A Novel Robust-DEA Approach for Efficiency Measurement of Heterogeneous Hybrid Networks under Uncertainty

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Abstract

Over the past two decades, the use of network data envelopment analysis in real-world issues has attracted the attention of many researchers. This analysis is used when the output results of a decision unit are used as input to later units. Uncertainty in the inputs and outputs of each decision unit complicates the performance evaluation of such systems. In this paper, a new model of heterogeneous hybrid network data envelopment analysis is developed to measure the efficiency of decision units assuming the open structure of each decision unit, as well as the existence of interlayer relationships. In this model, constraints are defined in such a way that the number of units on the efficient boundary is limited. As a result, there is no need to use super-efficient models to determine overall performance. A robust approach has been used to deal with uncertainties in inputs, outputs and, interlayers. The application of the proposed model was studied to evaluate the performance of pistachio orchards in Yazd province, Iran, and the results were compared with traditional models. With the implementation of the proposed model, no unit was placed on the efficient boundary and there is no need for super-efficient models. The results were approved by agricultural experts.

Keywords: DEA; Heterogeneous network; Robust optimization; Uncertainty

1. Introduction

Performance evaluation identifies efficient and inefficient units and enables decision-making towards improved efficiency. Efficiency is the ratio of output or return of a system to the input given to it. To determine the efficiency, various methods and models have been presented, each of which has its strengths and weaknesses. Being based on mathematical planning, Data Envelopment Analysis (DEA) model is one of the most important models and powerful techniques used in this field to evaluate the performance and optimization of homogeneous units (Sabouhi Sabouni and Mardani, 2017). Homogeneous units are those that use the same inputs to produce similar outputs, including universities, hospitals, banks, etc. (Cook et al., 2010). In traditional DEA models, efficiency is calculated without considering the relationships among the internal components of the DMUs, while, in reality, there may be many internal processes that challenge the calculation of performance (Yang et al., 2008). In order to solve this problem, Färe and Grosskopf (2000) introduced network data envelopment analysis (NDEA) and described its important role in enabling a more accurate performance analysis of DMUs. Network DMUs are structurally classified into three basic groups, the first of which is parallel DMUs. The second group is serial DMUs and the third one is hybrid DMUs, which include both serial and parallel structures (Barat et al., 2019). There are many cases of DMUs with parallel and serial, internal structures, and heterogeneous processes. Heterogeneous hybrid models are classified into closed and open structure models. In a heterogeneous hybrid NDEA model with a closed structure, intermediate outputs are transferred from one stage to the next without any alterations. Moreover, internal processes do not produce final outputs. In heterogeneous hybrid NDEA models with open structure, part of the intermediate outputs is taken out of the system as final outputs while part of it enters the next stage as inputs in the same layer or the other layer (Maghbouli et al., 2014). Another challenge in applying DEA models is data uncertainty. Indeed, many real-life applications face uncertain data, which may influence the results of performance evaluation. In the worst case, the optimal solutions offered by the models may even become infeasible, and the ranking of DMUs can be invalid, especially when the efficiency scores of DMUs are close to each other. Thus, different uncertain DEA (UDEA) models have been proposed in recent years (Peykani et al., 2020). Peykani et al. (2020) investigated some of the problems in UDEA models. They concluded that DEA robust optimization (RDEA), as a new tool, could resolve existing problems in DEA uncertainty optimization approaches (Stochastic DEA ،Fuzzy DEA ،Bootstrap DEA ،Imprecise DEA). RDEA is the last UDEA approach that is applied for the performance assessment of DMUs when uncertain data exist (Peykani et al., 2020). Note that the robust optimization (RO) approach is one of the most applicable and popular approaches to deal with uncertainty in real-life applications. Also, the approach does not require historical data or probability distribution function (Bertsimas et al., 2011). In this paper, a new model of heterogeneous hybrid NDEA with open structure and interlayer connection is presented. And a UDEA, RO approach has been used to deal with simultaneous uncertainty in the inputs, outputs, and intermediate inputs/outputs.

2. Literature Review

These two stages located in a closed network structure were evaluated independently. Li et al. (2016) examined the heterogeneous inputs that usually occur in manufacturing plants. An example is when the outputs are packaged using a complex set of machines, robots, and workers, where the input configuration in one DMU can vary from that of the other one. They developed a new NDEA model and used it to evaluate the agricultural efficiency of Chinese provinces. Barat et al. (2019) presented a model of closed hybrid NDEA with heterogeneous inputs and outputs. This method enables the calculation of total efficiency and components in DMUs. Lo Storto (2020) introduced the two-stage heterogeneous NDEA model to measure the efficiency of 103 large

Italian municipalities. The proposed model has a closed structure. Inputs and outputs in the proposed model are considered definitive. Shi et al. (2021) Proposed a heterogeneous NDEA with undesirable outputs and a closed structure. The DMUs in their model consist of two series stages, the second stage of which consists of two parallel processes. They used their proposed approach to evaluate China's commercial banks during 2012-2016. Ebrahimnejad et al. (2014) presented a model in which a DMU has three sub-networks. In the first part, two sub-networks are parallel and their outputs are used as inputs in the third sub-network. This model is used to evaluate the efficiency yielded by Tejarat Bank branches. Having considered adverse output, Zhou et al. (2018) presented a heterogeneous hybrid NDEA model. The structure of this network is two serial stages that the first of which has two parallel processes. They assumed there is no connection between two parallel processes. This model was used to measure the efficiency of the water industry in China. Stefaniec et al. (2020) presented a heterogeneous parallel NDEA model with common inputs without interlayer connections and used it to calculate the efficiency of China's domestic transportation industry. Zhang et al. (2021) proposed a heterogeneous parallel NDEA model, assuming openness and the presence of undesirable outputs in the processes that form the DMUs. In their model, the layers consist of only one process and the inputs and outputs are definite. The robust optimization approach, which has recently received attention, has solved the problems that other approaches suffer when facing uncertainty. Therefore, researchers in the DEA studies have widely used this approach to deal with uncertainty. Sadjadi and Omrani (2008) proposed robust DEA models to evaluate the efficiency of electricity distribution companies. The results showed that the efficiency of the units appears more realistically in the model as uncertainty increases. Studies in the field of robust NDEA models can be summarized in the following four cases: Khademi Zare et al. (2016) presented a robust model of two-stage heterogeneous NDEA with a closed structure and used it to evaluate the efficiency of regional power companies in Iran. They considered the input-oriented and considered the uncertainty only in the final outputs and intermediate inputs/ outputs. Fathollah Bayati and Sadjadi (2017) presented a robust model of heterogeneous series NDEA with an open structure to evaluate the efficiency of Iranian regional power companies. Using Stackelberg's (leader-follower) approaches, Esfandiari et al. (2017) introduced two robust two-stage NDEA models with a closed structure. In the models presented by them, inputs, outputs, and intermediate products are assumed to be uncertain at the same time. The proposed models have been used to evaluate the performance of 20 banking branches in East Virginia. Shakouri et al. (2019) as in the previous article, presented robust two-stage NDEA models with closed structure using Stackelberg's approaches.

In this paper, a new model of heterogeneous hybrid NDEA with open structure is presented to evaluate the efficiency of pistachio orchards in Yazd province in the presence of uncertain data and was robust using Bertsimas and Sim (2003).

3. Development of Heterogeneous Hybrid NDEA Model with Open Structure at each Stage and Interlayer Connections

Open hybrid network structures are frequently encountered in real-world research and production systems. The outputs of each stage can be either the inputs of the next stage or the final product. Each DMU has L layers, with P number of serial stages. In each layer, the output of each stage denotes the outputs that leave the process at other stage and do not enter the next stage.

3.1. Problem Modeling

3.1.1. Definition of Variables and Parameters

The following variables and parameters are defined for modeling heterogeneous hybrid NDEA structure with independent inputs and final outputs at each stage and interlayer connection:

$x_{i_{lp}j}$: i th independent input of stage p for DMU$_j$ in layer l ($i_{lp} \in I_{lp}$)

$y_{r_{lp}j}$: r th exiting output of stage p for DMU$_j$ in layer l ($r_{lp} \in R_{lp}$)

$\bar{y}_{t_{lp}j}$: t th output of stage p for DMU$_j$ in layer l that as an input enters stage p+1 ($t_{lp} \in T_{lp}$)

$z^{al}_{k_{lp}j}$: k th input of stage p+1 that enters layer l from step p in layer a ($k_{lp} \in k_{lp}$)

$z^{lb}_{t_{lp}j}$: t th output of stage p in layer l, that enters stage p+1 in layer b ($t_{lp} \in T_{lp}$)

$v_{i_{lp}}$: the weights of $x_{i_{lp}j}$

$u_{r_{lp}}$: the weights of $y_{r_{lp}j}$

$u_{t_{lp}}$: the weights of $\bar{y}_{t_{lp}j}$ and $z^{lb}_{t_{lp}j}$

$v_{k_{lp}}$: the weights of $z^{al}_{k_{lp}j}$

3.1.2. Calculations for the Total Efficiency

Suppose the aim is to obtain the efficiency score of n DMUs, each of which uses the value of $x_j = (x_{1j}, \dots, x_{mj}) \in R^m_+$ as input to produce $y_j = (y_{1j}, \dots, y_{sj}) \in R^s_+$ as output. To obtain the efficiency score of DMU$_0$, the following formula can be used ettehadi et al. (2021).

$$\text{Maximum} \qquad \theta_{DMU_o} = \sum_{l=1}^{L}\left(\sum_{p=1}^{m-1}\left(\sum_{t_{lp}\in T_{lp}} u_{t_{lp}} \sum_{b=1}^{L} z^{lb}_{t_{lp}o} + \sum_{r_{lp}\in R_{lp}} u_{r_{lp}} y_{r_{lp}o}\right) + \sum_{r_{lm}\in R_{lm}} u_{r_{lm}} y_{r_{lm}o}\right)$$

Subject to:

$$\sum_{l=1}^{L}\left(\sum_{i_{l1}\in I_{l1}} v_{i_{l1}} x_{i_{l1}o} + \sum_{p=2}^{m}\left(\sum_{k_{lp}\in K_{lp}} v_{k_{lp}} \sum_{a=1}^{L} z^{al}_{k_{lp}o} + \sum_{i_{lp}\in I_{lp}} v_{i_{lp}} x_{i_{lp}o}\right)\right) = 1$$

$$\sum_{t_{l1}\in T_{l1}} u_{t_{l1}} \sum_{b=1}^{L} z^{lb}_{t_{l1}j} + \sum_{r_{l1}\in R_{l1}} u_{r_{l1}} y_{r_{l1}j} - \sum_{i_{l1}\in I_{l1}} v_{i_{l1}} x_{i_{l1}j} \le 0,$$

$$j = 1,2,\dots,n, l = 1,2,\dots,L$$

$$\sum_{t_{lp}\in T_{lp}} u_{t_{lp}} \sum_{b=1}^{L} z_{t_{lp}j}^{lb} + \sum_{r_{lp}\in R_{lp}} u_{r_{lp}} y_{r_{lp}j} - \sum_{k_{lp}\in K_{lp}} v_{k_{lp}} \sum_{a=1}^{L} z_{k_{lp}j}^{al} - \sum_{i_{lp}\in I_{lp}} v_{i_{lp}} x_{i_{lp}j} \le 0,$$

$$j = 1,2,\dots,n,\quad p = 2,3,\dots,m-1,\quad l = 1,2,\dots,L$$

$$\sum_{r_{lm}\in R_{lm}} u_{r_{lm}} y_{r_{lm}j} - \sum_{k_{lm}\in K_{lm}} v_{k_{lm}} \sum_{a=1}^{L} z_{k_{lm}j}^{al} - \sum_{i_{lm}\in I_{lm}} v_{i_{lm}} x_{i_{lm}j} \le 0,$$

$$j = 1,2,\dots,n,\quad l = 1,2,\dots,L$$

$$t_{lp} \in T_{lp},\ k_{lp} \in K_{lp}, r_{lp} \in R_{lp}, i_{lp} \in I_{lp},\quad v_{k_{lp}}, u_{r_{lp}}, u_{t_{lp}}, v_{i_{lp}} \ge \varepsilon,$$

$$p = 1,2,\dots,m,\quad l = 1,2,\dots,L$$

4. Developing a "Robust" Model of Heterogeneous Hybrid NDEA with Open Structure

In order to make the model robust, it is necessary to turn the equality constraint of this model into an unequal constraint. Using that the model is changed as:

Maximum $\quad \theta_{DMU_o}$

Subject to:

$$\theta_{DMU_o} - \sum_{l=1}^{L}\left(\sum_{p=1}^{m-1}\left(\sum_{t_{lp}\in T_{lp}} u_{t_{lp}} \sum_{b=1}^{L} z_{t_{lp}o}^{lb} + \sum_{r_{lp}\in R_{lp}} u_{r_{lp}} y_{r_{lp}o}\right) + \sum_{r_{lm}\in R_{lm}} u_{r_{lm}} y_{r_{lm}o}\right) + \sum_{l=1}^{L}\left(\sum_{p=1}^{m-1}\left(\sum_{t_{lp}\in T_{lp}} \sum_{b=1}^{L} g_{t_{lp}o}^{lb} + \sum_{r_{lp}\in R_{lp}} g_{r_{lp}o}\right) + \sum_{r_{lm}\in R_{lm}} g_{r_{lm}o}\right) + z_0\Gamma \le 0$$

$$z_0 + g_{t_{lp}o}^{lb} \ge \hat{\alpha}_{t_{lp}o}^{lb} u_{t_{lp}},\quad z_0 + g_{r_{lp}o} \ge \hat{\alpha}_{r_{lp}o} u_{r_{lp}},\quad z_0 + g_{r_{lm}o} \ge \hat{\alpha}_{r_{lm}o} u_{r_{lm}},\quad b, l = 1,2,\dots,L,\quad p = 1,2,\dots,m-1$$

$$\sum_{l=1}^{L}\left(\sum_{i_{l1}\in I_{l1}} v_{i_{l1}} x_{i_{l1}o} + \sum_{p=2}^{m}\left(\sum_{k_{lp}\in K_{lp}} v_{k_{lp}} \sum_{a=1}^{L} z_{k_{lp}o}^{al} + \sum_{i_{lp}\in I_{lp}} v_{i_{lp}} x_{i_{lp}o}\right)\right) + \sum_{l=1}^{L}\left(\sum_{i_{l1}\in I_{l1}} h_{i_{l1}o} + \sum_{p=2}^{m}\left(\sum_{k_{lp}\in K_{lp}} \sum_{a=1}^{L} h_{k_{lp}o}^{al} + \sum_{i_{lp}\in I_{lp}} h_{i_{lp}o}\right)\right) + z_1\Gamma \le 1$$

$$z_1 + h_{i_{l1}o} \ge \hat{\alpha}_{i_{l1}o} v_{i_{l1}},\quad z_1 + h_{k_{lp}o}^{al} \ge \hat{\alpha}_{k_{lp}o}^{al} v_{k_{lp}},\quad z_1 + h_{i_{lp}o} \ge \hat{\alpha}_{i_{l1}o} v_{i_{lp}},\quad a, l = 1,2,\dots,L,\quad p = 2,3,\dots,m,$$

$$\sum_{t_{l1}\in T_{l1}} u_{t_{l1}} \sum_{b=1}^{L} z_{t_{l1}j}^{lb} + \sum_{r_{l1}\in R_{l1}} u_{r_{l1}} y_{r_{l1}j} - \sum_{i_{l1}\in I_{l1}} v_{i_{l1}} x_{i_{l1}j} + \sum_{t_{l1}\in T_{l1}} \sum_{b=1}^{L} w_{t_{l1}j}^{lb} + \sum_{r_{l1}\in R_{l1}} w_{r_{l1}j} + \sum_{i_{l1}\in I_{l1}} w_{i_{l1}j} + z_{lj}^{1}\Gamma \le 0,$$

$z_{lj}^{1}+w_{t_{l1}j}^{lb}\ge \hat{\alpha}_{t_{l1}j}^{lb}u_{t_{l1}},\quad z_{lj}^{1}+w_{r_{l1}j}\ge \hat{\alpha}_{r_{l1}j}\,u_{r_{l1}},\quad z_{lj}^{1}+w_{i_{l1}j}\ge \hat{\alpha}_{i_{l1}j}\,v_{i_{l1}},\quad l,b=1,2,\dots,L,\quad j=1,2,\dots,n,$

$$\sum_{t_{lp}\in T_{lp}} u_{t_{lp}}\sum_{b=1}^{L} z_{t_{lp}j}^{lb}+\sum_{r_{lp}\in R_{lp}} u_{r_{lp}}y_{r_{lp}j}-\sum_{k_{lp}\in K_{lp}} v_{k_{lp}}\sum_{a=1}^{L} z_{k_{lp}j}^{al}-\sum_{i_{lp}\in I_{lp}} v_{i_{lp}}x_{i_{lp}j}+\sum_{t_{lp}\in T_{lp}}\sum_{b=1}^{L} w_{t_{lp}j}^{lb}+\sum_{r_{lp}\in R_{lp}} w_{r_{lp}j}$$
$$+\sum_{k_{lp}\in K_{lp}}\sum_{a=1}^{L} w_{k_{lp}j}^{al}+\sum_{i_{lp}\in I_{lp}} w_{i_{lp}j}+z_{lj}^{p}\Gamma\le 0,$$

$z_{lj}^{p}+w_{t_{lp}j}^{lb}\ge \hat{\alpha}_{t_{lp}j}^{lb}u_{t_{lp}},\quad z_{lj}^{p}+w_{r_{lp}j}\ge \hat{\alpha}_{r_{lp}j}\,u_{r_{lp}},\quad z_{lj}^{p}+w_{k_{lp}j}^{al}\ge \hat{\alpha}_{k_{lp}j}^{al}v_{k_{lp}},\quad z_{lj}^{p}+w_{i_{lp}j}\ge \hat{\alpha}_{i_{lp}j}\,v_{i_{lp}},\quad j$
$=1,2,\dots,n,\quad p=2,3,\dots,m-1,\quad l,b=1,2,\dots,L,$

$$\sum_{r_{lm}\in R_{lm}} u_{r_{lm}}y_{r_{lm}j}-\sum_{k_{lm}\in K_{lm}} v_{k_{lm}}\sum_{a=1}^{L} z_{k_{lm}j}^{al}-\sum_{i_{lm}\in I_{lm}} v_{i_{lm}}x_{i_{lm}j}$$
$$+\sum_{r_{lm}\in R_{lm}} w_{r_{lm}j}+\sum_{k_{lm}\in K_{lm}}\sum_{a=1}^{L} w_{k_{lm}j}^{al}+\sum_{i_{lm}\in I_{lm}} w_{i_{lm}j}+z_{lj}^{m}\Gamma\le 0,$$

$z_{lj}^{m}+w_{r_{lm}j}\ge \hat{\alpha}_{r_{lm}j}u_{r_{lm}},\quad z_{lj}^{m}+w_{k_{lm}j}^{al}\ge \hat{\alpha}_{k_{lm}j}^{al}v_{k_{lm}},\quad z_{lj}^{m}+w_{i_{lm}j}\ge \hat{\alpha}_{i_{lm}j}u_{i_{lm}},\qquad j=1,2,\dots,n,\ \ a,l=1,2,\dots,L$
$t_{lp}\in T_{lp},\ k_{lp}\in K_{lp},\ r_{lp}\in R_{lp},\ i_{lp}\in I_{lp},\quad v_{k_{lp}},\ u_{r_{lp}},\ u_{t_{lp}},\ v_{i_{lp}}\ge \varepsilon,$

In this equation, α_{ij} is a nominal value and ξ_{ij} represents independent random variables that are uniformly distributed between 1 and -1.

5. Numerical Example

The proposed models are used to evaluate the efficiency of pistachio orchards in Yazd province in Iran. More than 44,000 hectares of the province's lands have been allocated to pistachio orchards, of which about 12,600 hectares are infertile and 31,400 hectares are fertile. On average, about 45,000 tons of pistachios are harvested annually from the fertile orchards of the province (Mohammadi Mohammadabadi et al., 2020). Most pistachio products of the province belong to jumbo pistachio (Kalleghoochi) (type 1) and long pistachio (Ahmad Aghaei) (type 2). The production process of each of these pistachios consists of two stages, which are planting and processing pistachios. Each stage has independent inputs, outputs, and intermediate products. In the planting stage, the area of land, the amount of water used and the number of pistachios seedlings are independent inputs of this stage. Part of the harvested pistachio is sold raw (as a final product), and the leftovers enter the processing stage (as an intermediate product). Finally, the sales revenue and tonnage of pistachios produced, the final output, and the number of workers required are independent inputs for the processing stage. The data in the present study were obtained from Ettehadi, et.al. (2021).

5.1. Results and discussion

Table 1 presents the results obtained from the implementation of the heterogeneous hybrid NDEA model for 10 DMUs. As can be seen units 2, 3, 4, 8, and 10 out of 10 DMUs, are located in the efficiency borderline using the CCR model. Using Model 1, only Unit 8 is on the efficient frontier. In columns 1 to 6 of Table 1, the efficiency, rank, and weight of each layer in the DMU are shown by implementing Model 1. As can be seen, DMU 8, with an efficiency of one, has constituent layers with the efficiency of one as well. A DMU will be efficient when all its constituent sub-networks are also fully efficient.

Table 1. Results of Rank and Total Efficiency and Layers of DMUs

DMU	Model 1 Results - Considering the relationships within DMUs								Model 1 Results	
	The First Layer			The Second Layer			Total DMU		Total DMU	
	Efficiency	Rank	Weight	Efficiency	Rank	Weight	Efficiency	Rank	Efficiency	Rank
1	0.908	8	0.307	0.556	10	0.692	0.829	10	0.719	10
2	1	1	0.903	0.858	8	0.104	0.986	2	1	1
3	0.989	4	0.464	0.983	2	0.535	0.986	2	1	1
4	0.947	7	0.262	0.977	3	0.737	0.969	6	1	1
5	0.828	10	0.097	0.961	5	0.902	0.948	7	0.773	9
6	0.884	9	0.101	0.935	6	0.898	0.931	9	0.856	8
7	0.971	6	0.763	0.827	9	0.236	0.937	8	0.926	7
8	1	1	0.541	1	1	0.458	1	1	1	1
9	0.976	5	0.461	0.969	4	0.539	0.973	4	0.941	6
10	0.997	3	0.761	0.885	7	0.239	0.970	5	1	1
Average	**0.950**		**0.466**	**0.895**		**0.534**	**0.953**		**0.921**	

Table 2 shows the rank and efficiency (columns 2 to 9) for all stages of DMUs using model 1. The optimal values of decision variables are determined using this model while the optimal values of efficiency for each stage are determined by placing them in the objective function of

Models 2 and 4. For this example, Model 1 includes 18 variables and 60 constraints that are coded and solved by LINGO software. The highest levels of efficiency go to DMUs 2, 3, 7, 8, and 9 while decision unit 6, with an efficiency of 0.785, has achieved the lowest efficiency in the type 1 pistachio planting stage. Similarly, the status of other stages in terms of efficiency and rank can be reviewed and analyzed in columns 4 to 9 . The important point in this section for the relevant authorities to note is that the amount of efficiency extracted for the processing stage in the second layer, compared to other stages, for all DMUs is significantly reduced. This decrease is also when the average efficiency of this stage (0.794) is compared with the average efficiency of other stages (0.946, 0.913, and 0.953). Here, it is deemed necessary for the officials at the province's Ministry of Agriculture Jihad to carefully and radically investigate the reason for the decrease in the efficiency of this stage, and to take necessary decisions towards its elimination. This helps reduce waste by improving the efficiency of all DMUs.

Table 2. Results of Efficiency and Ranking of Processes that Make up DMUs

DMU	Planting Stage, Type 1		Processing Stage in the First Layer		Planting Stage, Type 2		Processing Stage in the Second Layer	
	efficiency	Rank	efficiency	Rank	Efficiency	Rank	Efficiency	Rank
1	0.876	9	0.911	8	0.668	10	0.511	10
2	1	1	1	1	0.989	6	0.817	5
3	1	1	0.953	6	1	1	0.931	4
4	0.935	7	1	1	1	1	0.788	6
5	0.95	6	0.732	9	1	1	0.595	8
6	0.785	10	1	1	0.932	9	1	1
7	1	1	0.602	10	1	1	0.581	9
8	1	1	1	1	1	1	1	1
9	1	1	0.931	7	0.974	7	0.963	3
10	0.913	8	1	1	0.966	8	0.749	7
Average	**0.946**		**0.913**		**0.953**		**0.794**	

5.2. Results of the proposed model

Model 2 is used to investigate the effects of uncertainty on the efficiency and rank of DMUs. To implement this robust model, which is developed based on Bertismas and Sim’s approach; there are 18 uncertain variables to consider (including inputs, outputs, and intermediate products).

Therefore, the variable Γ was assumed to be equal to 2 for all constraints, and the error percentage $\hat{\alpha}$, for uncertain variables, was assumed to be 0.01 and 0.10, respectively. A value of Γ = 18 means that 100% of uncertain parameters can get the worst-case value. Table 3 shows the results produced by implementing the robust model 2, with an error percentage of 0.01 ($\hat{\alpha}$ = 0.01) for 10 DMUs .

Table 3. Robust Model 2 Results$\hat{\alpha} = 0.01$

DMU	Processes that Makeup DMUs								First Layer		Second Layer		Total DMU	
	Planting Pistachio Type 1		Processing in the First Layer		Planting Pistachio Type 2		Processing in the Second Layer							
	Efficiency	Rank	efficiency	Rank	Efficiency	Rank	Efficiency	Rank	efficiency	Rank	Efficiency	Rank	Efficiency	Rank
1	0.854	7	0.813	5	0.667	10	0.491	10	0.817	8	0.531	10	0.775	10
2	0.999	1	0.804	6	0.956	7	0.81	5	0.985	2	0.818	8	0.961	4
3	0.999	1	0.944	2	0.999	1	0.925	4	0.987	1	0.982	2	0.985	2
4	0.825	10	0.934	3	0.995	5	0.779	6	0.845	6	0.804	9	0.951	6
5	0.845	8	0.697	7	0.996	4	0.586	8	0.822	7	0.956	3	0.937	7
6	0.882	6	0.999	1	0.891	9	0.983	2	0.865	5	0.907	5	0.905	9
7	0.999	1	0.591	10	0.999	1	0.562	9	0.951	3	0.82	7	0.912	8
8	0.960	4	0.663	9	0.999	1	0.999	1	0.802	9	0.999	1	0.985	1
9	0.950	5	0.68	8	0.988	6	0.959	3	0.747	10	0.939	4	0.955	5
10	0.832	9	0.918	4	0.939	8	0.721	7	0.878	4	0.855	6	0.962	3
Average	**0.915**		**0.804**		**0.943**		**0.782**		**0.870**		**0.861**		**0.933**	

Table 4 shows the results produced by implementing the robust model 2, with an error percentage of 0.1 ($\hat{\alpha}$ = 0.1) for 10 DMUs.

Table 4. Robust Model 2 Result $\hat{\alpha} = 0.1$

DMU	Processes that Makeup DMUs								First Layer		Second Layer		Total DMU	
	Planting Pistachio Type 1		Processing in the First Layer		Planting Pistachio Type 2		Processing in the Second Layer							
	Efficiency	Rank	efficiency	Rank	Efficiency	Rank	Efficiency	Rank	efficiency	Rank	efficiency	Rank	Efficiency	Rank
1	0.826	7	0.782	5	0.64	10	0.453	10	0.791	7	0.501	10	0.751	10
2	0.989	3	0.708	6	0.923	6	0.804	5	0.977	2	0.8	8	0.953	4
3	0.991	2	0.943	2	0.994	1	0.901	4	0.98	1	0.98	2	0.979	2
4	0.767	9	0.891	3	0.971	4	0.751	6	0.834	5	0.781	9	0.949	1
5	0.733	10	0.651	7	0.981	3	0.535	9	0.772	8	0.942	3	0.922	6
6	0.879	5	0.989	1	0.837	9	0.964	2	0.86	4	0.872	5	0.882	9
7	0.997	1	0.551	8	0.969	5	0.547	8	0.935	3	0.802	7	0.901	7
8	0.95	4	0.538	9	0.994	1	0.981	1	0.638	10	0.997	1	0.969	3
9	0.859	6	0.536	10	0.896	8	0.939	3	0.726	9	0.916	4	0.888	8
10	0.811	8	0.826	4	0.922	7	0.706	7	0.818	6	0.81	6	0.951	5
Average	**0.880**		**0.742**		**0.913**		**0.758**		**0.833**		**0.840**		**0.915**	

5.3. Comparison of Traditional and Robust Model Results

The results obtained from implementing robust models show that an increase in the deviation of uncertain data, from 0.01 to 0.1, causes a decrease in the average total efficiency from 0.933 to 0.915 the average efficiency of the layers and the processes that constitute the DMUs. This decrease can also be seen in the efficiency score of each DMU and the processes constituting them. Using different deviations, the results show that, with an increase in the deviation from 0.01 to 0.1, the average efficiency of the first layer decreases from 0.870 to 0.833 (Column 10, Tables 3 and 4). Similar results are observed for the second layer and the processes constituting DMUs.

5.4. Validation of the proposed models

Pearson correlation test (ρ) was used to confirm the results obtained from the proposed models. This test measures the linear correlation between two random variables. The value of this coefficient varies between -1 to 1, where 1 means complete positive correlation, 0 means no correlation, and -1 means complete negative correlation. In this study, the Pearson test was used to compare the proposed robust models with the traditional heterogeneous hybrid NDEA model. Pearson test for heterogeneous hybrid NDEA model and its robust models with deviations of 0.01 and 0.1 produces values of 0.992 and 0.949, respectively. The result indicates a direct relationship between the heterogeneous hybrid NDEA model and the proposed robust models. Therefore, hypothesis H0 is rejected at the level of 1%.

6. Conclusion and Future Research

Evaluating the performance of DMUs with a heterogeneous hybrid network structure is a complex and challenging topic, which has attracted attention from many researchers in recent years. A heterogeneous hybrid network consists of several processes and layers that are interrelated so that efficiency or inefficiency in one stage affects the efficiency or inefficiency of the total network. In a real-world situation, uncertainty in problem data is an unavoidable phenomenon.

This leads to a more complex problem in performance appraisal. This study is a framework for evaluating the efficiency of DMUs with a heterogeneous hybrid network structure with independent inputs and outputs at each stage and interlayer connection, under uncertainty in all data, based on Bertismas and Sim's approach. Proposed robust models control uncertainty in input and output data and intermediate products without the need to identify data distribution. The proposed models have been used to measure the efficiency of pistachio orchards in Yazd province. The results show (Tables 3 and 4) that an increase in the amount of deviation in uncertain data from 0.01 to 0.1 leads to a decrease in the average total efficiency from 0.933 to 0.915 (column 14 of Tables 3 and 4), the average efficiency of the layers (columns 10 and 12 of Tables 3 and 4) and the processes that make up the DMUs. This decrease can also be seen in the efficiency score of each of the DMUs, layers, and their constituent processes (Tables 3 and 4). It is more reliable to consider data uncertainty and make use of a robust model to evaluate the performance and ranking strategies of DMUs. The method presented in this paper is assumed to be constant returns to scale. Future research can continue to be carried out by assuming variable returns to scale. Undesirable outputs from the model presented in this paper can also be considered to develop a new model.

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