedback design with past error feedforward case, as well as the output described in (20) to form the following path using the singular values

$$(\sigma(G+P) < \sigma(G)) \times \sigma(u_k) \tag{23}$$

which can be further reformed to the following:

$$\sigma(Gu_k + Gu_{k-1} - Gu_{k-1} + Pu_k) < \sigma(Gu_k + Gu_{k-1} - Gu_{k-1})$$

$$\sigma(G\widetilde{u}_k + Gu_{k-1} + Su_k) < \sigma(G\widetilde{u}_k + Gu_{k-1})$$

$$\sigma(\Psi_k - d_k + Gu_{k-1} + \Psi_k - d_k) < \sigma(\Psi_k - d_k + Gu_{k-1}) \tag{24}$$

Maximizing the effect of the load disturbance, it is written in one side and the other parameters in the other side as

$$\overline{\sigma}(d_k) < \overline{\sigma}(\sum_{i=0}^{k}(\Psi_i) - \sum_{j=0}^{k-1}(d_j) - Gu_0) - \underline{\sigma}(Gu_{k-1}) \tag{25}$$

This condition clearly says that the maximum singular value of the load disturbance allowed acting on trial k has to be less than the maximum singular value of the difference of the sum of all previous trials output eigenvalues minus the sum of previous trial load disturbances and the initial input response as well as the minimum singular value to last trial control action. If this condition is not met, then the system will become unstable. This builds a range where the load disturbance acting on any trial k is very restrictive and has a small range of variation in terms of its maximum singular value. Equation (25) can also be modified if the second part of the right hand side was further investigated to give the form of

$$\underline{\sigma}(\sum_{h=0}^{k-1}(\Psi_h) - \sum_{v=0}^{k-1}(d_v) - Gu_0) \tag{26}$$

which makes the condition given in (25) as

$$\bar{\sigma}(d_k) < \bar{\sigma}(\sum_{i=0}^{k}(\Psi_i) - \sum_{j=0}^{k-1}(d_j) - Gu_0) - \underline{\sigma}(\sum_{h=0}^{k-1}(\Psi_h) - \sum_{v=0}^{k-1}(d_v) - Gu_0) \tag{27}$$

The above condition sets the upper limit of the load disturbance acting on trial k such that the overall performance does not lose stability. If this condition was met and the load disturbance has a smaller value, then the system will remedy the influence of the disturbance as the repetitive controller will tend to learn to produce the proper input signal such that the error tends to zero along the operation.

On the other hand, the current error feedback case tends to hold the same condition as that of the past error feedforward. It is left for the reader to follow the same steps by starting from the stability condition and follow the same direction. The conditions in past or current error feedback do make sense since the load disturbance can affect the operation at any trial k; thus it has to be of a form that contains its weight. Disturbance suppression can be made if its maximum singular value follows the condition found in (27).

SIMULATION RESULTS

A non-minimum phase plant (NMP) is simulated for the proposed conditions. The NMP had been tested in many reported works in different RC and ILC schemes experimentally (Freeman et al., 2005, Alsubaie et al., 2008, Cai et al., 2008). Such systems are very known to compromise problematic mathematical structures and controlling those systems is a challenging task. Here in this section, simulation results are presented that show the proposed conditions success in enhancing performance against uncertainty associated with system modelling mismatch.

Due to the presence of the right half plane zero in the NMP plant mathematical description, such presence makes this system hard to test in RC arena due to the instability associated with plant inversion. Thus, any sudden change in system dynamics would result in unstable response. The mathematical equation describing the NMP simulated can be found in Freeman et al. (2008) and is given as

$$P(s) = \frac{1.202(4-s)}{s(s+9)(s^2+12s+56.25)} \tag{28}$$

For a detailed description of the facility, refer to Freeman et al. (2005).

The NMP was sampled at a sampling frequency of $100Hz$ and a reference of 4 seconds is applied creating 400 samples to record in each cycle. The system is operated for 20 cycles. This system was tested in two different cases, with and without the weighting factor presence. Figure 2 shows a comparison of the mean squared error for different variations in the entries of matrix (A) for 20 cycles. The red line in the figure shows an error divergence for the case where the weighting parameter was omitted. The entries of matrix (A) were changed by 5%, and this means that the system will not learn to follow the reference with minor difference in dynamical model description. On the other hand, the other lines in the figure show an extension in system stability against uncertainty up to 10% change in matrix (A) entries and as low to almost 10% in such difficult system, where the weighting factor limit in past error case is found with equation (19) to be 0.8176 and the selected weighting parameter is chosen to be half of the limit found, 0.4088 . Thus it can be concluded that the design is more robust with the conditions found and the response is acceptable up to a mismatch of 10% change in the matrix (A) entries for the selected weighting factor compared to the case where weighting factor was omitted and applied as can be seen in Figure 2.

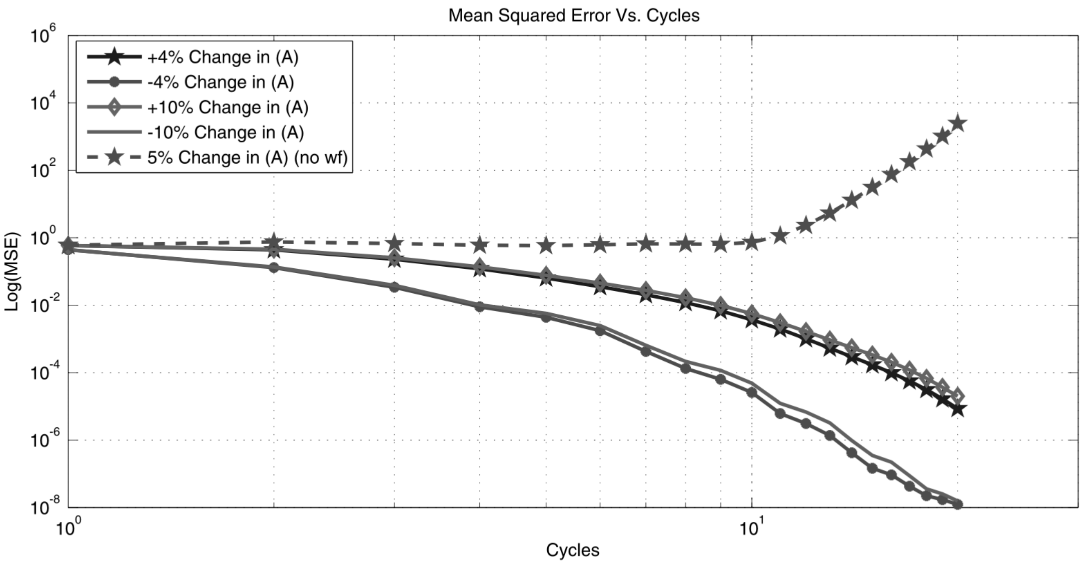

Figure 2. Mean squared error for the NMP output with/without the w.f.

Figure 3 shows the output response in different cases for 20 cycles with the presence of the weighting factor. It can be noticed that system uncertainty was extended up to 10% with stable response and decaying error along the cycle axis.

Figure 4 clarifies the norm of the input demand with the presence of the weighting factor where it forms an input region that gives the input high and low possibilities based on the model mismatch. This region is also compared to the case where no weighting factor is applied (red dots), where it shows a possible variety of no more than 4% change in system dynamics. Thus the conditions given and proposed in this reported work extend the range of learning for the reported design in the RC part to overcome system perturbation compared to the case where no weighting factor is applied.

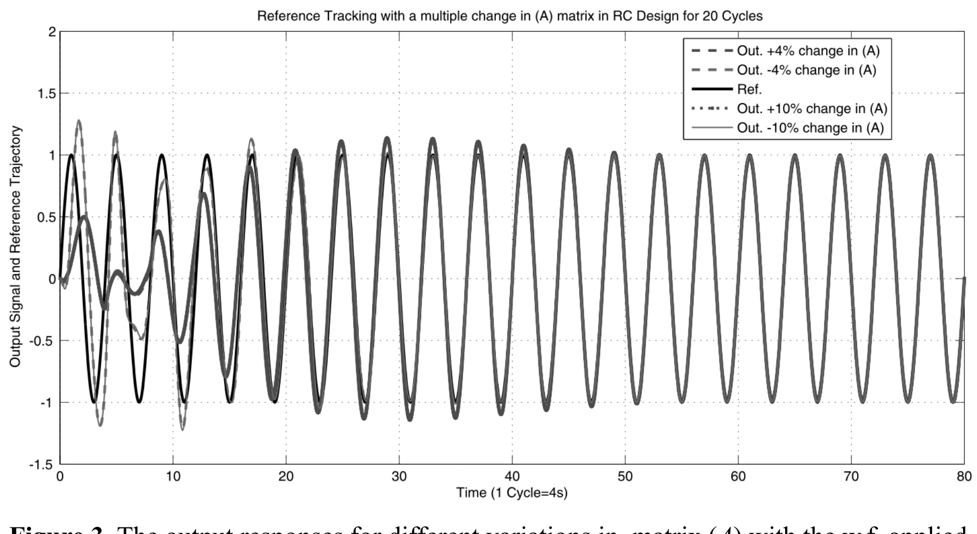

Figure 3. The output responses for different variations in matrix (A) with the w.f. applied.

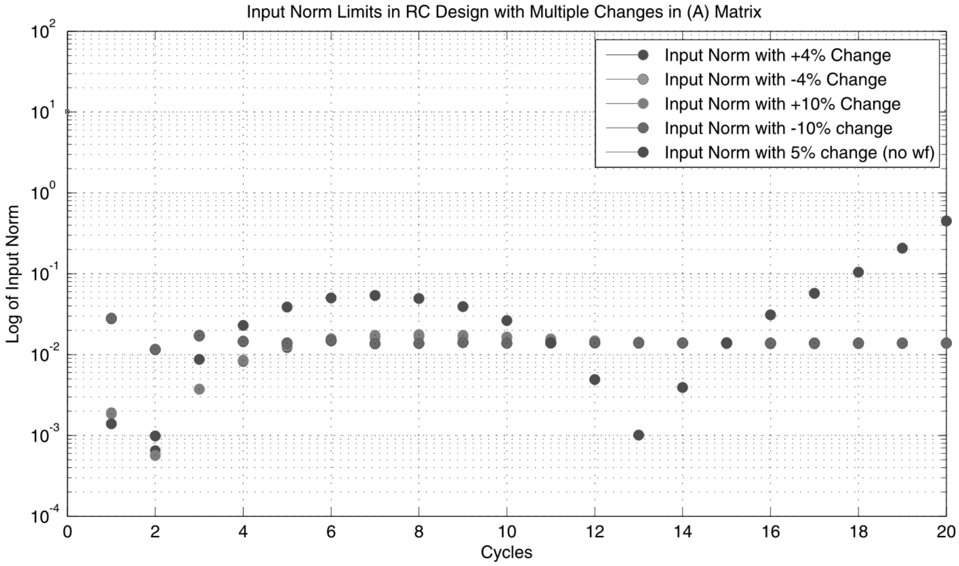

Figure 4. The input demand range for several variations in the matrix (A) with/without the weighting factor.

Conclusions and Future Work

In this paper conditions are set to extend linear system uncertainty based on the singular value principle for the RC design in state feedback. Different cases have been discussed and conditions are found that extend system robustness against system unmodelled dynamics. Load disturbance conditions also were investigated and found to set a limit where if violated will result in system instability. Reported simulation results show the weighting factor success to extending the range of uncertainty considered with the RC design via state feedback. High level of reference tracking is achieved for up to 10% change in system uncertainty in the NMP model. Future work will consider implementing those found conditions and verify their success experimentally.

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أنظمة التغذية المرتجعة لأنظمة التحكم بالتكرار: شروط عدم التيقن لتمثيل النظام والمؤثرات على الأحمال

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الخلاصة

تم بحث شروط عدم التيقن لتمثيل النظام والمؤثرات على الأحمال في أنظمة التغذية المرتجعة لأنظمة التحكم بالتكرار في الأنظمة الخطية غير المتأثرة بالزمن. الشروط الناتجة من البحث تضع إما حد أعلى أو حد أدنى للأوزان اعتماداً على الحالة التي تعمل بها الأنظمة. البحث المُقدم يدرس أثر تواجد تغذية الأخطاء السابقة والحالية للأنظمة بدلاً عن الأخطاء الحالية فقط. فكرة التصميم تعتمد على شمول نظام تأخير مثالي يؤثر في مدخلات النظام المتردد ويمثل المؤثرات المترددة. مع عزل نظام التأخير المتردد وإيجاد المحصلة النهائية للتمثيل الرياضي للدالة الانتقالية حول نظام التأخير باستخدام نظرية المحصلة الصغيرة، تم ايجاد شروط الثبات والتي تضمن ثبات النظام أمام المؤثرات المترددة. الشروط الناتجة وحسب النتائج بالرسومات تبين أثر هذه الشروط في تحجيم تأثير عدم اليقين في الناتج النهائي للنظام مقارنة مع حالة عدم الاعتداد بالشروط.