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A calculation method of transmission efficiency for RV reducer

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ABSTRACT

RV reducer is the core component of industrial robot. It is of great significance to raise the transmission efficiency of RV reducer for improving the transmission performance of industrial robot. RV reducer belongs to the 2K-V type planetary gear train and consists of an equal speed ratio mechanism. Therefore, according to the structural characteristics of RV reducer, the virtual power theory and split power theory are adopted, and a calculation method of transmission efficiency for RV reducer is proposed. Firstly, the structure of a common RV reducer is introduced. Related structural analysis, kinematic analysis, and loaded analysis are given. Secondly, RV reducer is represented by the method of graph theory. By the virtual power theory and split power theory, the power flow direction is determined, and the values of split powers are calculated. Finally, according to the graph representation for RV reducer and the calculation principle of meshing power loss of planetary gear, the power losses of meshing gear pairs are calculated, respectively. According to the input and output values described in graph representation of RV reducer, the formula of transmission efficiency for RV reducer is derived. The calculation of transmission efficiency for RV-40E reducer is considered as an example. The result is compared with previous work on the subject, and the relations between design parameters and transmission efficiency of RV reducer are discussed.

Keywords：RV reducer; Transmission efficiency; Graph theory; Virtual power; Split power.

Nomenclature

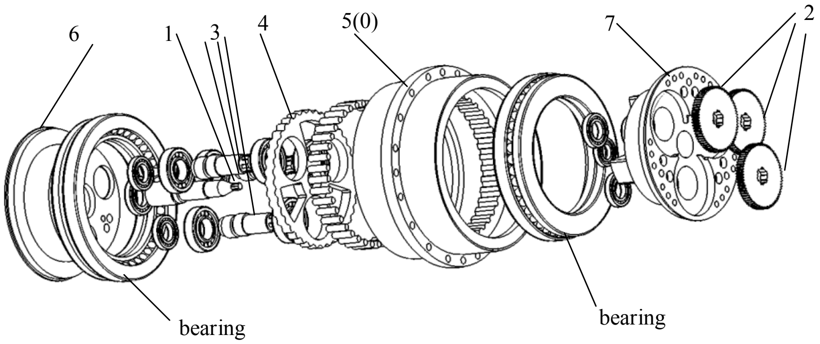
i_{12}^{7}	Transmission ratio between center gear 1 and planet gear 2 in the conversion gear train 0-1-2-7
i_{16}	Transmission ratio of RV reducer
i_{24}	Transmission ratio between planet gear 2 and cycloid gear 4
i_{45}^{3}	Transmission ratio between cycloid gear 4/4’ and needle gear/needle gear shell 5 in the conversion gear train 3-4/4’-5
L_1	Loss of gear meshing through planet gear 2

L_2	Loss of gear meshing through needle gear/needle gear shell 5
m	Number of cycloid gears
M_{13}	Torque applied to planet gear/crank shaft 2/3 by center gear 1
M_{43}, $M_{4'3}$, $M_{4''3}$	Torque applied to planet gear/crank shaft 2/3 by cycloid gear 4/4'/4''
M_{63}	Torque applied to planet gear/crank shaft 2/3 by output disk 6
M_{73}	Torque applied to planet gear/crank shaft 2/3 by planet frame 7
n	Number of planetary gears
P	Input power
P_4	Power through cycloid gear 4
P_4^3	Power through cycloid gear 4 in the transformation gear train
T_4	Torque exerted on the cycloid gear 4
U	Virtual power
V	Split power
w_1	Angular velocity of center gear 1
w_2	Angular velocity of planet gear 2
w_3	Angular velocity of crank shaft 3
$w_4/w_{4'}$	Angular velocity of cycloid gear 4/4'
w_5	Angular velocity of needle gear/needle gear shell 5
w_6	Angular velocity of output disk 6
w_7	Angular velocity of planet frame 7
z_1	Tooth number of center gear 1
z_2	Tooth number of planet gear 2
$z_4, z_{4'}$	Tooth number of cycloid gear 4/4'
z_5	Tooth number of needle gear/needle gear shell 5
$\alpha_{4(4')}$	Virtual power ratio on cycloid gear 4/4'
$\beta_{2(3)}$	Split power ratio on planet gear/crank shaft 2/3

η	Transmission efficiency for RV reducer
η_1	Meshing efficiency between center gear 1 and planet gear 2
η_2	Meshing efficiency between cycloid gear 4/4' and needle gear/needle gear shell 5

INTRODUCTION

The reducers in robot joint mainly include Rotate Vector (RV) reducer and harmonic reducer. RV reducer is a new type of two-stage deceleration drive developed on the basis of traditional cycloid pin gear drive. RV reducer is composed of cycloidal needle gear and planetary frame. It is mainly used in robot joints of more than 20 kg, and the harmonic reducer is used in robot joints less than 20 kg. Because the load-carrying strength of RV reducer is higher, its usage is much larger than that of harmonic reducer. The use ratio of the two is 6:4. Figure 1 is a three-dimensional explosion model of a common RV reducer. Figure 2 is the corresponding schematic drawing of the RV reducer corresponding to Fig.1. It should be pointed out that when the number of cycloid gears is more than one, they are represented by 4, 4', 4''···. In Fig.1, only one cycloid gear is drawn.



0-Frame, 1-center gear, 2-planet gear, 3-crank shaft, 4, 4'-cycloid gear, 5-needle gear/needle gear shell, 6-output disk, 7-planet frame

Figure 1. Three-dimensional explosion model of a common RV reducer.

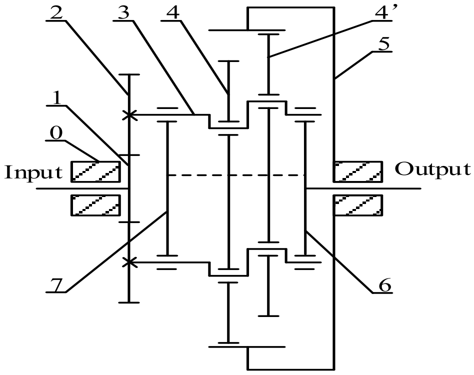


Figure 2. A schematic drawing of RV reducer corresponding to Fig. 1.

For the transmission performance of high precision reducer, the transmission efficiency is one of the important evaluation indexes. According to the classification method of planetary gear train proposed by B.H.Kyπρβ (He proposed a classification method of planetary gear train according to the difference of basis component in planetary gear train), RV reducer belongs to the 2K-V type planetary gear train. In the 2K-V type planetary gear train, K represents the center gear, and V represents the output mechanism (planetary frame in RV reducer); i.e., 2K-V type planetary gear train includes two center gears and planetary frame. Around the power flow and transmission efficiency of planetary gear train, a large number of researches have been carried out. A calculation method of transmission efficiency of the variable tooth thickness RV reducer was derived (Wang et al., 2015). A transmission efficiency of RV reducer with the knowledge of XP (single-loop system) was analyzed (Chen et al., 2004). According to the specific structure of the planetary gear trains and the transmission ratio of the basic components, a formula for calculating the transmission ratio of the planetary gear trains was proposed (Del, 2002). A calculation method of transmission efficiency of planetary gear trains and differential gear trains based on power flow maps was proposed (Salgado *et al*., 2005). A review of formulas for the mechanical efficiency analysis of two-degree-of-freedom epicyclic gear train was provided (Pennestrì *et al*., 2003). An analytical method for determining the power flow of automatic-transmission planetary gear system was proposed (Kahraman *et al*., 2004). A numeric approach for calculating the power flow and efficiency of the bevel gear planetary gear system was proposed (Nelson *et al*., 2005). The graph and screw theories were used to analyze the efficiency of planetary transmission (Laus *et al*., 2012). The power flow and transmission efficiency with multiple links based on graphical representation were analyzed (Salgado *et al*., 2014). In this study, a simple algorithm for determining the power flow direction and judging power backflow was proposed. Based on the transmission ratio of the basic components of differential gear trains and transformation gear trains, a simplified formula for calculating the transmission efficiency of 2K-H epicyclic gear trains was derived (Wang *et al*., 2013). In the 2K-H type planetary gear train, K represents the center gear, and H represents the planetary frame. The concept of virtual power and virtual power ratio, as well as the meshing power loss and transmission efficiency of simple planetary transmission, was proposed (Chen *et al*., 2017; Chen *et al*., 2011). The core idea of the virtual power theory is that the torque exerted on the component is invariant with respect to observer's frame, and the relationship of tooth number is established through the ratio between the power of gear in original gear transmission system and converted gear train. The concept of virtual power was used to analyze the transmission efficiency of eccentric paddle mechanism of planetary gear with double input and double output (Pu *et al*., 2013). The concept of split power ratio on the basis of theory of virtual power was proposed, and the analytical efficiency expression of a compound epicyclic gear train with split power was derived (Chen, 2013). The core idea of the split power theory is when the system is in a steady state, and the torque applied to component must be balanced.

Virtual power theory and split power theory can better solve the problem of transmission efficiency calculation for gear trains. However, the following problem should be paid attention. The geometric constraint, loaded analysis, and kinematics relation are related to the structure of gear trains. When the compound gear train is composed of multiple epicyclic gear trains, especially RV reducer includes an equal speed ratio mechanism, and how to use the virtual power theory and split power theory to calculate the efficiency is not given in the current literatures.

Therefore, in this paper, based on the theory of virtual power and split power (Chen *et al*., 2017; Chen *et al*., 2011; Chen, 2013) and in view of the structure characteristics of RV reducer, a calculation method of transmission efficiency for RV reducer is proposed. The analysis process of this paper is shown in Fig. 3.

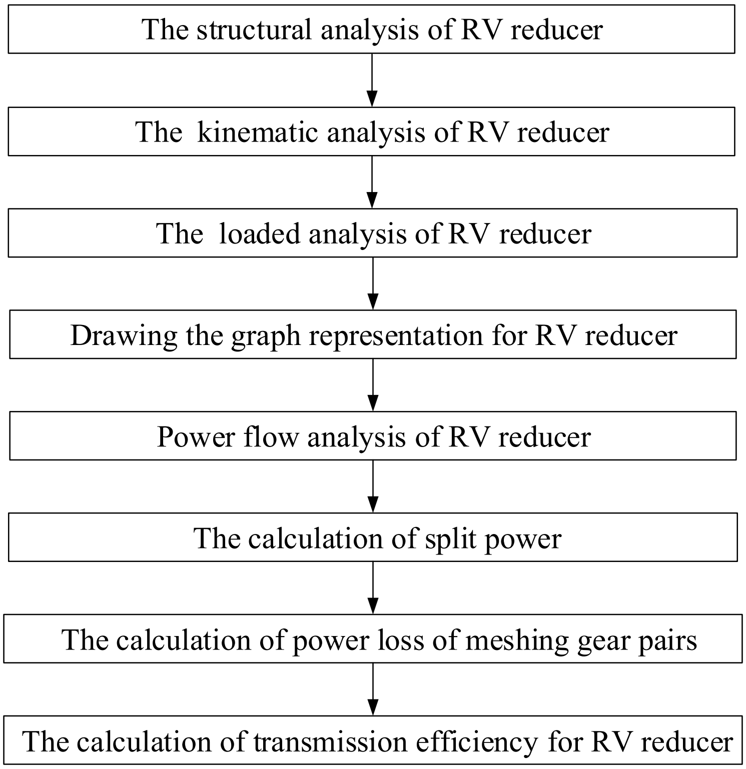


Figure 3. Flow diagram of transmission efficiency calculation for RV reducer.

THE STRUCTURAL, KINEMATIC, AND LOADED ANALYSIS OF RV REDUCER

The Structural Analysis of RV Reducer

The schematic drawing of a common RV reducer is shown in Fig. 2. RV reducer belongs to the compound gear train, which is composed of two epicyclic gear trains. In Fig. 2, the epicyclic gear trains are 0-1-2-7 and 3-4/4'-5, respectively. The needle gear/needle gear shell 5 is fixed in the frame. The output disk 6 and the planet frame 7 are fixed in the circumferential direction. The cycloid gear 4/4' and the output disk 6 are connected by an equal angle ratio mechanism and form the parallelogram structure (Zheng *et al.*, 2013); that is, their rotation speed is equal.

The Kinematic Analysis of RV Reducer

In the conversion gear train of epicyclic gear train 0-1-2-7, the transmission relationship can be represented as

$$i_{12}^{7} = \frac{w_1 - w_7}{w_2 - w_7} = -\frac{z_2}{z_1} \tag{1}$$

In the conversion gear train of epicyclic gear train 3-4/4'-5, the transmission relationship can be represented as

$$i_{45}^{3} = \frac{w_4 - w_3}{w_5 - w_3} = \frac{z_5}{z_4} \tag{2}$$

The connection condition between the two epicyclic gear trains can be represented as

$$w_2 = w_3,\ w_4 = w_6 = w_7 \tag{3}$$

From Eqs. (2) and (3), we obtain

$$\frac{w_3}{w_4} = \frac{z_4}{z_4 - z_5} \tag{4}$$

Besides, from Eqs. (1), (2), and (3), we can obtain

$$i_{24} = \frac{z_1 + z_2 - z_1 i_{16}}{z_2} \tag{5}$$

The Loaded Analysis of RV Reducer

When the system is in a steady state, the torque applied to component must be balanced. In this paper, the loaded condition of planet gear/crank shaft 2/3 is analyzed. Figure 4 shows the static analysis of planet gear/crank shaft 2/3.

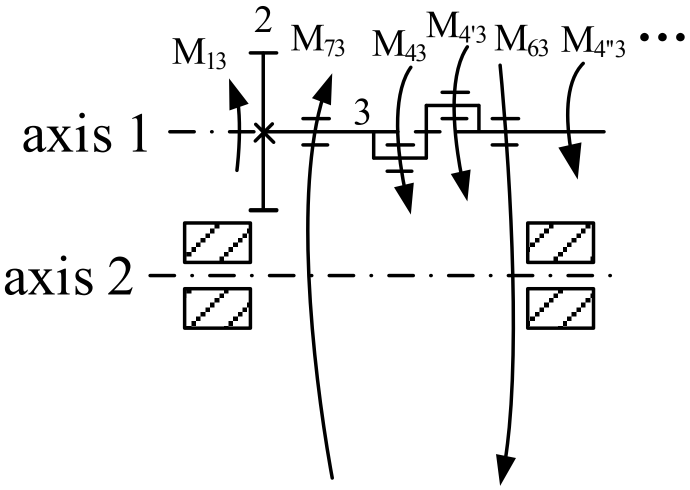


Figure 4. Torque analysis on component 2(3).

The planet gear/crank shaft 2/3 is applied to torque in the direction of the planetary gear axis (axis 1) and in the direction of the center axis (axis 2), respectively. They should be balanced, respectively, i.e.,

$$M_{73} = -M_{63} \tag{6}$$

$$M_{13} = -(M_{43} + M_{4'3} + M_{4''3} + \cdots) = -mM_{43} \tag{7}$$

where when there are multiple cycloid gears, they are represented by 4, 4', 4''….

POWER FLOW ANSLYSIS OF RV REDUCER

RV reducer is represented by the method of graph theory. Graph theory is a branch of mathematics. It takes graph as its object of study. Graph theory is a graph composed of a number of given points and lines connecting two points. This graph is usually used to describe a specific relationship between certain things. Points are used to represent things, and lines connecting two points are used to represent the corresponding relationship between two things. The converting principles are described as follows.

(1) When the power flow passes through the motion pair consisting of frame and active component, the power value is 0. When the power flow passes through the rotating pairs consisting of frame and multiple active components, the power value between active components is 0.

(2) The components are described in Arabia numbers, the motor is represented by the symbol $\odot$, the generator is represented by the symbol $\otimes$, and the gear meshing is represented by the symbol G.

According to the above converting principles, the schematic drawing of RV reducer shown in Fig. 2 can been converted into a graph representation shown in Fig. 5, where the power losses of meshing gear pairs are not considered. The direction of power flow *V* shown in Fig. 5 is from the planet gear/crank shaft 2/3 to the output disk/planet frame 6/7. As to whether it is reasonable, we will judge in the back.

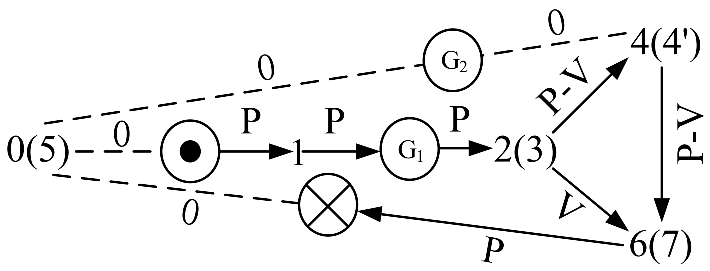


Figure 5. The graph representation for RV reducer without considering gear meshing losses.

THE CALCUALTION FOR TRANSMISSION EFFICIENCY OF RV REDUCER

The Calculation of Split Power V

According to the split power ratio (Chen, 2013), from Fig.5, we have

$$\beta_{2(3)} = \frac{P}{P - V} = \frac{M_{13} w_2}{m M_{43} w_4} = - \frac{w_2}{w_4} \tag{8}$$

According to Eqs. (3), (4), and (8), we have

$$V = \frac{2z_4 - z_5}{z_4} P \tag{9}$$

From Eq. (8) and according to the structure of RV reducer, it can be found that the value of V is positive; i.e., the power flow is from the planet gear/crank shaft 2/3 to the output disk/planet frame 6/7. Therefore, the direction of power flow shown in Fig. 5 is correct.

The calculation of split power *U*

RV reducer shown in Fig. 2 is applied to the rotational speed ($-w_3$), and the rotational speed of components changes to w_0 (w_5)-w_3, w_1-w_3, w_2 (w_3)-w_3, w_4 ($w_{4'}$) $-w_3$ and w_6 (w_7)-w_3. The obtained gear train is called the transformation gear train. The power is called the virtual power (Chen *et al*., 2013). The graph representation under the virtual power for RV reducer without considering gear meshing losses when the planet gear/crank shaft 2/3 becomes the frame is shown in Fig. 6.

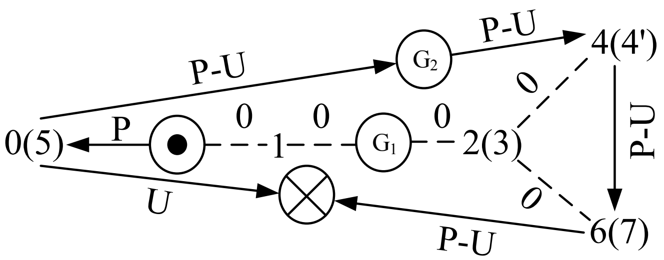


Figure 6. The graph representation under the virtual power for RV reducer without considering gear meshing losses when the planet gear/crank shaft 2/3 becomes the frame.

According to the theory of virtual power (Chen et al., 2013), the torque exerted on the component is invariant with respect to the observer's frame. Therefore, for the cycloid gear 4/4', we have

$$\alpha_{4(4')} = \frac{P_4^3}{P_4} = \frac{T_4(w_4 - w_3)}{T_4 w_4} = \frac{w_4 - w_3}{w_4} = \frac{P - U}{P - V} \tag{10}$$

According to Eqs. (3) and (4), we have

$$\frac{z_5}{z_5 - z_4} = \frac{P - U}{P - V} \Rightarrow U = \frac{z_4 - z_5}{z_4} P \tag{11}$$

From Eq. (11) and according to the structure of RV reducer ($z_4<z_5$), it can be found that the value of U is negative; i.e., the actual power flow direction of U is opposite to that shown in Fig. 6. The revised graph representation under the virtual power for RV reducer without considering gear meshing losses when the planet gear/crank shaft 2/3 becomes the frame is shown in Fig. 7.

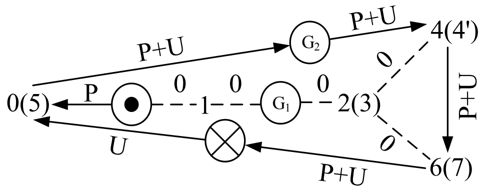


Figure 7. The revised graph representation under the virtual power for RV reducer without considering gear meshing losses when the planet gear/crank shaft 2/3 becomes the frame.

According to Fig.7, the virtual power ratio in the cycloid gear 4/4' is given by

$$\alpha_{4(4')} = \frac{P_4^3}{P_4} = \frac{T_4(w_4 - w_3)}{T_4 w_4} = \frac{w_4 - w_3}{w_4} = \frac{P + U}{P - V} \Rightarrow U = \frac{z_5 - z_4}{z_4} P \tag{12}$$

The Calculation of Power Loss of Meshing Gear Pair

The power loss at each gear mesh can only be evaluated by an observer standing on the corresponding gear carrier (Chen *et al.*, 2017; Chen, 2013). In RV reducer, meshing gear pairs are distributed in two epicyclic gear trains. Therefore, the transformation gear train of the two epicyclic gear trains should be first obtained, and then the power loss of meshing gear pair is calculated, respectively.

The Calculation of Power Loss L_1 of Meshing Gear Pair

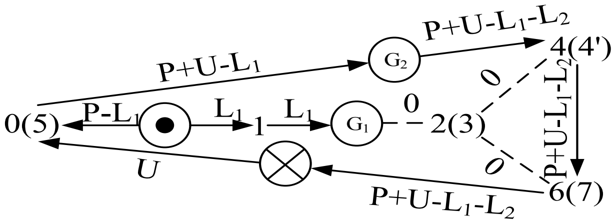


Figure 8. The graph representation under the virtual power for RV reducer with considering gear meshing losses when the output disk/planet frame 6/7 becomes the frame.

The meshing gear pair 1-2 causing power loss L_1 is located in the epicyclic gear train 0-1-2-7. The output disk/planet frame 6/7 is planetary frame. The RV reducer shown in Fig. 2 is applied to the rotational speed ($-w_7$), and the rotational speed of components changes to $w_0(w_5)$-w_7, w_1-w_7, $w_2(w_3)$-w_7, $w_4(w_{4'})-w_7$ and $w_6(w_7)$-w_7. The graph representation under the virtual power for RV reducer with considering gear meshing losses when the output disk/planet frame 6/7 becomes the frame is shown in Fig. 8.

In Fig. 8, the power flow that flows through the planet gear/crank shaft 2/3 is completely consumed by the gear pair meshing loss. Therefore, combined with Fig. 5, the power loss L_1 of meshing gear pair can be briefly represented as

$$L_1 = P(1-\eta_1) \tag{13}$$

The Calculation of Power Loss L_2 of Meshing Gear Pair

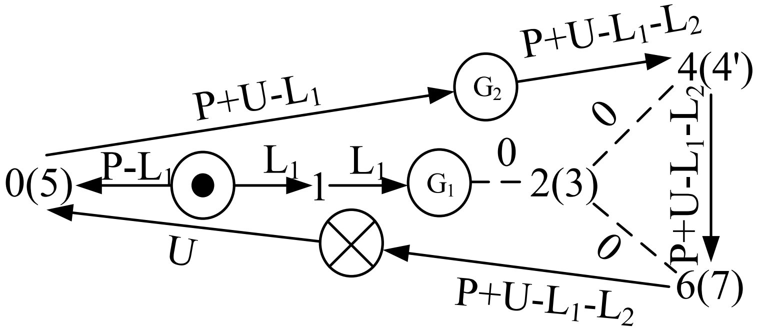


Figure 9. The graph representation under the virtual power for RV reducer with considering gear meshing losses when the planet gear/crank shaft 2/3 becomes the frame.

The meshing gear pair 4/4'-5 causing power loss L_2 is located in the epicyclic gear train 3-4/4'-5. The planet gear/crank shaft 2/3 is planetary frame. The RV reducer shown in Fig. 2 is applied to the rotational speed ($-w_3$), and the rotational speed of components changes to $w_0(w_5)$-w_3, w_1-w_3, $w_2(w_3)$-w_3, $w_4(w_{4'})-w_3$ and $w_6(w_7)$-w_3. The graph representation under the virtual power for RV reducer with considering gear meshing losses when the planet gear/crank shaft 2/3 becomes the frame is shown in Fig. 9.

The power loss of meshing gear pairs can be represented as

$$L_2 = (P + U - L_1)(1 - \eta_2) \tag{14}$$

The Calculation of Transmission Efficiency of RV Reducer

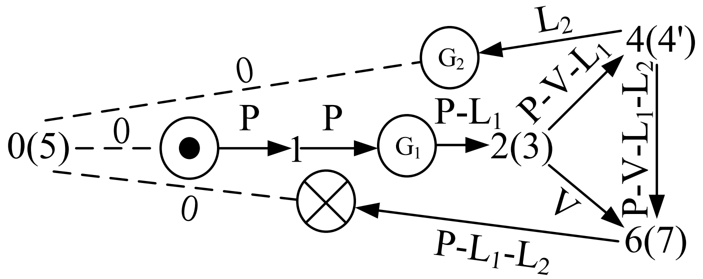


Figure 10. The graph representation for RV reducer with considering gear meshing losses

From Fig. 10, it can be clearly seen that the input power is P and the output power is $P - \sum_{i=1}^{2} L_i$. Therefore, the transmission efficiency of RV reducer can be represented as

$$\eta = \frac{P - \sum_{i=1}^{2} L_i}{P} \times 100\% \tag{15}$$

ILLUSTRATIVE EXAMPLE

The calculation of transmission efficiency for RV-40E reducer is considered as an example. (RV-40E reducer is a type of RV reducer. RV represents the formal name, 40 represents the model name, and E represents the built-in type of main bearing.) The parameters of RV-40E reducer are given in Tab. 1.

Table 1. Parameters of RV reducer (Lv, 2016).

Parameters	Numerical value
Z_1	12
Z_2	36
Z_4	39
$Z_{4'}$	39
Z_5	40
P	2.27kw

We assume that the meshing efficiency of all gear pairs is 0.94. According to Eq. (8), the split power V is

$$V = \frac{2z_4 - z_5}{z_4} P = 2211.8w \tag{16}$$

According to Eq. (12), the split power U is

$$U = \frac{z_5 - z_4}{z_4} P = 58.2051w \tag{17}$$

According to Eq. (13), the power loss L_1 of meshing gear pair is

$$L_1 = 2270 \times (1\text{-}0.94) = 136.2w \tag{18}$$

According to Eq. (14), the power loss L_2 of meshing gear pair is

$$L_2 = (2270 + 58.2051 - 136.2) \times (1 - 0.94) = 131.52w \tag{19}$$

According to Eq. (15), the transmission efficiency of RV-40E reducer is

$$\eta = \frac{2270 - (136.2 + 131.52)}{2270} \times 100\% = 88.2\% \tag{20}$$

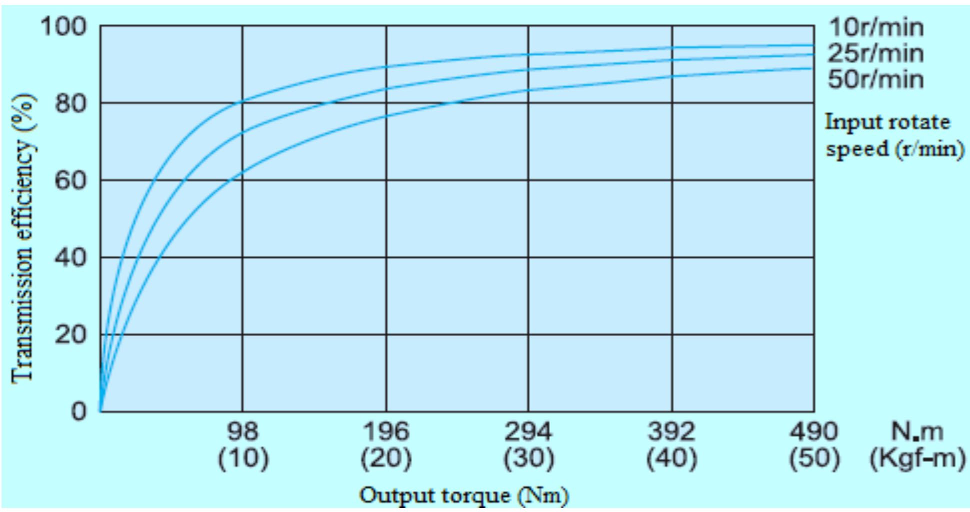


Figure 11. Transmission efficiency curve of RV-40E (Nabtesco, 2003).

Figure 11 gives the transmission efficiency curve of RV-40E reducer provided by the Nabtesco Co., Ltd. From Fig. 11, it can be found that transmission efficiency is related to input rotate speed and output torque. In the steady state, the change range of transmission efficiency is between 80% and 95%.

The transmission efficiency test-rig of RV-40E reducer was set up, and the transmission efficiency test was completed (Lv, 2016). The conclusions obtained by him include the following: (1) the transmission efficiency of RV reducer is influenced by out torque and input rotate speed; (2) the maximum transmission efficiency of RV-40E reducer is 73.23%, which is less than 75% qualified indicators. The reason for this is that there is an error in the measurement.

The result of this paper comes from the static state. Compared with the above results, the result of this paper is within the scope of transmission efficiency for RV reducer. Therefore, it can be used for the preliminary efficiency calculation of RV reducer.

The transmission efficiency of RV reducer is not only related to the out torque and input rotate speed, but also related to its design parameters. The corresponding relations are analyzed; they are shown in Fig. 12 and Fig. 13, respectively.

From Fig. 12 and Fig. 13, it can be found that (1) the relationship between meshing efficiency of a single pair of gears and the transmission efficiency of RV reducer is proportional. With the increase of meshing efficiency of a single pair of gears, the transmission efficiency increases rapidly; (2) the relationship between transmission ratio of RV reducer and the transmission efficiency of RV reducer is inversely proportional. With the increase of transmission ratio of RV reducer, the transmission efficiency of RV reducer decreases. According to the above analysis, the following conclusions can be obtained. Improving the meshing efficiency of a single pair of gears and decreasing transmission ratio of RV reducer under the premise of meeting the transmission requirements can effectively improve the transmission efficiency of RV reducer.

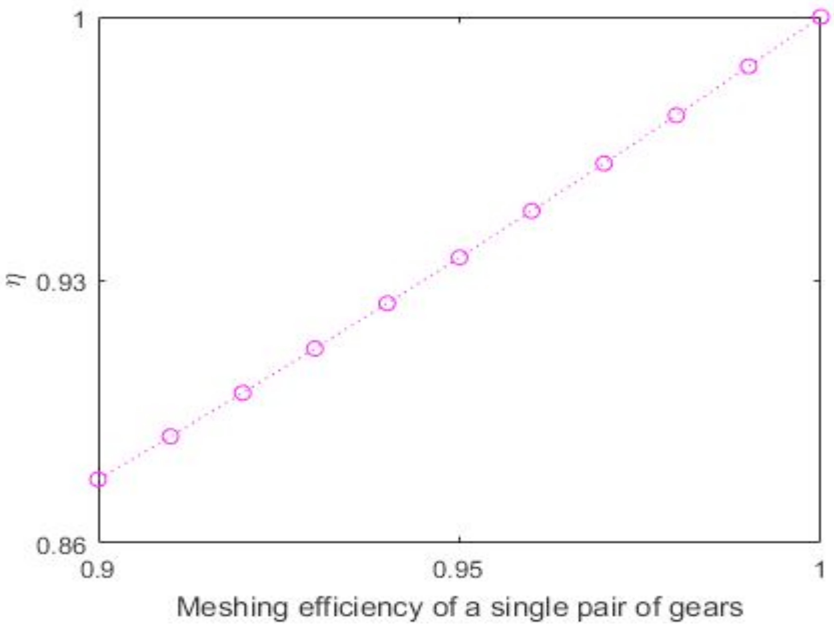


Figure 12. The relationship between meshing efficiency of a single pair of gears and the transmission efficiency of RV reducer.

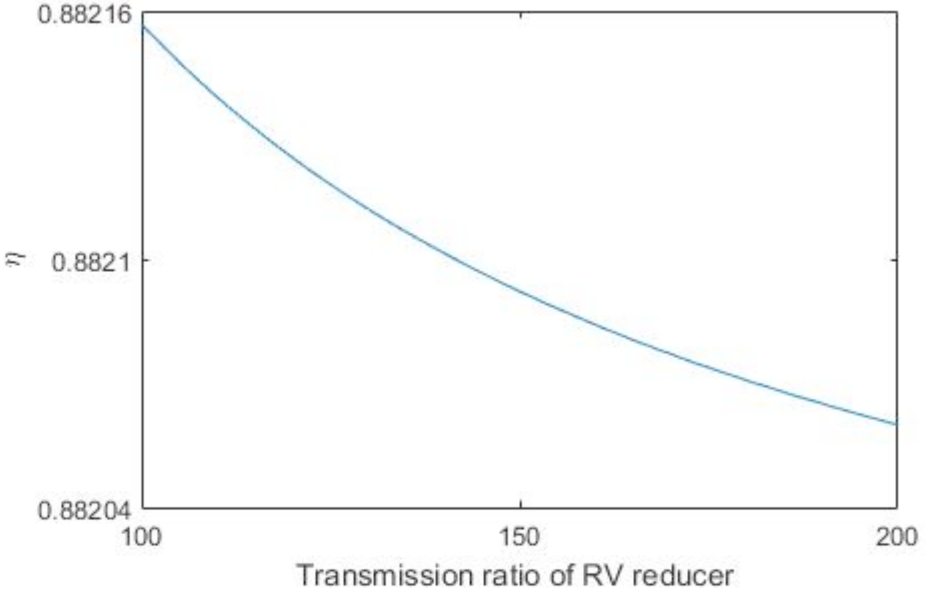


Figure 13. The relation between transmission ratio of RV reducer and transmission efficiency of RV reducer.

CONCLUSION

For the transmission performance of high precision reducer, the transmission efficiency is one of the important evaluation indexes. Therefore, a calculation method of transmission efficiency for RV reducer is proposed. Related structural analysis, kinematic analysis, and loaded analysis for RV reducer are completed. Based on the virtual power theory and the split power theory, power flow direction is determined, and the values of split powers are calculated. The power losses of meshing gear pairs are calculated, and the formula of transmission efficiency for RV reducer is given. The transmission efficiency for RV-40E reducer is calculated according to the proposed method. Compared with the existing results, the result of this paper is within the scope of transmission efficiency for RV reducer. Therefore, it can be used for the preliminary efficiency calculation of RV reducer.

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