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An EMQ-based system considering machine malfunction, allowable backlogging level, repairable items, and discontinuous issuing policy

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ABSTRACT

An economic manufacturing quantity (EMQ)-based system considering machine malfunction, allowable backlogging level, repairable nonconforming items, and discontinuous issuing policy is explored. Different from previous research (Chiu et al., 2017a), a discontinuous multi-shipment stock issuing policy is adopted in this study. With the aim of not only addressing the inevitable instances of machine failures and imperfect product quality, but also undertaking the practical matters of permissible backlogging and multi-delivery stock issuing, a complete procedure for problem solving is presented, which consists of problem modeling, formulation and integration of each separate situation, determination of convexity of system cost function, algorithm proposal for locating the optimal cycle length, and demonstration of the applicability of research result. Other than offering a way for production professionals to resolve such a specific problem, this study also reveals diverse in-depth system information for managerial decision making.

Keywords: Optimal cycle time; economic manufacturing quantity; machine failure; backlogging; service level constraint; discontinuous stock issuing; rework.

INTRODUCTION

The present study decides the optimal replenishment cycle length for an EMQ-based system considering machine malfunction, allowable backlogging level, repairable nonconforming items, and discontinuous issuing policy. Traditional EMQ model (Nahmias, 2009) decided the optimal fabrication batch size for an inventory system considering a perfect manufacturing condition and taking on a continuous stock issuing plan. Conversely, the imperfection of manufacturing process is inevitable in real fabrication systems due to diverse disruptive reasons. Random machine malfunction and production of nonconforming items are ordinary issues that must be cautiously handled by production managers to keep up smooth production schedule and retain desired product quality. Unsurprisingly, many prior studies have focused on diverse aspects of machine malfunction and imperfect products issues and their subsequent actions (e.g., machine repair, inventory control policy when a breakdown taking place, rework, and scrap). Shih (1980) investigated an inventory system with random defective products in the incoming batch, which obeys a known probability distribution. The stock-out situation occurs due to these unpredictable imperfect items. A mathematical model was developed to assist in analyzing the problem and deciding the best replenishment operating policy. Research result was compared to existing models via numerical examples. Gopalan & Kannan (1994) analyzed the expected time interval for a fabrication system featuring products screening and reworking in a two-stage transfer-line process. A stochastic model of a starting buffer of unlimited capacity was built, wherein inspection and rework are performed in both stages of the transfer-line route. In addition, the characteristics of the transient state and a few explicit system parameters were disclosed. Abboud (1997) explored a Poisson distributed machine malfunction rate associated with an economic manufacturing quantity model, wherein machine repair time is also random. The behavior of the proposed system was investigated by the use of a simple approximation approach. Precise formulas were provided for separate assumptions of machine repair times, namely, the exponential distribution and the constant ones. Simulations were employed to confirm the superior performance of solutions from the research. Giri & Dohi (2005) studied a stochastically unreliable economic manufacturing quantity (EMQ) model and provided an exact formula for its replenishment decision according to the net present value method. The obtained result was compared to the result from a cost-based conventional EMQ model to demonstrate its better performance. Pillai & Chandrasekharan (2008) employed a Markov chain approach to model a fabrication system's material flow, wherein the uncertain situations of reworking and scrap disposal were categorized as absorbing states. Through identifying and analyzing system variables of their Markov model, the required quantity of raw materials can be correctly calculated, and they claimed that their proposed model could also be used even when intangible cost data is inaccessible. Zied et al. (2011) investigated a manufacturing system featuring a maintenance plan and the random demand. A cost minimization production plan was first developed, then machine weakening status in relation to the rate of production was considered, and a most favorable (in terms of minimum cost) maintenance plan was derived. An example was used to demonstrate the applicability of the obtained results. It also indicated the important influence of fabrication rate on the machine weakening status, and subsequently on the integrated fabrication-maintenance policy. Extra works that focused on the aforementioned areas can also be found elsewhere (Balaji et al., 2016; Panizzolo, 2016; Rakyta et al., 2016; Zhang et al., 2016; Abubakar et al., 2017; Ajbar & Ali, 2017; Ashfaq et al., 2017; Buckova et al., 2017; Chiu et al., 2017b; Luong & Karim, 2017; Yadav, 2017; Yassen et al., 2017; Pearce et al., 2018; Saari & Odelius, J. 2018).

Moreover, with the aim of lowering stock holding cost (especially for items of high unit production cost), retaining an acceptable backlog level can be an effective strategy in production planning, that is, to allow a predetermined percentage of shortage/backorder in order to reduce holding cost and also meet customer's satisfaction in purchasing (i.e., the acceptable stock-out status). Schneider (1979) explored an inventory system with an (Q,s) ordering policy aiming at retaining a specific service level and minimizing the expected inventory cost per year. The best ordering lot size Q and reordering level s were decided. Fogarty & Aucamp (1985) studied the implied cost for backordering in order to assist management in determining whether their current backlogging strategy is working as planned. As a result, they figured out a critical backorder cost to help decide whether or not to implement a specific backorder plan. van der Sluis (1993) explored the combined ordering policies for periodic review multiproduct systems to meet stochastic demands. Under the service level constraint, their objective is to determine an optimal order policy that minimizes total purchase setup and stock holding cost. Due to the stochastic characteristic of product demands, they proposed to solve the problem as a deterministic model for the first N periods and implement decision obtained for the first period. Accordingly, under this rolling-horizon method, the behavior of the proposed multiproduct system was investigated. They also presented a procedure to amend their decision so as to reduce the unstable inventory status due to stochastic demand. As a result, cost reduction was confirmed through their simulation. Ouyang et al. (2003) used backorder discount and a so-called "protection time period" as controlling parameters to study a periodic review inventory system, wherein the protection time period includes the review time plus order lead time. Different demand distributions for the protection time period are assumed and analyzed; the first one follows the normal distribution and the second one obeys known first and second moments of the probability function. An algorithm was proposed to search for the optimal operating policies for these two separate problems. Results were exhibited via numerical examples. Toews et al. (2011) assumed backordering rate as an increasing linear function to time and reexamined the economic order quantity (EOQ) and economic production quantity (EPQ) models with partial backlogging. Additional works that focused on diverse aspects of backlogging and service level constraint can also be found elsewhere (Chiu et al, 2006; Fergany, 2016; Jaggi et al., 2016; Salemi, 2016; Oblak et al., 2017).

Furthermore, unlike the continuous stock issuing policy as assumed in the classic EMQ model, in real supply-chain environments, the discontinuous/periodic products shipping plan is frequently implemented. Goyal & Gupta (1989) examined diverse vendor-buyer integrated systems and introduced a scheme to categorize these different systems.

They also pointed out a few directions for future study. Viswanathan (1998) explored the optimal strategy for the vendor-buyer integrated systems. Two separate stock shipping policies from the existing studies were examined. The first plan is a fixed quantity for each delivery and the other plan is that at each delivery vendor supplies all available stock to the buyer. The research result indicated that an overview of performances of all system parameters no one policy outperforms the other. Sarker & Diponegoro (2009) determined the optimal fabrication and delivery plans for a supply-chain scheme that includes multiple suppliers, single maker, and multiple customers. In this particular integrated system, suppliers sell raw materials to a producer, then end products are made and distributed from producer to customers at a fixed time interval. The objectives were to simultaneously decide the most economic policies for raw materials purchase, fabrication, and stock shipping so as to keep the overall systems expenses minimum. Other studies that focused on the aforementioned areas can also be found elsewhere (Aboumasoudi, et al., 2016; Chiu et al., 2015, 2016; Kazemi et al., 2017; Radej et al., 2017; Settanni et al., 2017; Chiu et al., 2018). Little attention has been paid to the exploration of combined effects of machine malfunction, allowable backlogging level, repairable items, and discontinuous end products shipping plan on the optimal fabrication time decision for EMQ-based system; the present study aims at filling the gap.

THE PROPOSED EMQ-BASED SYSTEM

The optimal manufacturing uptime is determined for an EMQ-based system featuring allowable backlogging with service level constraint, rework of random nonconforming products, stochastic breakdown, and discontinuous products issuing policy. We extend a prior work (Chiu et al., 2017a) by considering discontinuous stocks issuing policy rather a continuous one as in the prior work. Definition of notation used in the present research is provided in Appendix A.

Assumption and modeling of the problem are given as follows: a particular EMQ-based system with manufacturing rate P_1 units per year is utilized to meet a flat demand rate λ per year for a particular product. Shortages are permitted and backordered with a service level constraint, to ensure that on-hand product availability stays at an acceptable level $(1 - \alpha)$% (where α denotes the percentage of shortage permitted time per cycle). The proposed EMQ-based system is subject to a random equipment failure and the breakdown rate complies with the Poisson distribution that its mean is β per year. The abort/resume (A/R) stock control policy is employed when an equipment failure occurs. Under such an A/R policy, the repair of failure starts right away and the interrupted lot continues once the equipment is restored. It is assumed that the failure repair time t_r is constant; a spare machine will be put in use in case the repair time is greater than t_r. Besides, an x portion of nonconforming items is randomly produced by this EMQ-based system, at a rate of d_1 (where $d_1 = P_1x$). All defective items produced in each cycle are assumed to be repairable thru a rework process immediately follows the regular fabrication process and the rework rate is P_2. Due to the assumption of discontinuous multi-shipment policy in this study, upon accomplishment of rework, n fixed-quantity installments of on-hand inventories are distributed at a fixed time interval t_n' during delivery time t_3'. In addition, a single shipment is made to satisfied backlogging B at the end of uptime t_5'. Moreover, as the basic assumption of EMQ model, $(P_1 - d_1 - \lambda)$ must be greater than zero to ensure positive inventory status in a replenishment cycle. Owing to the assumption of stochastic machine failure, the following situations must be explored, respectively.

Situation one: time to equipment failure $t < t_5'$

Situation one examines an equipment failure taking place in t_5' (Figure 1). It is noticed that, at the time a cycle's uptime starts, backordering level is at B. When an equipment failure takes place, the backordering level will reach H_0 and the equipment is under repair immediately. After the repair time t_r, the production equipment is resumed to fabricate the remaining of the interrupted batch. At the end of t_5', backlogging is satisfied by one single delivery. The stock level turns into positive and grows to H_1 when uptime ends. Then, the reworking of nonconforming products brings the on-hand stock level to H at the end of t_2'.

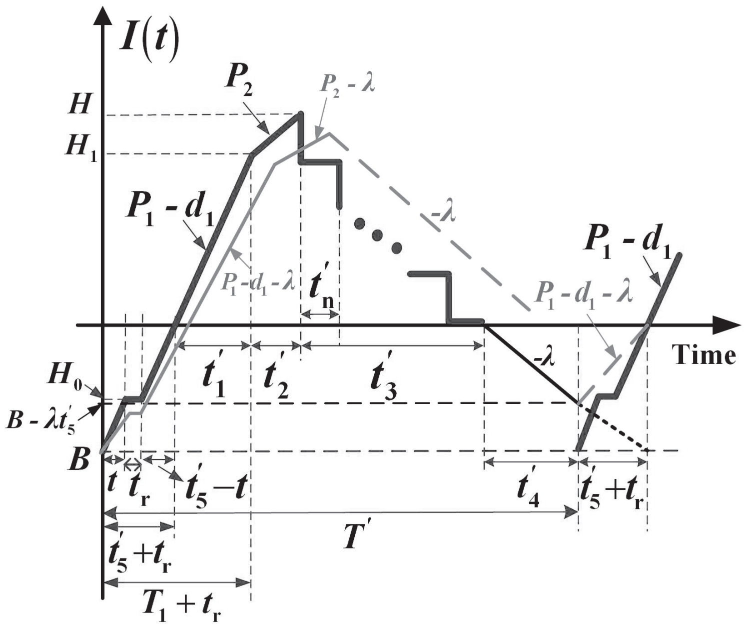

Figure 1. On-hand stock/backlog status at time t in the proposed EMQ system with an equipment failure taking place in t_5' as compared to that in (Chiu et al., 2017a) (in gray lines).

Figure 2 depicts the on-hand stock level of nonconforming products at time t. During t_3', n fixed-quantity installments of on-hand stocks are delivered (Figures 1 and 3). Finally, in t_4', any product demand becomes shortage and backordered, until backlogging level reaches ($B - \lambda t_5$'), then the next replenishment cycle commences. Because of the discontinuous product issuing policy, the maximum backlogging level B will be satisfied at the end of t_5'. From Figures 1 to 3, one can directly obtain the following basic equations:

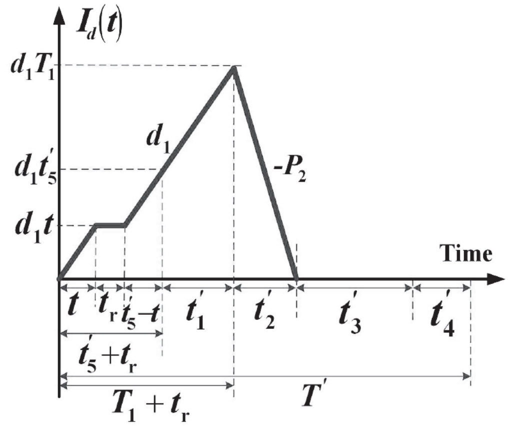

Figure 2. On-hand stock level of nonconforming products in the proposed EMQ system with an equipment failure taking place in t_5'.

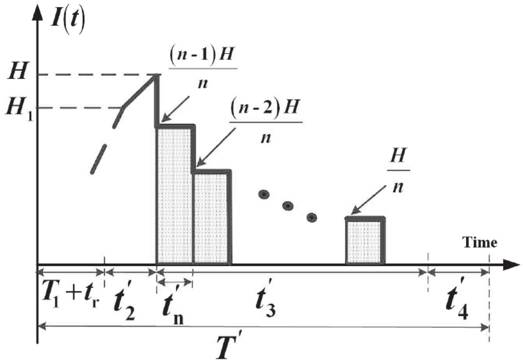

Figure 3. On-hand stock level of products in t_3' in the proposed EMQ system with an equipment failure taking place in t_5'.

$$B=(P_1-d_1)t_5^{'} \tag{1}$$

$$t_5^{'}=\frac{B}{(P_1-d_1)} \tag{2}$$

$$H_1=(P_1-d_1)t_1^{'} \tag{3}$$

$$t_1^{'}=\frac{H_1}{(P_1-d_1)} \tag{4}$$

$$T_1=\left(t_5^{'}+t_1^{'}\right)=\frac{Q}{P_1} \tag{5}$$

$$H=H_1+P_2t_2^{'} \tag{6}$$

$$t_2^{'}=\frac{H-H_1}{P_2}=\frac{d_1T_1}{P_2} \tag{7}$$

$$t_4^{'}=\frac{B-\lambda t_5^{'}}{\lambda} \tag{8}$$

$$T^{'}=\left(\sum_{i=1}^{5}t_i^{'}\right)+t_r \tag{9}$$

$$t_3^{'}=T^{'}-\left(t_1^{'}+t_2^{'}+t_4^{'}+t_5^{'}+t_r\right) \tag{10}$$

Total inventories during delivery time t_3' (Chiu et al., 2015) are as follows:

$$\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right)Ht_3' = \left(\frac{1}{n^2}\right)\left[\frac{n(n-1)}{2}\right]Ht_3' = \left(\frac{n-1}{2n}\right)Ht_3' \tag{11}$$

$$H_0 = B - \left(P_1 - d_1\right)t \tag{12}$$

Total relevant cost per cycle in case an equipment failure takes place in t_5', $TRC_1(T_1)$ contains variable manufacturing cost in uptime T_1, manufacturing setup cost, variable reworking and holding costs in t_2', repairing cost for broken equipment, procurement and holding for safety stocks, holding cost for perfect quality stocks and nonconforming products in t_1', t_2', t_3', t_5', and t_r', shortage backordering cost in t_4', t_5', and t_r, and the fixed and variable costs for finished stocks delivered in t_3' and in the end of t_5'. Hence, $TRC_1(T_1)$ is as follows:

$$\begin{aligned} TRC_1\left(T_1\right) &= C\left(T_1P_1\right) + K + C_R\left[x\left(T_1P_1\right)\right] + h_1\left(\frac{P_2t_2'}{2}\right)\left(t_2'\right) + M + \left[h_3(\lambda t_r)\left(t + \frac{t_r}{2}\right) + C_1\left(\lambda t_r\right)\right] \\ &+ h\left[\frac{H_1}{2}\left(t_1'\right) + \frac{\left(H_1 + H\right)}{2}\left(t_2'\right) + \left(\frac{n-1}{2n}\right)Ht_3' + (d_1t)t_r + \frac{d_1T_1}{2}\left(T_1\right) + \frac{B}{2}t_5' + \left[\left(P_1 - d_1\right)t\right]t_r\right] \\ &+ b\frac{B}{2}\left(t_4' + t_5' + t_r\right) + K_1 + nK_1 + C_T\left(T_1P_1 + \lambda t_r\right) \end{aligned} \tag{13}$$

Using expected values of $E[x]$ to deal with random nonconforming rate and substituting equations (1) to (12) in Eq. (13), $E[TRC_1(T_1)]$ can be derived as follows:

$$\begin{aligned} E\left[TRC_1(T_1)\right] &= CT_1P_1 + K + C_RE\left[x\right]\left(T_1P_1\right) + M + \left[h_3(\lambda t_r)\left(t + \frac{t_r}{2}\right) + C_1\left(\lambda t_r\right)\right] + h_1\frac{T_1^2P_1^2E\left[x\right]^2}{2P_2} + b\frac{B}{2}\left(\frac{B}{\lambda} + g\right) \\ &+ h\left[\delta_1 - \frac{T_1^2P_1^2E\left[x\right]^2}{2P_2} + P_1tg + \delta_2\right] - \frac{h}{n}\left[-\delta_1 - \frac{T_1P_1E\left[x\right]\lambda g}{2P_2} - \frac{T_1^2P_1}{2} + \delta_2\right] + K_1 + nK_1 + C_T\left(T_1P_1 + \lambda g\right) \end{aligned} \tag{14}$$

where

$$\delta_1 = -\frac{T_1B}{2} + \frac{B^2}{2P_1\left(1 - E\left[x\right]\right)} + \frac{T_1^2P_1^2E\left[x\right]}{2P_2} - \frac{T_1P_1BE\left[x\right]}{2P_2}$$

$$\delta_2 = \frac{B\lambda g}{2P_1\left(1 - E\left[x\right]\right)} - \frac{T_1\lambda g}{2} + \frac{\left(T_1P_1 - B\right)^2}{2\lambda} + \frac{T_1B}{2\left(1 - E\left[x\right]\right)} - \frac{\lambda g^2}{2}$$

Situation two: time to equipment failure t falls into $[t_5', T_1]$

Situation two investigates an equipment failure taking place in between t_5' and T_1 (see Fig. 4). Similarly, the on-hand stock level of nonconforming products at time t is illustrated in Figure B-1. By observing Fig. 4, one notices that, at the time an equipment failure takes place, the on-hand stock level is H_2, and the equipment is under repair immediately. After the repair time t_r, the equipment is resumed to fabricate the remaining of the interrupted batch. Then, the stock level goes up to H_1 at the end of uptime.

$$H_2 = \left(P_1 - d_1\right)\left(t - t_5'\right) \tag{15}$$

A rework process begins to repair the nonconforming products and stock level builds up to H at the end of t_5' (Fig. 4). All available stocks are distributed to the buyer under a multi-delivery policy, with equal-quantity on each shipment in t_3' (see Fig. 4). Then, any product demand comes in during t_4', becomes shortage, and is backordered to the backlogging level ($B - \lambda t_5'$), then the next replenishment cycle starts.

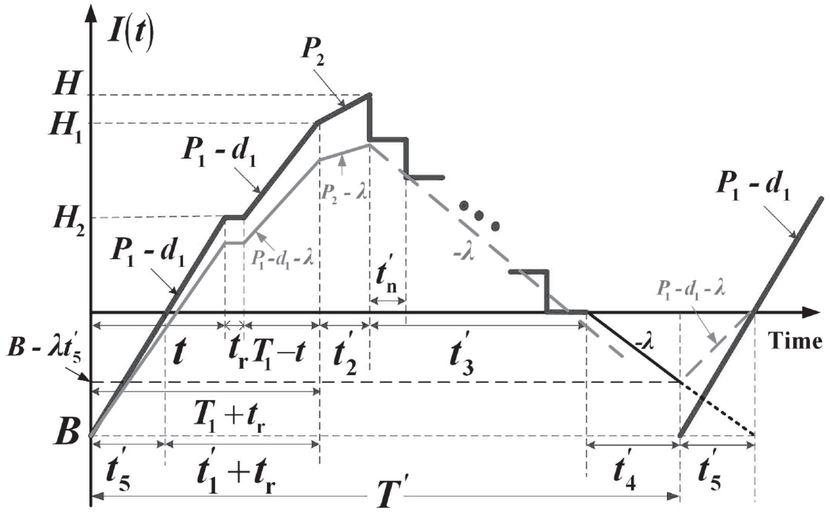

Figure 4. On-hand stock/backlog status at time t in the proposed EMQ system with an equipment failure taking place in between [t_5', T_1] as compared to that in (Chiu et al., 2017a) (in gray lines).

Aforementioned equations (1) to (11) are still valid for the situation two (see Figs. 4 and B-1) and total relevant cost per cycle in this case, $TRC_2(T_1)$, includes variable manufacturing cost in uptime T_1, manufacturing setup cost, variable reworking and holding costs in t_2', repairing cost for broken equipment, procurement and holding for safety stocks, holding cost for perfect quality stocks and nonconforming products in t_1', t_2', t_3', t_5', and t_r', shortage backordering cost in t_4' and t_5', and the fixed and variable costs for finished stocks delivered in t_3' and in the end of t_5'. Thus, $TRC_2(T_1)$ is as follows:

$$
\begin{aligned}
TRC_2(T_1) &= C(T_1P_1) + K + C_R\left[x(T_1P_1)\right] + h_1\left(\frac{P_2 t_2'}{2}\right)(t_2') + M + \left[h_3(\lambda t_r)\left(t+\frac{t_r}{2}\right) + C_1(\lambda t_r)\right] \\
&+ h\left[\frac{H_1}{2}(T_1 - t_5') + H_2 t_r + \frac{(H_1+H)}{2}(t_2') + \left(\frac{n-1}{2n}\right) H t_3' + (d_1 t)t_r + \frac{d_1 T_1}{2}(T_1) + \frac{B}{2}t_5'\right] \\
&+ b\frac{B}{2}(t_4' + t_5') + K_1 + nK_1 + C_T(T_1P_1 + \lambda t_r)
\end{aligned}
\tag{16}
$$

Similarly, using expected values of $E[x]$ to deal with random nonconforming rate and substituting equations (1) to (11), and (15) in Eq. (16), $E[TRC_2(T_1)]$ can be derived as follows:

$$
\begin{aligned}
E\left[TRC_2(T_1)\right] &= CT_1P_1 + K + C_R E[x](T_1P_1) + M + \left[h_3(\lambda t_r)\left(t+\frac{t_r}{2}\right) + C_1(\lambda t_r)\right] + h_1\frac{T_1^2P_1^2E[x]^2}{2P_2} + b\left(\frac{B^2}{2\lambda}\right) \\
&+ h\left[\delta_1 - \frac{T_1^2P_1^2E[x]^2}{2P_2} + P_1 t g + \delta_2\right] - \frac{h}{n}\left[-\delta_1 - \frac{T_1P_1E[x]\lambda g}{2P_2} - \frac{T_1^2P_1}{2} + \delta_2\right] + K_1 + nK_1 + C_T(T_1P_1 + \lambda g)
\end{aligned}
\tag{17}
$$

Situation three: time to equipment failure t is greater than T_1

Situation three explores time to equipment failure t that is greater than T_1 (Fig. 5). The on-hand stock level of nonconforming products at time t for situation three is displayed in Fig. C–1 (Appendix C), and the on-hand stock level of products in t_3 for situation three is shown in Fig. C–2 (Appendix C). The basic formulas for this situation can be observed from Figs. 5, C–1, and C–2 and are presented in Appendix C.

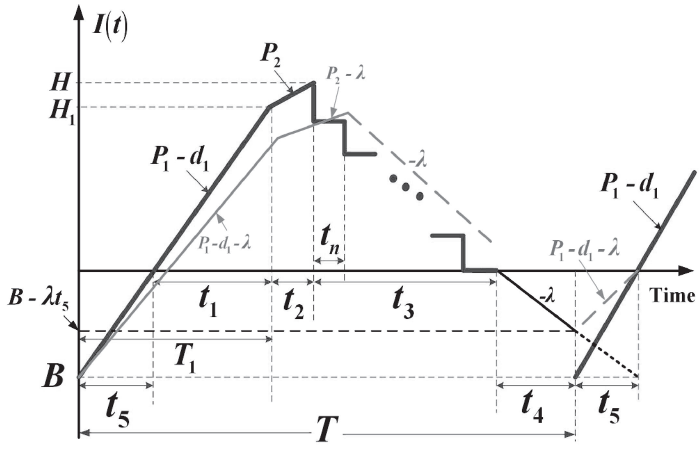

Figure 5. On-hand inventory/backlog status at time t in the proposed EMQ system with no equipment failure taking place in uptime as compared to that of [1] (in gray lines).

Total relevant cost per cycle in situation three, $TRC_3(T_1)$, comprises variable manufacturing cost in uptime T_1, manufacturing setup cost, variable reworking and holding costs in t_2, procurement and holding for safety stocks, shortage backordering cost in t_4 and t_5, holding cost for perfect quality stocks and nonconforming products in t_1, t_2, t_3, and t_5, and the fixed and variable costs for finished stocks delivered in t_3 and in the end of t_5. So, $TRC_3(T_1)$ is as follows:

$$TRC_3\left(T_1\right)=C\left(T_1P_1\right)+K+C_R\left[x\left(T_1P_1\right)\right]+h_1\left(\frac{P_2t_2}{2}\right)\left(t_2\right)+\left[h_3(\lambda t_r)T+C_1\left(\lambda t_r\right)\right]+b\frac{B}{2}\left(t_4+t_5\right)$$
$$+h\left[\frac{H_1}{2}\left(t_1\right)+\frac{\left(H_1+H\right)}{2}\left(t_2\right)+\left(\frac{n-1}{2n}\right)Ht_3+\frac{d_1T_1}{2}\left(T_1\right)+\frac{B}{2}t_5\right]+K_1+nK_1+C_T\left(T_1P_1\right) \tag{18}$$

Using expected values of $E[x]$ to deal with random nonconforming rate and substituting equations (B-1) to (B-12) in Eq. (18), $E[TRC_3(T_1)]$ can be derived as follows:

$$E\left[TRC_3(T_1)\right]=K+CT_1P_1+C_RE\left[x\right](T_1P_1)+\left[h_3(\lambda t_r)T+C_1\left(\lambda t_r\right)\right]+h_1\frac{T_1^2P_1^2E\left[x\right]^2}{2P_2}+b\left(\frac{B^2}{2\lambda}\right)$$
$$+K_1+nK_1+C_T\left(T_1P_1\right)+h\left[\delta_1-\frac{T_1^2P_1^2E\left[x\right]^2}{2P_2}+\delta_3\right]-\frac{h}{n}\left[-\delta_1-\frac{T_1^2P_1}{2}+\delta_3\right] \tag{19}$$

where

$$\delta_3=\frac{\left(T_1P_1-B\right)^2}{2\lambda}+\frac{T_1B}{2\left(1-E\left[x\right]\right)}$$

INTEGRATING SUB-MODELS & DECIDING THE OPTIMAL UPTIME

From prior literature (Chiu et al., 2006), the relationship between the maximum backlogging B and minimum acceptable service level $(1 - \alpha)$ is as follows:

$$\frac{\alpha}{1-\alpha}=\frac{t_4+t_5}{t_1+t_2+t_3}=\frac{B}{T_1P_1-B}\quad\therefore B=\alpha\left(T_1P_1\right) \tag{20}$$

In addition, for the reason that equipment failure rate follows the Poisson distribution with mean = β, the manufacturing time to failure obeys the Exponential distribution with $f(t) = \beta e^{-\beta t}$ and its cumulative density function $F(t) = (1 - e^{-\beta t})$. Therefore, the expected total relevant system cost per unit time whether or not an equipment failure takes place, $E[TRCU(T_1)]$ is as follows:

$$E\left[TRCU\left(T_1\right)\right]=\frac{\left\{\int_0^{t_5'} E\left[TRC_1\left(T_1\right)\right]f\left(t\right)dt+\int_{t_5'}^{T_1} E\left[TRC_2\left(T_1\right)\right]f\left(t\right)dt+\int_{T_1}^{\infty} E\left[TRC_3\left(T_1\right)\right]f\left(t\right)dt\right\}}{\mathrm{E}[\boldsymbol{T}]} \tag{21}$$

where

$$\mathrm{E}\left[\boldsymbol{T}\right]=\frac{T_1P_1}{\lambda} \tag{22}$$

Substitute $\mathrm{E}[TRC_1(T_1)]$, $\mathrm{E}[TRC_2(T_1)]$, $\mathrm{E}[TRC_3(T_1)]$, B, and $\mathrm{E}[\boldsymbol{T}]$ in Eq. (21), and with extra derivations, $E[TRCU(T_1)]$ can be found as follows:

$$E\left[TRCU\left(T_1\right)\right]=\left\{\begin{array}{l}\dfrac{\lambda z_1}{T_1}+\lambda T_1 z_2+\dfrac{\lambda n K_1}{T_1P_1}+\dfrac{\lambda T_1 z_3}{n}+\lambda C+\lambda C_R E\left[x\right]+\lambda C_T+\dfrac{\lambda bvg}{2P_1}\\ +\lambda h\left[-\dfrac{\lambda g}{2P_1}+\dfrac{v\lambda g}{2P_1^2(1-E\left[x\right])}\right]-\dfrac{\lambda h}{n}\left(\dfrac{v\lambda g}{2P_1^2(1-E\left[x\right])}-\dfrac{\lambda g}{2P_1}-\dfrac{\lambda g E\left[x\right]}{2P_2}\right)\\ +\lambda h_3 g+\dfrac{\lambda w_1}{T_1}+\lambda w_2 e^{-\beta T_1}+\dfrac{\lambda w_3 e^{-\beta T_1}}{T_1}+\lambda w_4 e^{-\beta T_1 s}+\dfrac{\lambda w_5}{T_1 n}+\dfrac{\lambda w_6 e^{-\beta T_1}}{n}+\dfrac{\lambda w_7 e^{-\beta T_1}}{T_1 n}\\ +\lambda h_3\left\{\dfrac{\lambda g^2}{2T_1P_1}\left(1-e^{-\beta T_1}\right)-\dfrac{\lambda g e^{-\beta T_1}}{P_1}+\dfrac{\lambda g}{T_1P_1\beta}\left(1-e^{-\beta T_1}\right)\right\}\end{array}\right\} \tag{23}$$

where s, v, w_1, w_2, w_3, w_4, w_5, w_6, w_7, z_0, z_1, z_2, and z_3 stand for the following:

$$s=\frac{v}{P_1\left(1-E\left[x\right]\right)};\ v=\alpha P_1;\ w_1=\frac{M}{P_1}+\frac{C_T\lambda g}{P_1}-\frac{h\lambda g^2}{2P_1}+\frac{hg}{\beta} \tag{24}$$

$$w_2=h\left(\frac{\lambda g}{2P_1}-\frac{v\lambda g}{2P_1^2(1-E\left[x\right])}-g\right);\ w_3=-\frac{M}{P_1}-\frac{C_T\lambda g}{P_1}+\frac{h\lambda g^2}{2P_1}-\frac{hg}{\beta} \tag{25}$$

$$w_4=-\frac{bvg}{2P_1};\ w_5=h\left(\frac{\lambda g^2}{2P_1}\right);\ w_6=-h\left(-\frac{v\lambda g}{2P_1^2(1-E\left[x\right])}+\frac{\lambda g}{2P_1}+\frac{\lambda g E\left[x\right]}{2P_2}\right) \tag{26}$$

$$w_7=-h\left(\frac{\lambda g^2}{2P_1}\right);\ z_0=\left(\frac{h_3\lambda g}{2P_1}\right)\left(g+\frac{2}{\beta}\right);\ z_1=\frac{K}{P_1}+\frac{K_1}{P_1}+\frac{C_1\lambda g}{P_1} \tag{27}$$

$$\begin{aligned}z_2=&\frac{P_1E\left[x\right]^2\left(h_1-h\right)}{2P_2}+\frac{bv^2}{2P_1\lambda}\\ &+h\left[\frac{v}{2P_1}\left(\frac{v}{\lambda}-1\right)+\frac{E\left[x\right]\left(P_1-v\right)}{2P_2}+\frac{v}{2P_1(1-E\left[x\right])}\left(1+\frac{v}{P_1}\right)+\frac{P_1}{2\lambda}-\frac{v}{\lambda}\right]\end{aligned} \tag{28}$$

$$z_3 = -h\left[-\frac{1}{2}+\frac{v}{2P_1(1-E[x])}\left(1-\frac{v}{P_1}\right)-\frac{E[x](P_1-v)}{2P_2}+\frac{v}{2P_1}\left(1+\frac{v}{\lambda}\right)+\frac{P_1}{2\lambda}-\frac{v}{\lambda}\right] \tag{29}$$

It follows that the first- and second-derivatives of $E[TRCU(T_1)]$ (with respect to decisional variable T_1) can be obtained as follows:

$$\frac{dE\left[TRCU\left(T_1\right)\right]}{dT_1} = \left\{\begin{array}{l} -\frac{\lambda z_1}{T_1^2}+\lambda z_2-\frac{\lambda nK_1}{T_1^2 P_1}+\frac{\lambda z_3}{n}-\frac{\lambda w_1}{T_1^2}-\lambda\beta w_2 e^{-\beta T_1}-\frac{\lambda\beta w_3 e^{-\beta T_1}}{T_1}-\frac{\lambda w_3 e^{-\beta T_1}}{T_1^2}-\lambda\beta s w_4 e^{-\beta T_1 s}-\frac{\lambda w_5}{T_1^2 n} \\ -\frac{\lambda\beta w_6 e^{-\beta T_1}}{n}-\frac{\lambda\beta w_7 e^{-\beta T_1}}{T_1 n}-\frac{\lambda w_7 e^{-\beta T_1}}{T_1^2 n}-\frac{\lambda z_0}{T_1^2}+\lambda\beta e^{-\beta T_1}\left(\frac{h_3\lambda g}{P_1}\right)+\lambda z_0\left(\frac{\beta e^{-\beta T_1}}{T_1}+\frac{e^{-\beta T_1}}{T_1^2}\right) \end{array}\right\} \tag{30}$$

and

$$\frac{d^2E\left[TRCU\left(T_1\right)\right]}{dT_1^2} = \left\{\begin{array}{l} \frac{2\lambda z_1}{T_1^3}+\frac{2\lambda nK_1}{T_1^3 P_1}+\frac{2\lambda w_1}{T_1^3}+\lambda\beta^2 w_2 e^{-\beta T_1}+\frac{\lambda\beta^2 w_3 e^{-\beta T_1}}{T_1}+\frac{2\lambda\beta w_3 e^{-\beta T_1}}{T_1^2}+\frac{2\lambda w_3 e^{-\beta T_1}}{T_1^3} \\ +\lambda\beta^2 s^2 w_4 e^{-\beta T_1 s}+\frac{2\lambda w_5}{T_1^3 n}+\frac{\lambda\beta^2 w_6 e^{-\beta T_1}}{n}+\frac{\lambda\beta^2 w_7 e^{-\beta T_1}}{T_1 n}+\frac{2\lambda\beta w_7 e^{-\beta T_1}}{T_1^2 n} \\ +\frac{2\lambda w_7 e^{-\beta T_1}}{T_1^3 n}+\lambda z_0-\lambda\left(\beta^2 e^{-\beta T_1}\right)\left(\frac{h_3\lambda g}{P_1}\right)-\lambda z_0\left(\frac{\beta^2 e^{-\beta T_1}}{T_1}+\frac{2\beta e^{-\beta T_1}}{T_1^2}+\frac{2e^{-\beta T_1}}{T_1^3}\right) \end{array}\right\} \tag{31}$$

From Eq. (31), it is noted that if the right-hand side term is positive, then $E[TRCU(T_1)]$ is convex. With additional derivations, we obtain that $E[TRCU(T_1)]$ is convex if Eq. (32) holds.

$$\frac{2\left(z_1+\frac{nK_1}{P_1}+w_1+w_3 e^{-\beta T_1}+\frac{w_5}{n}+\frac{w_7 e^{-\beta T_1}}{n}\right)+z_0\left(1-e^{-\beta T_1}\right)}{\left(\begin{array}{l} -T_1^2\beta^2 w_2 e^{-\beta T_1}-T_1\beta^2 w_3 e^{-\beta T_1}-2\beta w_3 e^{-\beta T_1}-T_1^2\beta^2 s^2 w_4 e^{-\beta T_1 s}-\frac{T_1^2\beta^2 w_6 e^{-\beta T_1}}{n} \\ -\frac{T_1\beta^2 w_7 e^{-\beta T_1}}{n}-\frac{2\beta w_7 e^{-\beta T_1}}{n}-T_1\beta e^{-\beta T_1}z_0-T_1^2\beta e^{-\beta T_1}\left(\frac{h_3\lambda g}{P_1}\right) \end{array}\right)} > T_1 > 0 \tag{32}$$

Once $E[TRCU(T_1)]$ can be confirmed as convex, the optimal T_1* can be found by letting the first-derivative of $E[TRCU(T_1)] = 0$. That is,

$$\left\{\begin{array}{l} -\frac{\lambda z_1}{T_1^2}+\lambda z_2-\frac{\lambda nK_1}{T_1^2 P_1}+\frac{\lambda z_3}{n}-\frac{\lambda w_1}{T_1^2}-\lambda\beta w_2 e^{-\beta T_1}-\frac{\lambda\beta w_3 e^{-\beta T_1}}{T_1}-\frac{\lambda w_3 e^{-\beta T_1}}{T_1^2}-\lambda\beta s w_4 e^{-\beta T_1 s}-\frac{\lambda w_5}{T_1^2 n} \\ -\frac{\lambda\beta w_6 e^{-\beta T_1}}{n}-\frac{\lambda\beta w_7 e^{-\beta T_1}}{T_1 n}-\frac{\lambda w_7 e^{-\beta T_1}}{T_1^2 n}-\frac{\lambda z_0}{T_1^2}+\lambda\beta e^{-\beta T_1}\left(\frac{h_3\lambda g}{P_1}\right)+\lambda z_0\left(\frac{\beta e^{-\beta T_1}}{T_1}+\frac{e^{-\beta T_1}}{T_1^2}\right) \end{array}\right\} = 0 \tag{33}$$

With further rearrangements from Eq. (33), one has the following:

$$e^{-\beta T_1} = \frac{\left[-\frac{z_1}{T_1^2}+z_2-\frac{nK_1}{T_1^2 P_1}+\frac{z_3}{n}-\frac{w_1}{T_1^2}-\beta s w_4 e^{-\beta T_1 s}-\frac{w_5}{T_1^2 n}-\frac{z_0}{T_1^2}\right]}{\left[\beta w_2+\frac{\beta w_3}{T_1}+\frac{w_3}{T_1^2}+\frac{\beta w_6}{n}+\frac{\beta w_7}{T_1 n}+\frac{w_7}{T_1^2 n}-\beta\left(\frac{h_3\lambda g}{P_1}\right)-z_0\left(\frac{\beta}{T_1}+\frac{1}{T_1^2}\right)\right]} \tag{34}$$

Equation (33) can also be rearranged as follows:

$$q_2 T_1^2 + q_1 T_1 + q_0 = 0 \tag{35}$$

where q_0, q_1, and q_2 refer to the following:

$$q_0 = -z_1 - \frac{nK_1}{P_1} - w_1 - w_3 e^{-\beta T_1} - \frac{w_5}{n} - \frac{w_7 e^{-\beta T_1}}{n} - z_0\left(1 - e^{-\beta T_1}\right) \tag{36}$$

$$q_1 = -\beta w_3 e^{-\beta T_1} - \frac{\beta w_7 e^{-\beta T_1}}{n} + \beta e^{-\beta T_1} z_0 \tag{37}$$

$$q_2 = z_2 + \frac{z_3}{n} - \beta w_2 e^{-\beta T_1} - \beta s w_4 e^{-\beta T_1 s} - \frac{\beta w_6 e^{-\beta T_1}}{n} + \beta e^{-\beta T_1}\left(\frac{h_3 \lambda g}{P_1}\right) \tag{38}$$

By applying the square root solution to Eq. (35), one obtains T_1* as follows:

$$T_1^* = \frac{-q_1 \pm \sqrt{q_1^2 - 4q_2 q_0}}{2q_2} \tag{39}$$

The proposed algorithm for searching T_1*

As $F(T_1) = (1 - e^{-\beta T_1})$ is the cumulative density function of Exponential distributed mean time to failure, which falls within an interval of [0, 1], its complement $e^{-\beta T_1}$ also falls in a range of [0, 1]. The proposed algorithm for finding T_1* includes the following steps:

Step 1: We start with letting $e^{-\beta T_1} = 0$ and $e^{-\beta T_1} = 1$. Apply Eq. (39) with these initial values of $e^{-\beta T_1}$ to find the starting values of upper bound T_{1U} and lower bound T_{1L} for cycle time T_1.

Step 2: Compute $e^{-\beta T_1}$ using the current values of T_{1U} and T_{1L} to update the values for $e^{-\beta T_{1U}}$ and $e^{-\beta T_{1L}}$.

Step 3: Apply Eq. (39) with the current values of $e^{-\beta T_{1U}}$ and $e^{-\beta T_{1L}}$ to obtain a new set of upper and lower bounds (i.e., T_{1U} and T_{1L}) for T_1. Check the difference between current values of T_{1U} and T_{1L}: If the difference is insignificant, then go to Step 4; otherwise, go to Step 2.

Step 4: Stop. T_1^* is found (i.e., $T_1^* = T_{1U} = T_{1L}$).

NUMERICAL EXAMPLE

Consider the following values of parameters for an EMQ-based system (as assumed in Chiu et al., (2017a)): $\lambda = 4{,}000$, $P_1 = 10{,}000$, $K = \$450$, $C = \$2$, $h = \$0.8$, $x \in [0, 0.2]$, $\beta = 0.5$, $M = \$500$, $t_r = g = 0.018$, $(1 - \alpha)\% = 80\%$, $b = \$0.1$, $P_2 = 5{,}000$, $C_R = \$0.5$, $h_1 = \$0.8$, $C_1 = \$2$, $h_3 = \$0.6$, $C_T = \$0.01$, $n = 4$, and $K_1 = \$100$.

At $\beta = 0.5$, we first find $T_{1U} = 0.5304 < 3.448$ and $T_{1L} = 0.3747 < 3.131$ (where T_{1U} and T_{1L} are obtained from the Step 1 of the proposed algorithm, and we apply Eq. (32) with these values of T_{1U} and T_{1L}) to confirm the convexity of E[$TRCU(T_1)$]. To demonstrate that our model can be applied to a wider range of average number of equipment breakdowns (other than $\beta = 0.5$), additional tests for convexity of system cost function have been conducted and results are exhibited in Table D-1 in Appendix D.

Then, applying the aforementioned algorithm for deriving, on the 5th step the results of $e^{-\beta T_{1U}} = e^{-\beta T_{1L}} = 0.8165$, $T_{1U} = T_{1L} = 0.4054$, and E[$TRCU(T_{1U})$] = E[$TRCU(T_{1L})$] = \$10,519.16 are obtained (see Table 1). So, the optimal manufacturing cycle length T_1* is 0.4054 and optimal expected system cost per unit time E[$TRCU(T_1$*)] is \$10,519.16. The effect of different manufacturing cycle length T_1 on E[$TRCU(T_1)$] is depicted in Fig. 6.

Table 1. Analytical results from applying the algorithm for searching T_1* at $\beta = 0.5$.

Step	$e^{-\beta T_{1U}}$	T_{1U}	E[$TRCU(T_{1U})$]	$e^{-\beta T_{1L}}$	T_{1L}*	E[$TRCU(T_{1L})$]
1st	0	0.5304	$10,590.41	1	0.3747	$10,527.61
2nd	0.7671	0.4134	$10,519.12	0.8292	0.4033	$10,519.31
3rd	0.8132	0.4059	$10,519.14	0.8174	0.4052	$10,519.17
4th	0.8163	0.4054	$10,519.16	0.8166	0.4054	$10,519.17
5th	0.8165	0.4054	$10,519.16	0.8165	0.4054	$10,519.16

Comparisons of the effects of different T_1 on E[$TRCU(T_1)$] in this study (in green) and in the prior study (Chiu et al., 2017a) (in gray) are demonstrated in Fig. 7. It is noted that, due to the inclusion of fixed delivery cost in system cost of the present study, our E[$TRCU(T_1* = 0.4054)$] is greater than E[$TRCU(T_1* = 0.3858$] in prior study (Chiu et al., 2017a).

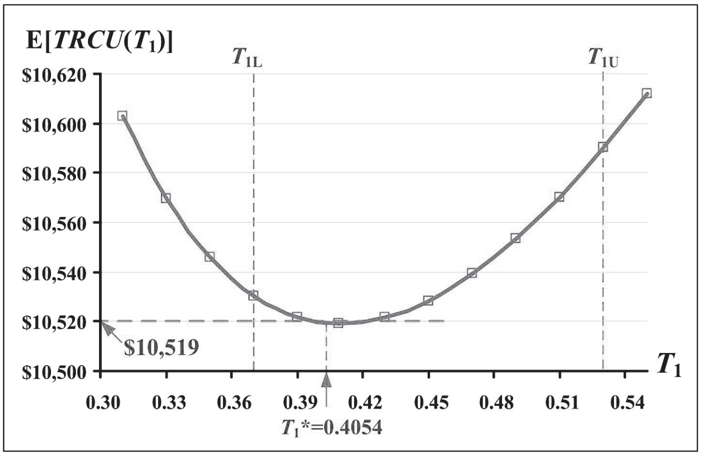

Figure 6. The effect of different manufacturing cycle length T_1 on E[$TRCU(T_1)$].

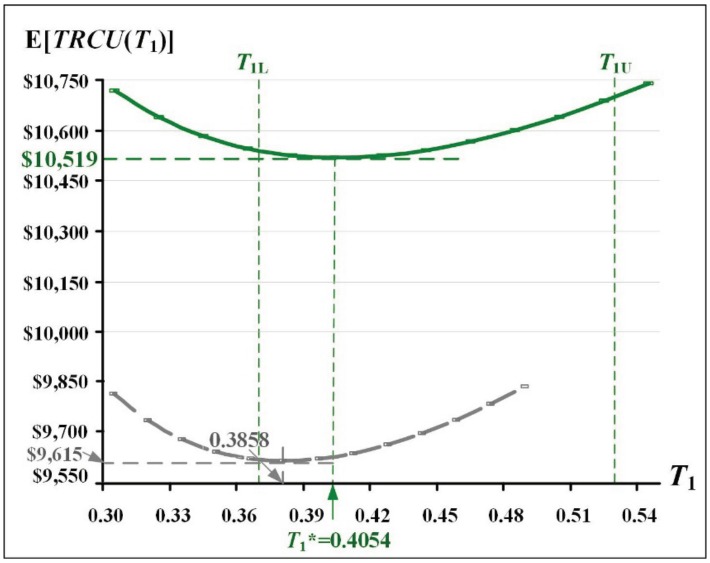

Figure 7. Comparison of the effects of different T_1 on E[$TRCU(T_1)$] in this study (in green) and that in a prior study (Chiu et al., 2017a) (in gray dash line).

Detailed analysis (Fig. 8) exposes the main differences in their cost components (i.e., stock holding and fixed shipping cost).

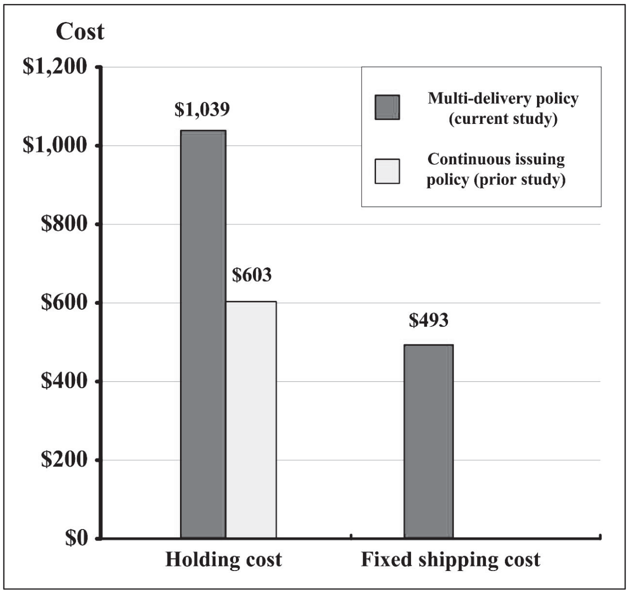

Figure 8. Main differences in cost components of the present study as compared to that in the prior study (Chiu et al., 2017).

Because we adopt the multi-delivery policy for end products, its impacts on the proposed system have been explored, and the results are illustrated in Figs. 9 and 10. Fig. 9 shows as n goes up and above two per cycle, the optimal cycle length T_1* becomes longer accordingly, and from Fig. 10 the analytical result indicates that as n raises, E[$TRCU(T_1)$] increases significantly.

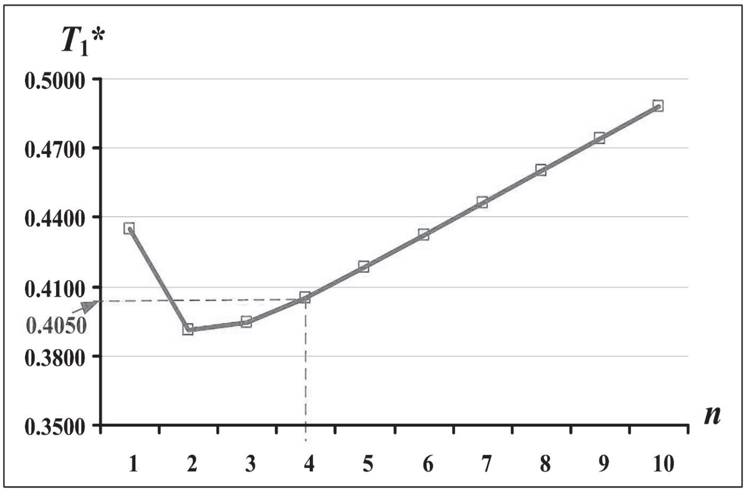

Figure 9. The impact of number of deliveries per cycle on T_1*.

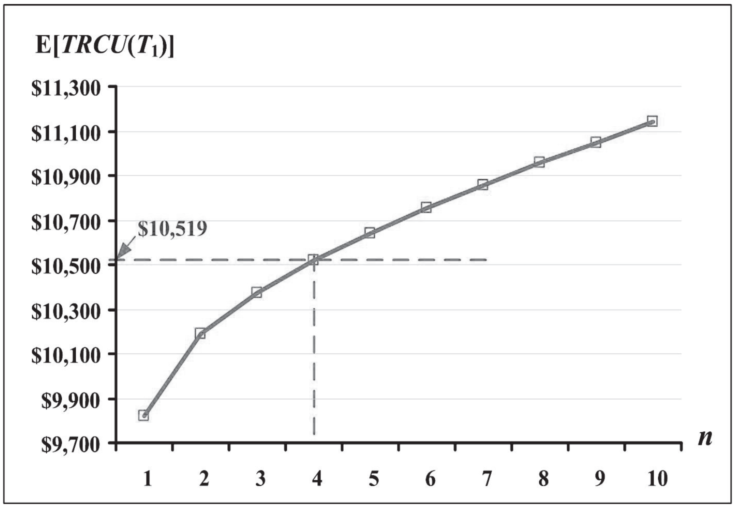

Figure 10. The effect of variations in n on E[$TRCU(T_1)$].

In order to meet basic customer satisfaction, we have set a minimum service level as 80% (i.e., $(1 - \alpha)$). Through a further analysis, the impact of diverse values of service level on E[$TRCU(T_1)$] is depicted in Fig. 11. It points out that as $(1 - \alpha)$ is set higher, E[$TRCU(T_1)$] raises significantly.

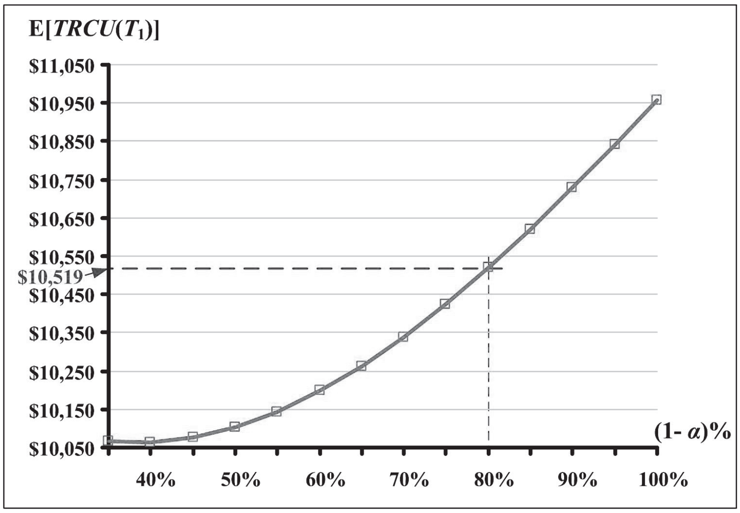

Figure 11. The impact of various service level percentage $(1 - \alpha)$ on E[$TRCU(T_1)$].

Inevitable machine failure is assumed in this study, and an extra investigation of the effect of diverse values in mean-time-to-failure $1/\beta$ on E[$TRCU(T_1)$] is shown in Fig. 12. At $\beta = 0.5$ per year or $1/\beta = 2$ years, E[$TRCU(T_1)$] = \$10,519 (i.e., the optimal value of our example); and as $1/\beta$ increases to 100 (or ∞), our proposed model turns into a model with no breakdown occurrence.

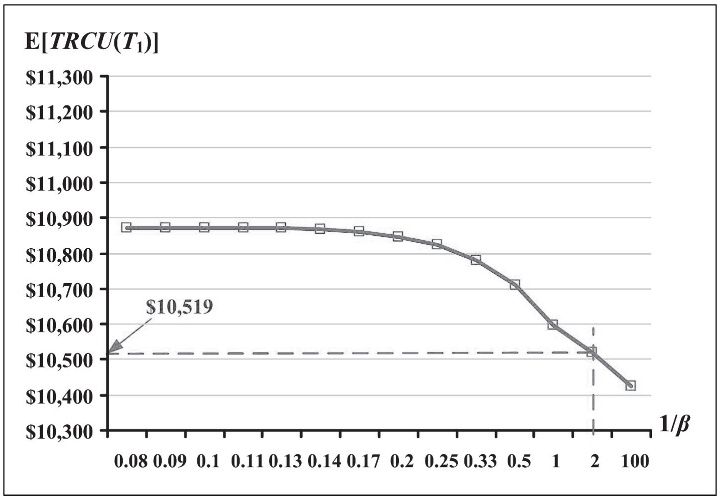

Figure 12. The effect of diverse values in mean-time-to-failure $1/\beta$ on $E[TRCU(T_1)]$.

Moreover, diverse combined effects of the aforementioned system factors on $E[TRCU(T_1)]$ can also be explored. For example, combined effects of variations in mean-time-to-failure $1/\beta$ and service level percentages $(1-\alpha)\%$ on $E[TRCU(T_1)]$ can be investigated as illustrated in Fig. 13. It can be seen that as $1/\beta$ goes up, $E[TRCU(T_1)]$ declines due to the less chance of having a machine failure, and as $(1-\alpha)\%$ moves higher, $E[TRCU(T_1)]$ increases notably.

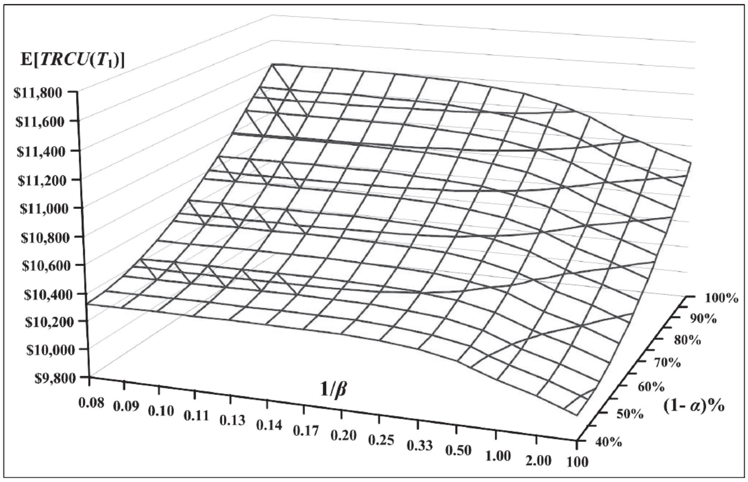

Figure 13. Combined effects of variations in mean-time-to-failure $1/\beta$ and service level percentages $(1-\alpha)\%$ on $E[TRCU(T_1)]$.

Finally, additional investigation reveals the effects of various service level percentages on system holding and backlogging quantities and their related costs, as well as $E[TRCU(T_1^*)]$, respectively (see Table 2).

Table 2. Effects of various service level percentages on diverse system parameters/costs.

$(1 - \alpha)\%$	H	B	*Holding cost*	*Backordering Cost*	$E[TRCU(T_1^*)]$
100%	3439	0	$1,268	$0	$10,959
90%	3398	370	$1,150	$2	$10,728
80%	3315	811	$1,039	$8	$10,519
70%	3168	1327	$936	$20	$10,339
60%	2931	1906	$846	$38	$10,198
50%	2584	2521	$774	$63	$10,103
39%	2128	3084	$724	$93	$10,063

CONCLUSIONS

This study explores an EMQ model with machine malfunction, allowable backlogging level, repairable defective items, and discontinuous issuing policy. Different from a prior work (Chiu et al., 2017a), a discontinuous multi-shipment stock issuing policy is adopted in this study. A complete procedure for problem-solving is presented, which consists of problem modeling, formulation and integration of each separate situation, determination of convexity of the system cost function, algorithm proposal for locating the optimal cycle length, and demonstration of the applicability of the research result. This study not only provides a way for production professionals to resolve this specific problem (see Sections 2 and 3), but also reveals in-depth system information for managerial decision making (Figs. 6-13 and Tables 1-2). Incorporation of stochastic demand rate into the present model could be an interesting direction for future study.

ACKNOWLEDGMENT

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APPENDIX A

Definition of notation is as follows:

α = proportion of permissible stock-out time in a cycle,

β = the average number of equipment breakdown in a year, which is a Poisson distributed variable,

x = random defective rate in uptime,

B = maximum allowable level of backordering,

b = unit backordering cost,

M = a fixed repair cost to fix the failure machine,

t = random time to an equipment failure,

t_1' = part of uptime that produces positive stocks,

t_2' = reworking time,

t_3' = end-item delivery time,

t_4' = shortage permitted time prior to the launch of next uptime,

t_5' = part of uptime that produces sufficient stocks to satisfy maximum backlogging B,

t_n' = fixed interval of time (i.e., t_3'/n) between two consecutive shipments in t_3',

t_r = repair time per equipment failure,

T_1 = the uptime – a decision variable of the proposed EMQ system,

λ = annual product demand rate,

Q = lot size in a cycle,

P_1 = annual manufacturing rate,

P_2 = annual reworking rate,

T' = manufacturing cycle time in the case that a breakdown occurs,

H = stock level in the end of the rework process,

H_0 = backordering level at the time a breakdown occurs,

H_1 = stock level when uptime ends,

H_2 = stock level at the time an equipment failure takes place,

K = setup cost,

K_1 = fixed delivery cost per shipment,

C = manufacturing cost per product,

C_R = unit cost for reworked item,

h = unit holding cost,

h_1 = unit holding cost for reworked item,

h_3 = unit holding cost for safety stock,

C_1 = unit procurement cost for safety stock,

n = number of deliveries per cycle,

C_T = unit delivery cost,

g = t_r, time needed to fix a breakdown,

d_1 = production rate of defective items in T_1,

$I(t)$ = on-hand stock/backlog status at time t,

$I_d(t)$ = on-hand level of defective items at time t,

t_1 = part of uptime that produces positive stocks, in an EMQ-based system with no equipment failures,

t_2 = reworking time in an EMQ-based system with no equipment failures,

t_3 = end items delivery time in an EMQ-based system with no equipment failures,

t_4 = backlogging permissible time prior to the launch of next uptime, in an EMQ-based system with no equipment failures,

t_5 = part of uptime that produces sufficient stocks to satisfy maximum backlogging B, in an EMQ system with no equipment failures,

t_n = fixed time interval (i.e., t_3/n) between two consecutive shipments in t_3,

T = cycle time in an EMQ-based system with no equipment failures,

$\boldsymbol{T}$ = cycle time of the EMQ system (whether an equipment failure taking place or not),

$TRC_1(T_1)$ = total relevant cost per cycle in the case that a breakdown occurs during t_5',

$TRC_2(T_1)$ = total relevant cost per cycle in the case that a breakdown occurs in $[t_5', T_1]$,

$TRC_3(T_1)$ = total relevant cost per cycle in the case that breakdown does not occur,

$E[TRC_1(T_1)]$ = expected total relevant cost per cycle in the case that a breakdown occurs during t_5',

$E[TRC_2(T_1)]$ = expected total relevant cost per cycle in the case that a breakdown occurs in $[t_5', T_1]$,

$E[TRC_3(T_1)]$ = expected total relevant cost per cycle in the case that breakdown does not occur,

$TRCU(T_1)$ = total system relevant cost per unit time whether a breakdown occurs or not,

$E[TRCU(T_1)]$ = expected total system relevant cost per unit time whether a breakdown occurs or not.

APPENDIX B

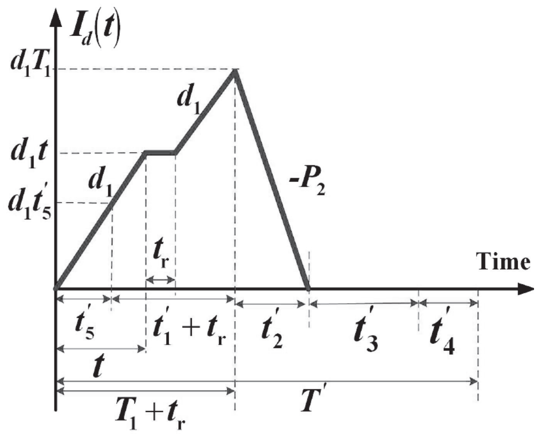

Figure B–1. On-hand stock level of nonconforming products in the proposed EMQ system with an equipment failure taking place in between $[t_5', T_1]$.

APPENDIX C

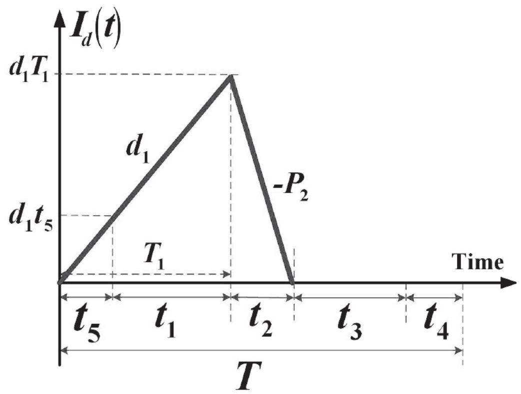

Figure C–1. On-hand inventory level of nonconforming products in the proposed EMQ system with no equipment failure taking place in uptime.

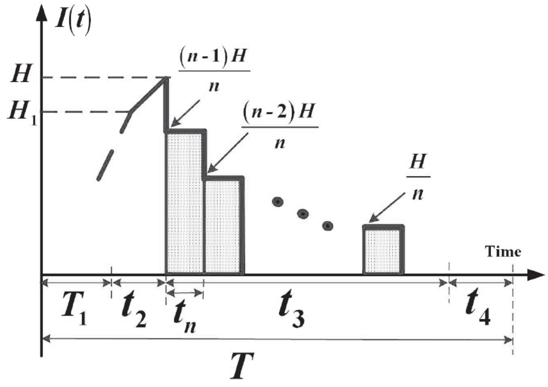

Figure C–2. On-hand stock level of products in t_3 in the proposed EMQ system with no equipment failure taking place in uptime.

By observing Figures 5, C–1, and C–2, we can obtain the following basic equations for situation three of the proposed EMQ-based system with no equipment failure taking place in uptime:

$$B=\left(P_1-d_1\right)t_5 \tag{C–1}$$

$$t_5=\frac{B}{\left(P_1-d_1\right)} \tag{C–2}$$

$$t_1=\frac{H_1}{\left(P_1-d_1\right)} \tag{C–3}$$

$$H_1=\left(P_1-d_1\right)t_1 \tag{C–4}$$

$$T_1=\left(t_5+t_1\right)=\frac{Q}{P_1} \tag{C–5}$$

$$H=H_1+P_2t_2 \tag{C–6}$$

$$t_2=\frac{H-H_1}{P_2}=\frac{d_1T_1}{P_2} \tag{C–7}$$

$$t_4=\frac{B-\lambda t_5}{\lambda} \tag{C–8}$$

$$T=\sum_{i=1}^{5}t_i \tag{C–9}$$

$$t_3=T-\left(t_1+t_2+t_4+t_5\right) \tag{C–10}$$

$$t_n=\frac{t_3}{n} \tag{C–11}$$

Total stocks in delivery time t_3 (Chiu et al., 2015) are as follows:

$$\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1}i\right)Ht_3=\left(\frac{1}{n^2}\right)\left[\frac{n(n-1)}{2}\right]Ht_3=\left(\frac{n-1}{2n}\right)Ht_3 \tag{C–12}$$

APPENDIX D

Additional test results for convexity of E[$TRCU(T_1)$] with diverse values of β are displayed in Table D-1. The results show that Eq. (32) holds for a wider range of breakdown rate βs, this implies that our proposed model is appropriate for solving most real production systems that have diverse numbers of equipment breakdowns.

Table D-1. Additional test results for convexity of E[$TRCU(T_1)$] with diverse values of β

β	T_{1U}	Outcome of applying T_{1U} to Eq.(32)	T_{1L}	Outcome of applying T_{1L} to Eq.(32)
10	0.4899	10.198	0.1667	0.756
9	0.4901	7.488	0.1785	0.798
8	0.4904	5.592	0.1918	0.845
7	0.4907	4.265	0.2069	0.898
6	0.4912	3.337	0.2240	0.960
5	0.4919	2.698	0.2437	1.035
4	0.4929	2.280	0.2661	1.134
3	0.4947	2.051	0.2920	1.278
2	0.4983	2.043	0.3217	1.535
1	0.5092	2.521	0.3558	2.181
0.5	0.5304	3.448	0.3747	3.131
0.01	1.5619	10.936	0.3945	7.317

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نظام يرتكز على EMQ ويراعي أعطال الماكينة ومستوى التراكم المسموح به والعناصر القابلة للإصلاح وسياسة الإصدار المتقطعة

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الخلاصة

تم استكشاف نظام يرتكز على كمية التصنيع الاقتصادية (EMQ) ويراعي أعطال الماكينة ومستوى التراكم المسموح به والعناصر غير المطابقة القابلة للإصلاح وسياسة الإصدار المتقطعة. وخلافاً عن الأبحاث السابقة (Chiu et al., 2017a)، تم اعتماد سياسة إصدار أسهم متعددة الشحنات متقطعة في هذه الدراسة. ليس فقط بهدف معالجة الحالات الضرورية لأعطال الماكينة وجودة المنتج المعيبة، ولكن أيضاً لمعالجة المسائل العملية المتعلقة بالتراكم المسموح به وإصدار الأسهم المتعددة التسليم. تم تقديم إجراءات كاملة لحل المشكلات، والتي تتكون من نمذجة المشكلة وصياغتها وتكاملها لكل موقف منفصل، وتحديد مدى تحدب دالة تكلفة النظام، واقتراح خوارزمية لتحديد طول الدورة المثلى، وإظهار قابلية تطبيق نتائج البحث. بخلاف تقديم طريقة للعاملين في مجال الإنتاج لحل مثل هذه المشكلة المحددة، تكشف هذه الدراسة أيضاً عن معلومات متنوعة ومتعمقة لنظام اتخاذ القرارات الإدارية.