Journal of Engg. Research Vol. 9 No. (2) June 2021 pp. 149-160 https://doi.org/10.36909/jer.v9i2.8513

Optimal location of UPFC based on sensitivity of CDF with power system's stochasticity

Ningyu Zhang*, Jingbo Zhao and Qian Zhou

State Grid Jiangsu Electric Power CO. LTD Research Institute, Nanjing 211103, China
**Corresponding Author: zhangny190@163.com*

Submitted: 10/08/2019
Revised: 05/12/2019
Accepted: 12/12/2019

ABSTRACT

In this paper, a stochastic sensitivity algorithm is introduced to optimize the location of unified power flow controller (UPFC) in large scale power grid. The stochastic sensitivities are defined as the total operation cost of power system to control parameters of UPFC. Firstly, probability optimal power flow (POPF) model with power system's randomness is established. Then point estimate method (PEM) is utilized to solve the above problem, so that the stochastic sensitivities of UPFC in all possible transmission lines could be obtained. Finally, by sorting the influence degree of UPFC at different locations to cumulative distribution function (CDF) of operation cost, the optimal location for UPFC could be selected correspondingly. To this end, IEEE-5 and IEEE-14 systems are employed to verify our proposed approach. The results show that installing UPFC by the method in this paper could significantly reduce the probability distribution of operation cost in higher region.

Keywords: Unified Power Flow Controller; Stochastic Sensitivity; Probability Optimal Power Flow; Location Optimization.

INTRODUCTION

Unified power flow controller (UPFC) is one of the most powerful devices in the flexible AC transmission systems (FACTS) field (Rajbongshi et al., 2017). It has the ability to not only control voltage magnitude and phase angle, but also independently export or absorb reactive power to power grid. Since the first UPFC projection was built at the AEP Inez substation in 1998, several power electrical companies have put four other UPFC projections into the operation in the world. These projections have effectively solved the problem of insufficient power supply and insufficient voltage support capability. However, the cost of UPFC is still higher than that of other transmission equipment, so that it is impossible to install UPFC in each candidate site. Therefore, it is urgent to study the optimization method of UPFC installation location.

In the last few decades, researchers have developed a lot of algorithms for placing UPFC in the most suitable site, which could be divided into sensitivity analysis method (SAM) and artificial intelligence algorithm (AIA). SAM, firstly, makes use of power flow (PF) or optimal power flow (OPF) to acquire the most serious over-limit transmissions. Then, by calculating and comparing the sensitivities of UPFC at different stations to the highest load lines, reasonable installation location for UPFC could be decided finally. For example, Singh et al. (2005) and Song et al. (2004) proposed a method based on the sensitivity of branch load to determine the suitable locations of UPFC. In the literature Fang et al. (1999), location and capacity of UPFC are optimized by establishing multiobject model with the cost of UPFC and marginal saving on network losses. To improve SAM's efficiency, an ideal transformer model for UPFC is suggested in the literature An et al. (2007), and then the authors used a kind of the first-order sensitivity method to identify the most promising UPFC location by running only one-time OPF. In Verma et al. (2001), the sensitivity with respect to control parameters of UPFC is researched for locating UPFC. With consideration of electricity market, Tiwari et al. (2012) analyzed all possible locations and compensation degrees of UPFC by OPF. Also, the locational marginal pricing (LMP) at all buses has been calculated with or without UPFC. Furthermore, Taher et al. (2009) researched the impact of UPFC's critical parameters to operation objective of electricity market. The probability density function (PDF), describing the influence of randomness, is added to explore the most suitable location for UPFC in Faried et al. (2009). However, the Monte Carlo simulation consumes long calculation minutes.

AIA belongs to the modern methods, which can deal with nonlinear constraints preferably in optimization problem. Existing AIA include genetic algorithm (GA), particle swarm optimization (PSO), and differential evolution (DE) (Bhowmik et al., 2012; Balamurugan et al., 2018; Zahid et al., 2017; Kumar et al., 2017). In Shaheen et al. (2008), Baghaee et al. (2008), and Hassan et al. (2013), GA and PSO techniques are applied to search the optimal location and parameters of UPFC. Kumar et al. (2015) used the ABC technique to find the maximum power loss bus, which is identified as the most favorable location for UPFC. Taher et al. (2012) attempted to apply immune genetic algorithm (IGA) and immune particle swarm algorithm (IPSO) in finding the optimal location of UPFC. In Shaheen et al. (2010), after determining the most critical contingency scenarios, the DE method was applied to find out the optimal location and parameter setting of UPFC. Generation cost and UPFC installation cost are treated as multiobjectives to find the optimal location and capacity of UPFC simultaneously in the literature Dutta et al. (2015). In Ippolito (2005), based on a multiobjective power flow, a methodology is proposed to identify the optimal number and location of UPFC devices by using Generalized Algebraic Modeling of System (GAMS).

In summary, existing optimal algorithms for UPFC placement and capacity are almost based on certainty parameters of power system. This could not accurately reflect the influence of fluctuating elements such as load, wind power to UPFC's allocation and capacity. Even though a few papers have considered randomness in the optimization model, long computing time is needed, so that it could not be entirely applicable to large scale power grid. In this paper, a stochastic method to determine the suitable location of UPFC, with uncertainty, has been suggested based on the cumulative distribution function's (CDF) sensitivities of the system operating cost to control the parameters of UPFC. The probability optimal flow power (POPF) model and point estimating method (PEM) are adopted to calculate these sensitivities together. Using this analysis, we can easily identify the potential UPFC locations and also filter out ineffective UPFC candidates. The proposed method is implemented on IEEE-5 and IEEE-14 test systems to find the most promising location for a UPFC.

DEFINITION OF SENSITIVITY BASED ON CDF

(1) Sensitivity based on CDF

In this section, the definition of sensitivity based on CDF is discussed in detail. Define y as a function of multivariable, shown as follows:

$$y = \ell(x, \tilde{z}) \tag{1}$$

Suppose that x represents a certain variable, and $\tilde{z}$ is a random variable with PDF $f_z(\tilde{z})$ and CDF $F_z(\tilde{z})$. The uncertainty of $\tilde{z}$ causes output variable y to be uncertain. Therefore, PDF and CDF of y are $f_y(y)$ and $F_y(y)$, respectively.

In order to measure influence degree of certain variable x to CDF $F_y(y)$, we define the CDF's sensitivities to x as shown in (2).

$$\gamma = \partial F_y(y) / \partial x = \int_{-\infty}^{y} [\partial f_y(t) / \partial x] dt \tag{2}$$

For a given value y_0, sensitivities γ could be able to evaluate how the measurement of x influences the magnitude of $F_y(y_0)$. It means that the larger the absolute value of γ is, the greater the degree x is affected.

(2) Sensitivities on CDF of the operation cost to UPFC

In the power system with UPFC, the operation cost (*OC*) not only is related to generators' output and UPFC's control variables, but also is affected by innate randomness, such as load forecast error and wind power's volatility. Consequently, *OC* would be represented as

$$OC = \hbar(X_C, X_U, \tilde{X}_S) \tag{3}$$

Vector X_C can be written as

$$X_C = [P_G Q_G V_B] \tag{4}$$

The input vector X_C contains real and reactive power of generators (P_G, Q_G), and buses voltage V_B. X_U represents all UPFC's control variables. In this paper, an ideal transformer and shunt susceptance are control variables of UPFC, which will be introduced later. $\tilde{X}_S$ indicate power system's uncertainties specially, which is caused by forecasting error, and so on.

As analyzed before, *OC* would be a stochastic variable due to $\tilde{X}_S$. Hence, suppose that the mean value of *OC* is μ_{oc} and the variance of *OC* is σ_{oc}. If we ignored the third and higher moment, CDF of *OC* could be described as follows:

$$F_{oc}(x_{oc}) = \int_{-\infty}^{x_{oc}} f_{oc}(t)dt = 1/(\sigma_{oc}\sqrt{2\pi})\int_{-\infty}^{x_{oc}} \mathrm{e}^{-(t-\mu_c)^2/(2\sigma_{oc}^2)}\,dt \tag{5}$$

In the above formulation, x_{oc} represents the *OC* variable. If x_{oc} is known, F_{oc} is decided by μ_{oc} and σ_{oc}, as shown in

$$F_{oc}(x_{oc}) = \chi(\mu_{oc}, \sigma_{oc}) \tag{6}$$

With *Eq*. (3), we know that X_C, X_U and $\tilde{X}_S$ have significant influence on *OC*, which would be substituted by μ_{oc} and σ_{oc}. To study the effect of UPFC parameters on CDF of *OC* precisely, we can proceed partial differential calculation to (3) and (6).

Accordingly, CDF's sensitivities of *OC* to X_U could be obtained as

$$\gamma_{oc} = \partial F_{oc} / \partial x_U = \frac{\partial \chi}{\partial \mu_{oc}}\frac{\partial \mu_{oc}}{\partial X_U} + \frac{\partial \chi}{\partial \sigma_{oc}}\frac{\partial \sigma_{oc}}{\partial X_U} \tag{7}$$

In *Eq*. (7), $\partial\chi/\partial\mu_{oc}$ and $\partial\chi/\partial\sigma_{oc}$ could be realized directly through (6). As a result, it is essential to establish probability optimal power flow (POPF) with UPFC and randomness elements for computing $\partial\mu_{oc}/\partial X_U$, $\partial\sigma_{oc}/\partial X_U$ accurately.

POPF model with UPFC

In this section, the relationship between *OC* and X_C, X_U, $\tilde{X}_S$, as shown in (3), will be studied by POPF model with UPFC and load stochastic variables.

(1) Modeling of UPFC

The UPFC injection model and the uncoupled model have been proposed for steady-state power flow analysis in some published papers. These models can be easily incorporated into steady-state power flow or OPF problems. However, control variables of UPFC are not explicitly declared so that these models are undesirable for researching CDF's sensitivities to UPFC.

To overcome the above difficulties, a kind of UPFC model with a complex turns ratio and variable shunt admittance is introduced for POPF. In that model, UPFC control variables could not depend on input voltages and current of

UPFC anymore. This means that the UPFC model could be combined with transmission line models by using two-port representations conveniently. Besides, the Jacobian matrix of power system only needs to be modified in four positions if one UPFC is installed in power system.

Fig. 1 shows a simplified UPFC circuit with two extra buses *I* and *O*. U_I, θ_I, U_o, θ_o are amplitude and phase of bus *I* and *O*. $\dot{U}_{se}$ is the series voltage source of UPFC, while $\dot{I}_{sh}$ is the shunt current source of UPFC.

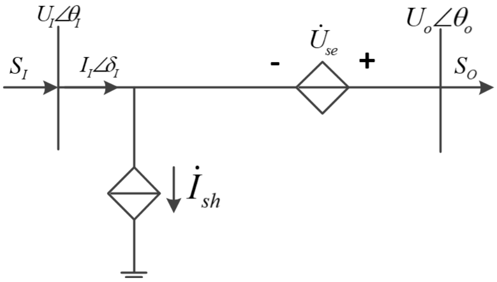

Fig. 1. Simplified UPFC circuit.

Assume that UPFC is located between buses *i* and *k*. Transmission line could be represented as an equivalent Π circuit, as shown in Fig. 2. The currents injected into bus *I* and *O* are as follows:

$$\dot{I}_I = (Y_i/2 + 1/Z_i)\dot{U}_I - \dot{U}_i/Z_i \tag{8}$$

$$\dot{I}_o = (Y_k/2 + 1/Z_k)\dot{U}_o - \dot{U}_k/Z_k \tag{9}$$

UPFC can be modeled through an ideal transformer and a shunt branch, as shown in Fig. 3. The advantage of this model is that ideal transformer turns ratio and the variable shunt susceptance are independent variables, which are not directly associated with the UPFC outputs. We define the UPFC variables as follows:

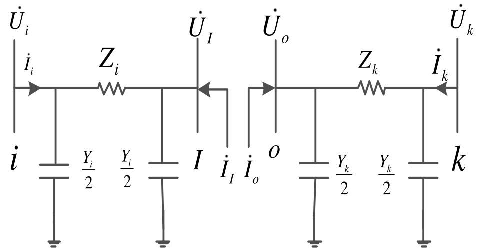

Fig. 2. UPFC model in a transmission line.

T transformer voltage magnitude turns ratio (real);

ϕ phase shifting angle;

ρ shunt susceptance;

and the ideal transformer turns ratio can be written by

$$\dot{T} = Te^{j\phi} \tag{10}$$

The UPFC input-output voltage and current relationship can be expressed as

$$\dot{U}_I = \dot{U}_o \dot{T} \tag{11}$$

$$\dot{I}_I = \dot{I}_o / \dot{T}^* + j\rho \dot{U}_o \dot{T} \tag{12}$$

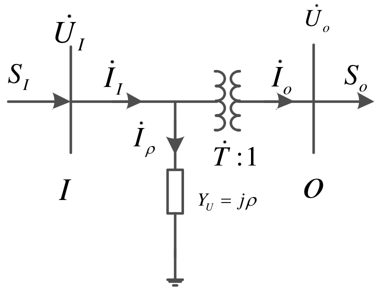

Fig. 3. UPFC ideal transformer model.

The UPFC can be represented by a matrix as

$$\begin{bmatrix} \dot{U}_I \\ \dot{I}_I \end{bmatrix} = J_U \begin{bmatrix} \dot{U}_o \\ \dot{I}_o \end{bmatrix} \tag{13}$$

where

$$J_U = \begin{bmatrix} \dot{T} & 0 \\ j\rho\dot{T} & 1/\dot{T}^* \end{bmatrix} \tag{14}$$

(2) POPF Model

Suppose that a UPFC is installed in the transmission line between buses i and k. The mathematical formulation of the POPF problem with the UPFC can be expressed as the following nonlinear programming problem:

$$OC = \min f_{\cos t}(X_C, X_U, \tilde{X}_S) \tag{15}$$

Subject to

$$h_m(X_C, X_U, \tilde{X}_S) = 0, \qquad m = 1,...,M \tag{16}$$

$$g_n(X_C, X_U, \tilde{X}_S) \le 0, \qquad n = 1,...,N \tag{17}$$

where

$f_{\cos t}(X_C, X_U, \tilde{X}_S)$	total operation cost or consumer benefit;
$X_U = [T_{ik}\phi_{ik}\rho_{ik}]$	vector of the UPFC control variables in line;
$\tilde{X}_S = [\tilde{P}_D]$	vector of stochastic load variables;
$\{h_m : m = 1,...,M\}$	set of equality constraint functions;
$\{g_n : n = 1,...,N\}$	set of inequality constraint functions.

Since the UPFC control variables could not depend on UPFC's voltage and current, the number of equality constraints is the same as that of the base case POPF without UPFC, while ignoring UPFC operational limits.

(3) PEM for POPF problem

The basic idea of PEM is to estimate the moment of operation cost, bus voltage, branch power flows, and other random output by considering a few specific points on the probability distributions of input variables including load's random disturbances.

It is assumed that there are S random variables in the POPF. The stochastic variables are as follows:

$$\tilde{X}_S = [\tilde{x}_1, \tilde{x}_2, \ldots \tilde{x}_S] \tag{18}$$

where $\tilde{x}_i (i = 1,2,\ldots S)$ is the i^{th} random variable and denotes different bus load in this paper. $\tilde{x}_i$ can be estimated by two-point estimate method using two variables $x_{i,1}$ and $x_{i,2}$, which are defined as

$$x_{i,k} = \mu_{x_i} + \xi_{i,k}\sigma_{x_i}, k = 1,2 \tag{19}$$

where μ_{x_i} and σ_{x_i} denote the mean and standard deviation of random variable x_i.$\xi_{i,k}$ denotes the location, which is expressed as follows:

$$\xi_{i,k} = \lambda_{i,3}/2 + (-1)^{3-k}\sqrt{m + (\lambda_{i,3}/2)^2}, k = 1,2 \tag{20}$$

The skewness $\lambda_{i,3}$ of x_i can be expressed as follows:

$$\lambda_{i,3} = E[(x_i - \mu_{x_i})^3]/(\sigma_{x_i})^3 \tag{21}$$

The variable x_i is replaced by the values of both sides of mean, $x_{i,1}$ and $x_{i,2}$, respectively, and other uncertainty variables are replaced by the means, such as (μ_{x_1}, μ_{x_2},…, $x_{i,k}$,…, $\mu_{x_{S-1}}$, μ_{x_S}). Based on the path following interior point method, the deterministic OPF calculation is operated for each variable. Then, two estimated values $H(i,1)$ and $H(i,2)$ of output random variables are obtained.

Let $\omega_{i,k}$ denote the concentrations (or weights) located at point (μ_{x_1}, μ_{x_2},…, $x_{i,k}$,…, $\mu_{x_{S-1}}$, μ_{x_S}). That can be expressed as follows:

$$\omega_{i,k} = (-1)^k \xi_{i,3-k}/(S \times \varsigma_i) \tag{22}$$

where $\varsigma_i = 2\sqrt{S + (\lambda_{i,3}/2)^2}$, the value of $\omega_{i,k}$ ranges from 0 to 1, and the sum of all $\omega_{i,k}$ is 1.

Based on PEM, deterministic OPF calculation will run two times for each input random variable. It means that the deterministic OPF calculation will run *2S* times if there are S uncertainty variables. Once all the concentrations $(\xi_{i,k}, \omega_{i,k}) (i = 1,..S, k = 1,2)$ are acquired, the function $\hbar$ of *OC* can be evaluated at the point (μ_{x_1}, μ_{x_2},…, $x_{i,k}$,…, $\mu_{x_{S-1}}$, μ_{x_S}) yielding $H(i,k)$. Using $\omega_{i,k}$ and $H(i,k)$, the j^{th} raw moment of the output random variable can be obtained according to the expression

$$E[H^j] \cong \sum_{i=1}^{S}\sum_{k=1}^{2}\omega_{i,k}[H(i,k)]^j \tag{23}$$

Furthermore, the mean and standard deviation of *OC* can be easily obtained from

$$\mu_{oc} = E[OC] = \sum_{i=1}^{S}\sum_{k=1}^{2}\omega_{i,k}[OC(i,k)] \tag{24}$$

$$\sigma_{oc} = \sqrt{\mathrm{var}(OC)} = \sqrt{E(OC^2) - \mu_{oc}^2} \tag{25}$$

As shown in (24) and (25), since the mean and standard deviation expressions of *OC* have been obtained, CDF's sensitivities of *OC* to UPFC, as shown in (7), could be reformed as follows:

$$\begin{aligned}\gamma_{oc} &= \frac{\partial F_{oc}}{\partial \mu_{oc}}\frac{\partial \mu_{oc}}{\partial X_U}+\frac{\partial F_{oc}}{\partial \sigma_{oc}}\frac{\partial \sigma_{oc}}{\partial X_U}=\frac{\partial F_{oc}}{\partial \mu_{oc}}\sum_{i=1}^{S}\sum_{k}^{2}\omega_{i,k}\frac{\partial OC(i,k)}{\partial X_U}\\ &+\frac{1}{2}\frac{\partial F_{oc}}{\partial \sigma_{oc}}(E[OC^2]-\mu_{oc}^2)^{-1/2}\sum_{i=1}^{S}\sum_{k}^{2}2\omega_{i,k}OC(i,k)\frac{\partial OC(i,k)}{\partial X_U}\\ &-\mu_{oc}\frac{\partial F_{oc}}{\partial \sigma_{oc}}(E[OC^2]-\mu_{oc}^2)^{-1/2}\sum_{i=1}^{S}\sum_{k}^{2}\omega_{i,k}\frac{\partial OC(i,k)}{\partial X_U}\end{aligned} \tag{26}$$

To calculate CDF's sensitivities of *OC* to UPFC, all components but $\partial OC/\partial X_U$ could be calculated by using PEM to solve POPF problem. How to compute the partial differential function $\partial OC/\partial X_U$ will be discussed in the next section.

(4) SENSITIVITIES of OC to UPFC ANALYSIS (move)

we deduce the expression of $\partial OC/\partial X_U$, which could be obtained by using interior point method to solve OPF problem.

Now, let us construct the Lagrangian for the OPF problem, in which the random variable $\tilde{x}_s$ has been transferred to certain ones ($\bar{X}_S$) by PEM.

$$L(X_C,X_U,\bar{X}_S,\kappa,\nu)=f_{\cos t}(X_C,X_U,\bar{X}_S)+\sum_{m=1}^{M}\kappa_m h_m(X_C,X_U,\bar{X}_S)+\sum_{n=1}^{N}\nu_n g_n(X_C,X_U,\bar{X}_S) \tag{27}$$

where κ_m and ν_n are the Lagrange multipliers for the equality and inequality constraints, respectively. Therefore, if an inequality constraint is binding, we could treat it as an equality constraint. Otherwise, ignore it. Then, we rewrite (27) as

$$L(X_C,X_U,\bar{X}_S,\kappa)=f_{\cos t}(X_C,X_U,\bar{X}_S)+\sum_{m\in A}\kappa_m h_m(X_C,X_U,\bar{X}_S) \tag{28}$$

where A is the set of active constraints. We consider the case where the UPFC is installed between buses *i* and *k*. The derivation $\partial OC/\partial X_U$ is simply the amounts by which the operation cost could be changed by allowing a small change of the UPFC control variables. Then, obtain the $\partial OC/\partial X_U$ by assuming that there is a UPFC, which is not put into operation. Thus, three additional constraints are added into the original OPF problem.

$$T_{ik}=T,\phi_{ik}=\phi,\rho_{ik}=\rho \tag{29}$$

Then, using KKT condition for OPF problem with the constraints, we can obtain

$$\frac{\partial f_{\cos t}}{\partial X_U}=-\sum_{m\in A}\kappa_m^*\frac{\partial h_m}{X_U} \tag{30}$$

where κ_m^* is the Lagrange multipliers of equality constraints in the optimal situation.

Note that (30) is easy to compute. Then, combining (26) and (30), we could get CDF's sensitivities of *OC* to UPFC finally. Thus, we can obtain CDF's sensitivities of *OC* to UPFC control variables for each possible transmission line by solving only the base case POPF.

RESULTS AND ANALYSIS

The proposed CDF's sensitivity method was tested on IEEE-5 and IEEE 14 systems to validate its effectiveness. These systems have 7 and 15 lines, respectively. Fig. 4 shows the five-bus system. The line data of the five-bus system are given in Ref. 6.

(1) IEEE-5 bus system

The system consists of two generators and buses 1 and 2. The generation cost function, measured in \$/h, is defined as follows:

$$f_{cost}(P_{Gi}) = a_i P_{Gi}^2 + b_i P_{Gi} + c_i \tag{31}$$

where P_{Gi} is the generator's real power generation, measured in MW. a_i, b_i, and c_i, cost parameters for generator i, are summarized in Table 1. The load data and statistic characteristic are given in Table 2.

Table 1. Generator Data.

generator	ai	bi	ci	Pmax/MW	Pmin/MW	Qmax/MVar	Qmin/MVar
1	0.00082	11	692.32	250	45	150	-100
2	0.000776	12	692.32	150	15	50	-40

Table 2. Load data and statistic characteristic.

Bus	Pload/MW	Qload/MVar	Variance of Pload	Variance of Qload
1	0	0	--	--
2	21.7	12.7	2.31	1.12
3	94.2	19	43.4	2.32
4	47.8	3.9	11.22	1.01
5	7.6	1.6	1.44	0.67

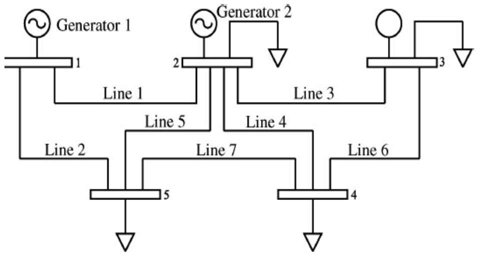

Fig. 4. Diagram of five-bus system.

To analyze the impact of the UPFC installed in different lines to CDF of *OC*, we calculate CDF's sensitivities of *OC* to control variables of UPFC. The sensitivities of UPFC in all possible line are shown in Table 3. By analyzing the results, CDF's ranges of UPFC control variables are 0.00069~0.05477, 0.00747~0.22716, and 1.07×10^{6}~5.99×10^{6}, respectively. Obviously, phase shifting angle has minimal impact on CDF of *OC* totally. From the view of UPFC located in different branches, when UPFC is installed in Line 1 (0.00587, 0.11569, 3.15×10^{6}), Line 2 (0.07926, 0.32861, 3.15×10^6), or Line 5 (0.02887, 0.0939, 4.59×10^5), the probability distribution of *OC* in high value region is lower than other lines. Among these three candidates, Line 2 is the most suitable line for UPFC installed.

Table 3. Sensitivities of UPFC control variables.

UPFC location	Sensitivities to T_{ij} of UPFC	Sensitivities to ϕ_{ij} of UPFC	Sensitivities to ρ_{ij} of UPFC
Line 1	0.00587	0. 11569	3.15×10^6
Line 2	0.07926	0.32861	3.15×10^6
Line 3	0.00147	0.01068	4.59×10^6
Line 4	0.00156	0.01112	4.59×10^5
Line 5	0.02887	0.09390	4.59×10^5
Line 6	0.00099	0.01067	3.60×10^6
Line 7	0.00228	0.021790	1.54×10^4

To illustrate the effectiveness of stochastic sensitivity to optimal location of UPFC, CDF of *OC* is calculated in the situation of all branches with equal capacity of UPFC (T_{ij}, φ_{ij}, ρ_{ij} are set to 0.98p.u., 0.004p.u.,0.3p.u. partly). The corresponding results are shown in Table 4. Before UPFC is installed, the operation cost of the system is 3282.46$. With UPFC being installed in different lines, the operation cost will be lower than 3282.46, and the larger reduction margin is 22.46%, 8.04%, and 6.3%, when UPFC is located in Line 2, Line 5, and Line 1 accordingly.

Table 4. Operation cost of CDF when UPFC is located in different lines.

UPFC location	Operation Cost		CDF when OC is mean value without UPFC	CDF's Reduction margin with UPFC
	Mean /$	standard deviation		
Without UPFC	3282.46	63.48	0.5	--
Line 1	3277.4	63.42	0.5315	6.30%
Line 2	3264.8	61.79	0.6123	22.46%
Line 3	3282.1	63.47	0.5021	0.43%
Line 4	3282.1	63.47	0.5022	0.44%
Line 5	3276.1	63.39	0.5402	8.04%
Line 6	3282.2	63.48	0.5017	0.34%
Line 7	3281.5	63.46	0.5058	1.16%

Compared to the *OC* without UPFC, PDF of *OC* with UPFC in Line 1, Line 2, and Line 5 is shown in Fig. 5. All possible installed methods could move PDF of *OC* towards lower value region, so as to reduce CDF of *OC* in higher value area.

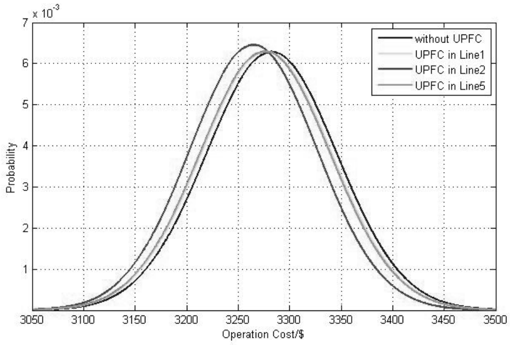

Fig. 5. Operation cost of PDF when UPFC is located in different lines.

(2) IEEE-14 bus system

There are two generators and 15 lines in IEEE-14 bus system. Generator data are the same as IEEE-5 bus system. Based on the sensitivity of *OC*'s CDF calculation, the result is obtained and shown in Table 5. It could be seen that line 2 is the most acceptable position for UPFC, because the CDF's sensitivity of UPFC's control variables is 0.07206, 0.25671, and 8.01×10^6.

Table 5. Sensitivities of UPFC control variables.

UPFC location	Stochastic sensitivity of UPFC's variables		
	T_{ij}	φ_{ij}	ρ_{ij}
Line 1	0.00625	0. 09511	8.01×10^6
Line 2	0.07206	0.25671	8.01×10^6
Line 3	0.00407	0.01144	1.02×10^5
Line 4	0.00881	0.01276	1.02×10^5
Line 5	0.00604	0.07089	1.02×10^5
Line 6	0.00332	0.01146	7.71×10^6
Line 7	0.00211	0.01328	6.17×10^6
Line 8	0.00958	0.00728	8.60×10^5
Line 9	0.00367	0.00240	8.60×10^5
Line 10	0.00370	0.00606	8.60×10^4
Line 11	0.00946	0.00728	2.22×10^4
Line 12	0.00012	0.00366	2.22×10^4
Line 13	0.00956	0.00728	3.42×10^4
Line 14	0.00364	0.00240	2.29×105
Line 15	0.00013	0.00366	1.18×104

In order to explain the principle that *OC* is affected by UPFC, real powers' PDF of generators is got when UPFC is installed in line 2, and control variables are set to 0.98p.u., 0.003 p.u., 0.3 p.u. As shown in Fig. 6 and Fig. 7, the mean real powers of generators are 171.88MW and 88.87 MW without UPFC, while the mean active powers change to 212.72MW and 48.69MW due to UPFC in line 2. Because the generation marginal cost is lower, it is profitable to obtain more real power from generator 1, as long as its adjusted marginal cost stays lower, and no operational limits are reached. Therefore, for the optimal location of the UPFC in this system, it is profitable to reduce the total generation cost rather than the transmission line loss since two generators have different generation costs. After UPFC is installed, generator 1 output would increase, while generator 2 output decreases. As a result, the operation cost of the power system could be lower than that of the original situation.

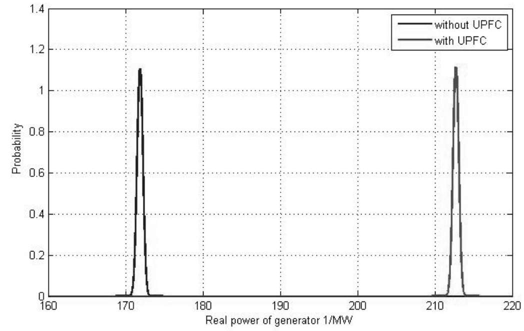

Fig. 6. Comparison of PDF of Generator 1 active power.

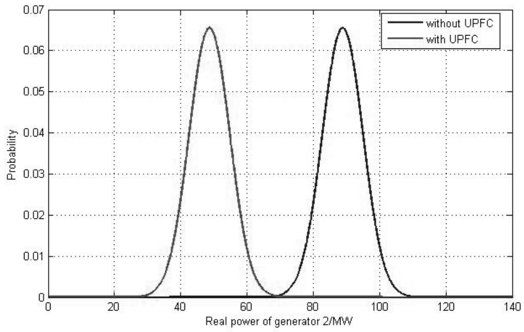

Fig. 7. Comparison of PDF of Generator 2 active power.

CONCLUSION

We have proposed a kind of stochastic method to optimize UPFC's installing location. Different from the existing algorithms, the proposed method considers randomness in power system, such as load's uncertainty. As a result, the operation cost would be a stochastic variable, not a certain value any more. To evaluate the impact of UPFC at different transmission lines to CDF of *OC*, CDF' sensitivity is introduced in this paper. POPF and PEM models are also used to calculate the sensitivities. Finally, based on CDF' sensitivity, the optimal UPFC location would be decided by comparison. IEEE-5 and IEEE-14 test systems are employed to illustrate the validity and efficiency of the proposed method. In future research, we will take the voltage regulation ability of UPFC into consideration and optimize UPFC's installation by dealing with active power imbalance and lack of voltage stability.

ACKNOWLEDGMENT

The authors gratefully acknowledge the support of the State Grid Project of Research on Optimization and Operation Control Technology of Large-scale Offshore Wind Power to Adapt the AC/DC Hybrid Receiving Grid.

REFERENCES

An, S., Condren, J. & Gedra, T.W. 2007. An ideal transformer UPFC model, OPF First-Order sensitivities, and application to screening for optimal UPFC locations. IEEE Transactions on Power Systems, **22**(1): 68-75.

Baghaee, H.R., Jannati, M., Vahidi, B., Hosseinian, S.H. & Jazebi, S. 2008. Optimal multi-type FACTS allocation using genetic algorithm to improve power system security. 12th International Middle East Power System Conference.

Balamurugan, K. & Muthukumar, K. 2019. Differential evolution algorithm for contingency analysis-based optimal location of FACTS controllers in deregulated electricity market. Soft Computing, **23**(1):163-179.

Bhowmik, A.R., Chakraborty, A.K. & Das, P. 2012. Optimal location of UPFC based on PSO algorithm considering active power loss minimization. IEEE 5th Power India Conference.

Dutta, S., Roy, P.K. & Nandi, D. 2015. Optimal location of UPFC controller in transmission network using hybrid chemical reaction optimization algorithm. International Journal of Electrical Power & Energy Systems, **64**: 194-211.

Fang, W.L. & Ngan, H.W. 1999. Optimising location of unified power flow controllers using the method of augmented Lagrange multipliers. IEE Proceedings - Generation, Transmission and Distribution, **146**(5):428-434.

Faried, S.O., Billinton, R. & Aboreshaid, S. 2009. Probabilistic technique for sizing FACTS devices for steady-state voltage profile enhancement IET Generation, Transmission and Distribution, **3**(4):385-392.

Hassan, L.H., Moghavvemi, M., Haider, A.F. & Steinmayer, O. 2013. Application of genetic algorithm in optimization of unified power flow controller parameters and its location in the power system network. International Journal of Electrical Power and Energy Systems, **46**(1): 89-97.

Ippolito, L. 2005. A novel strategy for selection of the optimal number and location of UPFC devices in deregulated electric power systems. IEEE Russia Power Tech.

Kumar, B.V. & Srikanth, N.V. 2015. Optimal location and sizing of unified power flow controller (UPFC) to improve dynamic stability: a hybrid technique. International Journal of Electrical Power & Energy Systems, **64**: 429-438.

Kumar, B.V. & Srikanth, N.V. 2017. A hybrid approach for optimal location and capacity of UPFC to improve the dynamic stability of the power system. Applied Soft Computing Journal, **52**: 974-986.

Rajbongshi, R. & Saikia, L.C. 2017. Performance of coordinated FACTS and energy storage devices in combined multiarea ALFC and AVR system. Journal of Renewable and Sustainable Energy, **9**(6): 1-21.

Shaheen, H.I., Rashed, G.I. & Cheng, S.J. 2008. Optimal location and parameters setting of UPFC based on GA and PSO for enhancing power system security under single contingencies. IEEE Power and Energy Society General Meeting.

Shaheen, H.I., Rashed, G.I. & Cheng, S.J. 2010. Application and comparison of computational intelligence techniques for optimal location and parameter setting of UPFC. Engineering Applications of Artificial Intelligence, **23**(2): 203-216.

Singh.S.N. & Erlich.I. 2005. Locating unified power flow controller for enhancing power system loadability. International Conference on Future Power Systems.

Song, S.H., Lim, J.U. & Moon, S.I. 2004. Installation and operation of FACTS devices for enhancing steady-state security. Electric Power Systems Research, **70**(1): 7-15.

Taher, S.A. & Abrishami, A.A. 2009. UPFC location and performance analysis in deregulated power systems. Mathematical Problems in Engineering, **2019**: 1-20.

Taher, S.A. & Amooshahi, M.K. 2012. New approach for optimal UPFC placement using hybrid immune algorithm in electric power systems. International Journal of Electrical Power and Energy Systems, **43**(1): 899-909.

Tiwari, P.K. & Sood, Y.R. 2012. Efficient and optimal approach for location and parameter setting of multiple unified power flow controllers for a deregulated power sector. IET Generation, Transmission and Distribution, **6**(10): 958-967.

Verma, K.S., Singh, S.N. & Gupta, H.O. 2001. Location of unified power flow controller for congestion management. Electric Power Systems Research, **58**(2): 89-96.

Zahid, M., Chen, J.F. & Li, Y.H. 2017. New approach for optimal location and parameters setting of UPFC for enhancing power systems stability under contingency analysis. Energies, **10**(11):1738-1761.