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استقرار البندول المقلوب س– ص باستخدام متحكمات المكاسب المجدولة

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الخلاصـة

البندول المقلوب س–ص ويعرف أيضا بالكروي أو البندول المقلوب ذات بعدين يحتوي على قضيب أسطواني رقيق مثبت من القاعدة على موصل عام. الهدف من التحكم هو وضع البندول في وضع مستقيم مع الحفاظ على القاعدة على مسار الإشارة المعرفة. ويحقق هذا البحث من متحكمات المكاسب المجدولة في تصميم وحدات التحكم (PID) لاستخدامها لتحقيق الاستقرار للبندول المقلوب س– ص. والاختلافات في مكاسب (PID) تعتمد على الحالة المتغيرة والمستقرة مع الزمن للاستجابة. وقد تم مقارنة أداء التحكم المقترحة مع نظام (PID) التقليدية التي وردت في البحوث السابقة. كما تم دراسة فعالية التحكم المقترح تحت تأثير اضطراب والضوضاء والاحتكاك في نظام البندول المقلوب. وتبين نتائج المحاكاة أن وحدات تحكم المقترحة توفر أداء أفضل من وحدات التحكم (PID) التقليدية من مختلف نواحي مواصفات الأداء.

Stabilization of an X - Y inverted pendulum using adaptive gain scheduling PID controllers

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ABSTRACT

An $X-Y$ inverted pendulum, also known as a spherical or a two-dimensional inverted pendulum consists of a thin cylindrical rod attached to a base through a universal joint. The control objective is to place the pendulum in the upright position while keeping the base at some desired reference trajectory. This paper presents an adaptive gain scheduling method in designing PID controllers for the stabilization of an $X-Y$ inverted pendulum. The variations in PID gains depend upon the transient and the steady-state part of the response. The performance of the proposed scheme has been compared with the conventional PID scheme given in the literature. The effectiveness of the proposed scheme under the effect of disturbance, noise and friction in the inverted pendulum system has also been studied. Simulation results show that the proposed controllers provide better performance than the conventional PID controllers in terms of various performance specifications.

Keywords: Adaptive gain scheduling PID controller; disturbance; friction; PID controller; $X-Y$ inverted pendulum

INTRODUCTION

Inverted pendulums are under-actuated, nonlinear and non-minimum phase systems with lesser control inputs than the degrees of freedom (Astrom & Furuta, 1999). Designing a controller for such a system resembles the control of many applications such as robots, sage ways and space rockets. Due to the effect of gravity, stabilizing an inverted pendulum is a challenging task.

Inverted pendulums discussed within the literature are X inverted pendulum also known as the traditional inverted pendulum (Kumar & Jerome, 2013), $X-Y$ inverted pendulum or spherical inverted pendulum (Liu *et al.*, 2008) and $X-Z$ inverted pendulum (Maravall, 2004). The X inverted pendulum can move in x horizontal direction by a horizontal force, while the $X-Y$ inverted pendulum can move in xy

horizontal plane by two horizontal forces, and an $X - Z$ inverted pendulum can move in xz horizontal and vertical plane with the help of horizontal and vertical forces. Due to the effect of gravitational force, controlling an inverted pendulum is a difficult task. The control of an inverted pendulum has been divided into three parts, i.e. the swing up control (Muskinja & Tovornik, 2006; Mason *et al*., 2008), the stabilization control (Bloch *et al*., 2000; Muskinja & Tovornik, 2006) and the tracking control (Chang & Lee, 2007). The most widely used controller for controlling a number of systems is the PID controller due to its simple structure, easy implementation and lesser number of tuning parameters (Astrom & Hagglund, 1995). The stabilization and tracking control of X, $X - Y$ and $X - Z$ inverted pendulums using PID controllers have been discussed by Wang (2011). A two-loop PID controller was designed by Nasir (2007) for an inverted cart–pendulum system. PID controllers designed via pole placement technique have been used for stabilization of the inverted cart pendulum system (Ghosh *et al*., 2012). Sliding mode control strategy used in the stabilization of X, $X - Y$ and $X - Z$ inverted pendulums has been discussed by Wai & Chang (2006); Tsai *et al*., (2011) and Wang (2012) respectively. Other methods such as the linear quadratic regulator (Prasad *et al.* (2011), Lyapunov approach (Ibanez *et al*., 2006) and continuous state feedback (Angeli, 2001) approach has also been implemented for controlling an inverted pendulum. Intelligent control of inverted pendulum using an interval type 2 fuzzy PID controller has been proposed by Nagar *et al.,* (2014). The hybrid control for global stabilization of the cart-pendulum system has been presented by Zhao & Spoong (2001). However, only a little or no efforts have been made in designing PID controllers based on gain scheduling method for an ; inverted pendulum system. Moreover, the effect of friction and noise on the performance of $X - Y$ inverted pendulum has not been discussed throughout the literature. In this paper the stabilization of $X - Y$ inverted pendulum using adaptive gain scheduling PID controllers has been presented under the effect of uncertainties such as external disturbance, noise and friction.

The paper is organized as follows: Firstly, the state space modeling equations of the $X - Y$ inverted pendulum system have been presented. Secondly, the state equations in the presence of disturbance and friction have been obtained. Thereafter, the designing of adaptive gain scheduling PID controller is explained and finally the simulation results under different conditions are presented followed by the conclusion of the paper and references.

STATE SPACE MODELING OF $X - Y$ INVERTED PENDULUM SYSTEM

An $X - Y$ inverted pendulum mounted upon a base has been shown in Figure 1. The stabilization of the inverted pendulum in the upright position depends upon the horizontal displacement of the base, which in turn depends upon the applied forces F_x and F_y in the xy plane.

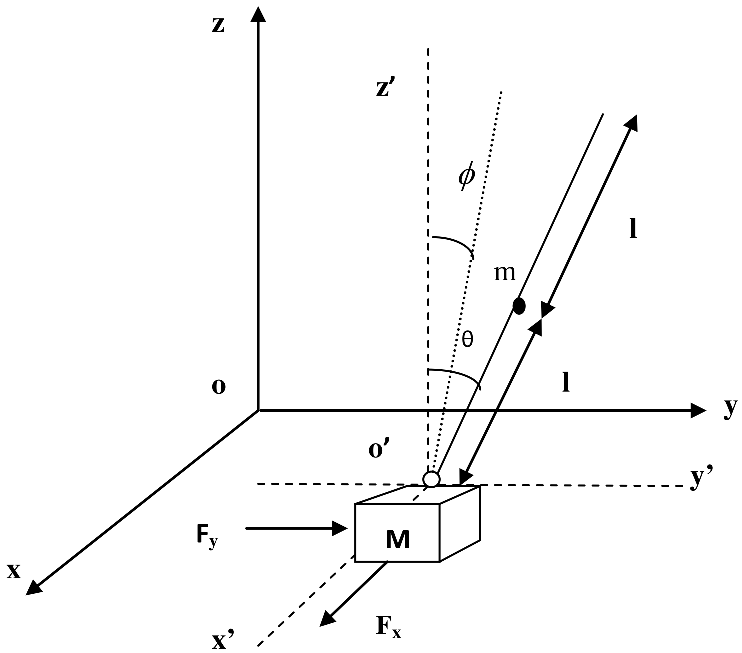


Fig. 1. *X – Y* inverted pendulum system

As shown in Figure 1, *x* and *y* are the positions of the base from the reference in the *xy* plane. *l* is the distance from the base to the mass center of the pendulum. M and m are the mass of the base and the pendulum respectively. θ and ϕ are the angles made by the inverted pendulum with the vertical axis, when the base moves in *x* and *y* directions respectively.

The parameters of the inverted pendulum system are given in Table 1

Table 1. Inverted pendulum system parameters (Wang, 2011)

M(kg)	m(kg)	*l* (m)	g(m/s²)
1	0.1	0.3	9.8

Defining the states as $x_1 = x, x_2 = \dot{x}, x_3 = y, x_4 = \dot{y}, x_5 = \theta, x_6 = \dot{\theta}, x_7 = \phi$ and $x_8 = \dot{\phi}$.

The state space model of the $X - Y$ inverted pendulum system as proposed by Wang (2011) is given by:

$$\dot{x}_1 = x_2 \tag{1}$$

$$\dot{x}_2 = \frac{\begin{array}{c} Mml\sin x_5 x_6^2 + Mml\cos^2 x_5 \sin x_5 x_8^2 - Mmg\cos^2 x_5 \sin x_5 \cos^2 x_7 + \\ m\cos^2 x_5 \sin^2 x_7 F_x + MF_x - m\cos^2 x_5 \sin x_5 \sin x_7 F_y \end{array}}{M^2 + Mm\sin^2 x_5 \cos^2 x_7 + Mm\sin^2 x_7} \tag{2}$$

$$\dot{x}_3 = x_4 \tag{3}$$

$$\dot{x}_4 = \frac{(M+m)F_y - m\cos^2 x_5 F_y + m\sin x_5 \cos x_5 \sin x_7 F_x - Mmg\cos^2 x_5 \sin x_7 \cos x_7 + Mml\cos^3 x_5 \sin x_7 x_8^2 - Mml\cos x_5 \sin x_7 x_6^2}{M^2 + Mm\sin^2 x_5 \cos^2 x_7 + Mm\sin^2 x_7} \tag{4}$$

$$\dot{x}_5 = x_6 \tag{5}$$

$$\dot{x}_6 = \frac{m\cos x_5 \cos^2 x_7 F_x - (M+m)\cos x_5 F_x + (M+m)\sin x_5 \sin x_7 F_y + M(m+M)g\sin x_5 \cos x_7 - Mml\sin x_5 \cos x_5 \cos^2 x_7 x_6^2 - M(m+M)l\cos x_5 \sin x_7 x_8^2}{(M^2 + Mm\sin^2 x_5 \cos^2 x_7 + Mm\sin^2 x_7)l} \tag{6}$$

$$\dot{x}_7 = x_8 \tag{7}$$

$$\dot{x}_8 = \frac{-m\sin^2 x_5 \cos x_7 F_y - M\cos x_7 F_y + m\sin x_5 \cos x_5 \sin x_7 \cos x_7 F_x + 2Mml\sin x_5 x_6 x_8 + M(m+M)g\sin x_7 + 2M^2 l\sin x_5 x_6 x_8 - 2Mml\sin x_5 \cos^2 x_5 \cos^2 x_7 x_6 x_8 - Mml\cos^3 x_5 \sin x_7 \cos x_7 x_8^2 - Mml\cos x_5 \sin x_7 \cos x_7 x_6^2}{M^2 + Mm\sin^2 x_5 \cos^2 x_7 + Mm\sin^2 x_7} \tag{8}$$

From Equation (1) to Equation (8), it can be concluded that the $X - Y$ inverted pendulum is a two input four output system. The two inputs are the applied forces F_x and F_y while the outputs are the positions x , y and the angles θ, ϕ.

STATE SPACE MODELING OF $X-Y$ INVERTED PENDULUM SYSTEM WITH DISTURBANCE AND FRICTION

The state equations of the $X - Y$ inverted pendulum after considering the effect of external disturbance (d) and friction (F_{xfric} and F_{yfric}) can be written as:

$$\dot{x}_1 = x_2 \tag{9}$$

$$\dot{x}_2 = \frac{Mml\sin x_5 x_6^2 + Mml\cos^2 x_5 \sin x_5 x_8^2 - Mmg\cos^2 x_5 \sin x_5 \cos^2 x_7 + m\cos^2 x_5 \sin^2 x_7 (F_x + F_{xfric}) + M(F_x + F_{xfric}) - m\cos^2 x_5 \sin x_5 \sin x_7 (F_y + F_{yfric})}{M^2 + Mm\sin^2 x_5 \cos^2 x_7 + Mm\sin^2 x_7} + d \tag{10}$$

$$\dot{x}_3 = x_4 \tag{11}$$

$$\dot{x}_4 = \frac{\begin{array}{l}(M+m)(F_y+F_{yfric})-m\cos^2 x_5(F_y+F_{yfric})+m\sin x_5\cos x_5\sin x_7(F_x+F_{xfric})-\\ Mmg\cos^2 x_5\sin x_7\cos x_7+Mml\cos^3 x_5\sin x_7 x_8^2-Mml\cos x_5\sin x_7 x_6^2\end{array}}{M^2+Mm\sin^2 x_5\cos^2 x_7+Mm\sin^2 x_7}+d \tag{12}$$

$$\dot{x}_5 = x_6 \tag{13}$$

$$\dot{x}_6 = \frac{\begin{array}{l}m\cos x_5\cos^2 x_7(F_x+F_{xfric})-(M+m)\cos x_5(F_x+F_{xfric})+(M+m)\sin x_5\sin x_7(F_y+F_{yfric})+\\ M(m+M)g\sin x_5\cos x_7-Mml\sin x_5\cos x_5\cos^2 x_7 x_6^2-M(m+M)l\cos x_5\sin x_7 x_8^2\end{array}}{(M^2+Mm\sin^2 x_5\cos^2 x_7+Mm\sin^2 x_7)l}+d \tag{14}$$

$$\dot{x}_7 = x_8 \tag{15}$$

$$\dot{x}_8 = \frac{\begin{array}{l}-m\sin^2 x_5\cos x_7(F_y+F_{yfric})-M\cos x_7(F_y+F_{yfric})+m\sin x_5\cos x_5\sin x_7\cos x_7(F_x+F_{xfric})\\ +2Mml\sin x_5 x_6 x_8+M(m+M)g\sin x_7+2M^2 l\sin x_5 x_6 x_8-\\ 2Mml\sin x_5\cos^2 x_5\cos^2 x_7 x_6 x_8-Mml\cos^3 x_5\sin x_7\cos x_7 x_8^2-Mml\cos x_5\sin x_7\cos x_7 x_6^2\end{array}}{M^2+Mm\sin^2 x_5\cos^2 x_7+Mm\sin^2 x_7}+d \tag{16}$$

The disturbance (d) and frictional forces (F_{xfric} and F_{yfric}) have been defined in Appendix A.

Based on the above state equations the non linear model of the $X-Y$ inverted pendulum has been developed using Matlab simulink in the present work.

PROPOSED ADAPTIVE GAIN SCHEDULING PID CONTROLLER

The output of a conventional PID controller is given by (Astrom & Hagglund, 1995):

$$u(t) = k_p e(t) + k_i \int e(t)dt + k_d \frac{de(t)}{dt} \tag{17}$$

Where $u(t)$ is the control signal. k_p, k_i and k_d are the proportional, the integral and the derivative gains of the controller respectively. The gains of conventional PID controller are fixed. However, a fixed gain PID controller cannot produce satisfactory performance under varying conditions. This limitation of the conventional controller can be overcome by using an intelligent controller or by adjusting the gains of PID controller depending on the operating conditions. The variation of PID gains for stabilizing the $X-Y$ inverted pendulum is explained as follows:

The value of k_p should be high during the initial part of the system response to reduce the rise time of the system and low during the later stages to overcome overshoot and the settling time. Similarly, the integral term is used to remove the offset. Hence a large value of k_i is required as time approaches infinity and minimum value is required during the initial stage. The derivative term is used to reduce the oscillations, which are more prone during the initial stage of the response. So, a high value for the derivative gain is required during the initial part of the response, while during the steady-state part of the response, a small value of k_d is sufficient.

In the present work the values of k_p, k_i and k_d have been designed to be a function of time and are given as follows:

$$k_p(t) = k_{p\min} + (k_{p\max} - k_{p\min})e^{-kt} \tag{18}$$

$$k_i(t) = k_{i\max} - (k_{i\max} - k_{i\min})e^{-kt} \tag{19}$$

$$k_d(t) = k_{d\min} + (k_{d\max} - k_{d\min})e^{-kt} \tag{20}$$

Where k is a constant for deciding the rate at which these gains vary between minimum and maximum values. The value of $k = 0.1$ for proportional and integral terms, while for the derivative term the value of $k = 0.05$ and 0.25 in case of angle and position controllers respectively. The parameters of the conventional PID controllers and the proposed adaptive gain scheduling PID controllers are given in Table 2 and Table 3 respectively.

Table 2. Parameters of conventional PID Controllers (Wang, 2011)

Parameter	k_p	k_i	k_d
PID1	25	15	3
PID2	-2.4	-1	-0.75
PID3	25	15	3
PID4	-2.4	-1	-0.75

Table 3. Parameters of adaptive gain scheduling PID controllers

Parameter	$k_{p\min}$	$k_{p\max}$	$k_{i\min}$	$k_{i\max}$	$k_{d\min}$	$k_{d\max}$
PID 1	15	25	3	15	2.8	3
PID2	-3	-2.4	-1.1	-1	-6.9	-0.75
PID3	15	25	3	15	2.8	3
PID4	-3	-2.4	-1.1	-1	-6.9	-0.75

These parameters have been obtained by using the trial-and-error method.

The Simulink diagram of the inverted pendulum system with angle and position PID controllers has been shown in Figure 2.

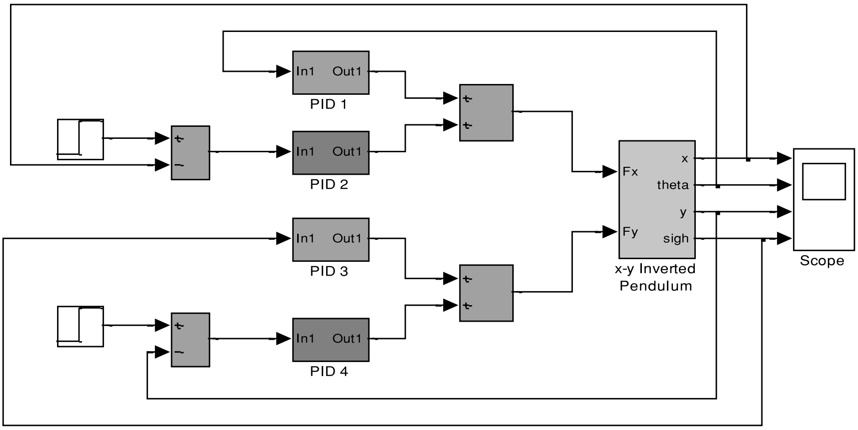


Fig. 2. Simulink model of the $X-Y$ inverted pendulum system control

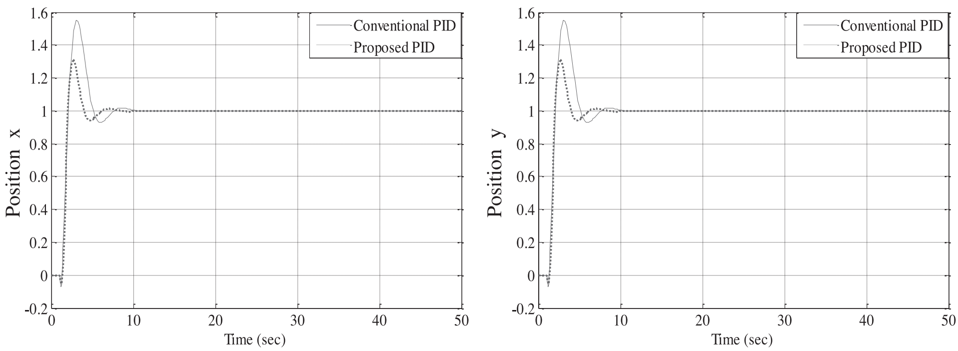
As shown in Figure 2, there are four controllers, two for position control and the remaining two for angle control. The set point for position control is the unit step input, while the set point for angle control is zero.

ANALYSIS AND VALIDATION OF THE PROPOSED METHODOLOGY

The performance analysis of the proposed adaptive gain scheduling PID controllers under various conditions is discussed as follows:

Performance analysis of adaptive gain scheduling PID controllers without disturbance

The performance of the proposed adaptive gain scheduling PID controller is compared to that of the conventional PID controller given by Wang (2011) as shown in Figure 3 and Figure 4 and quantitatively in Table 4 and Table 5.



(a) x Position (b) y Position

Fig.3. Angle Vs Time graph without disturbance

Table 4. Quantitative analysis for position control

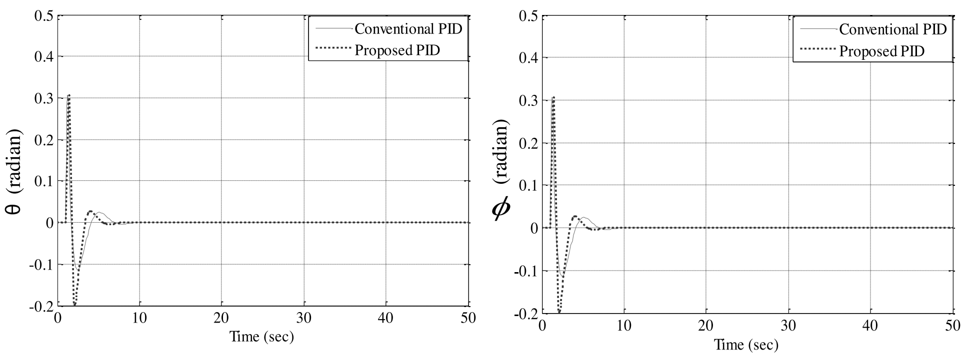
Parameters	Gain Scheduling PID	Conventional PID
M_P	0.31	0.55
M_U	0.056	0.07
t_r(sec)	1.97	1.98
t_S(sec)	5.71	7.18
e_{SS}	0	0

(a) x Position

Parameters	Gain Scheduling PID	Conventional PID
M_P	0.31	0.58
M_U	0.058	0.07
t_r(sec)	1.97	1.98
t_S(sec)	5.73	7.19
e_{SS}	0	0

(b) y Position

From Table 4, it can be observed that using proposed controllers the system response has become slightly fast with reduced overshoot and undershoot and has settled down rapidly to the steady-state value.



(a) Angle θ (b) Angle ϕ

Fig. 4. Angle Vs Time graph without disturbance

Table 5. Quantitative analysis for angle control

Parameters	Gain Scheduling PID	Conventional PID
M_P	0.3	0.31
M_U	-0.2	-0.11
t_S(sec)	4.52	5.64
e_{SS}	0	0

(a) Angle θ

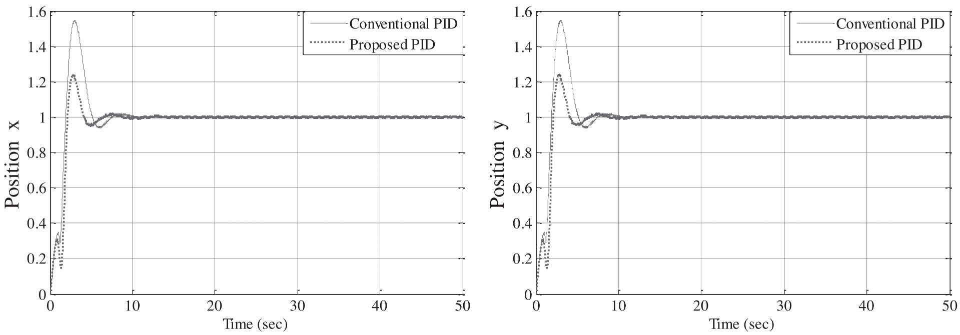
Parameters	Gain Scheduling PID	Conventional PID
M_P	0.3	0.32
M_U	-0.19	-0.12
t_S(sec)	4.55	5.69
e_{SS}	0	0

(b) Angle ϕ

Similarly as given in Table 5, there is a slight improvement in overshoot and a satisfactory improvement in settling time when using gain scheduling PID controllers for angle control.

Performance analysis of adaptive gain scheduling PID controller with disturbance

To check whether the proposed scheme is adaptive or not a disturbance *d* has been introduced into the inverted pendulum. The simulation results are compared in Figure 5 and Figure 6 for position and angle control, respectively, while Table 6 and Table 7 give the various performance specifications corresponding to position and angle control.



(a) *x* Position (b) *y* Position

Fig. 5. Position Vs Time graph with disturbance

Table 6. Quantitative analysis for position control

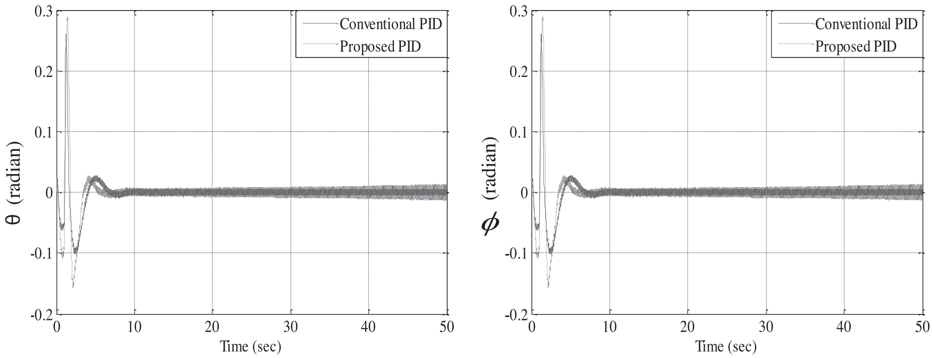
Parameters	Gain Scheduling PID	Conventional PID
M_P	0.24	0.55
M_U	0.05	0.063
t_r(sec)	2.02	1.81
t_S(sec)	5.94	7.13
e_{SS}	0	0

(a) *x* Position

Parameters	Gain Scheduling PID	Conventional PID
M_P	0.24	0.57
M_U	0.05	0.064
t_r(sec)	2.02	1.79
t_S(sec)	5.94	7.03
e_{SS}	0	0

(b) *y* Position

Table 6 shows that using the proposed scheme the system response has become a little bit slow with reduced overshoot and undershoot and has settled down rapidly to the steady-state value.



(a) Angle θ (b) Angle ϕ

Fig.6. Angle Vs Time graph with disturbance

Table 7. Quantitative analysis for angle control

Parameters	Gain Scheduling PID	Conventional PID
M_P	0.29	0.26
M_U	-0.16	-0.1
t_S(sec)	4.67	5.79
e_{SS}	0	0

(a) Angle θ

Parameters	Gain Scheduling PID	Conventional PID
M_P	0.29	0.27
M_U	-0.16	-0.11
t_S(sec)	4.67	5.79
e_{SS}	0	0

(b) Angle ϕ

As given in Table 7, there is a slight increase in the overshoot and undershoot with a satisfactory improvement in settling time, when using adaptive gain scheduling PID controllers for angle control.

Performance analysis of adaptive gain scheduling PID Controller with Noise in the Controller

A band limited white noise is added to the controllers as shown in Figure 7. The results of the proposed controllers are shown in Figure 8 and Figure 9 for position and angle control respectively.

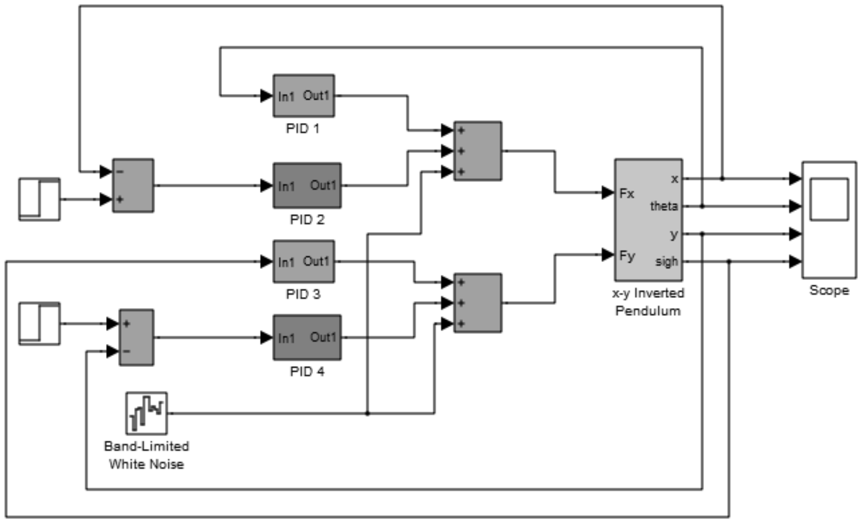
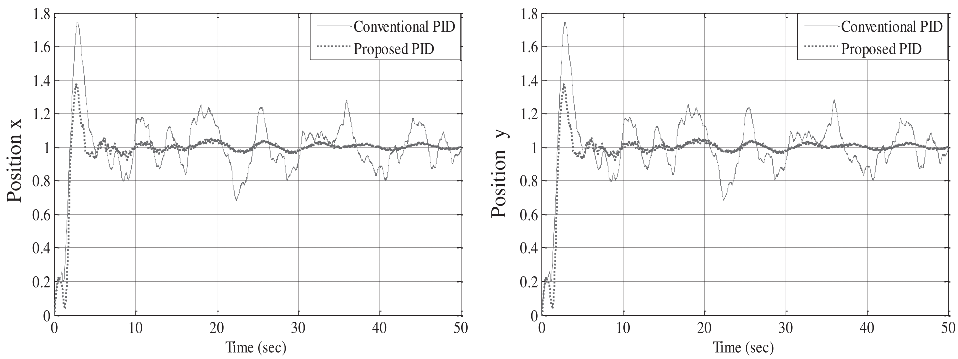


Fig. 7. Simulink model of the $X-Y$ inverted pendulum system with noise in the controller



(a) x Position (b) y Position

Fig. 8. Position Vs Time graph with noise in controller

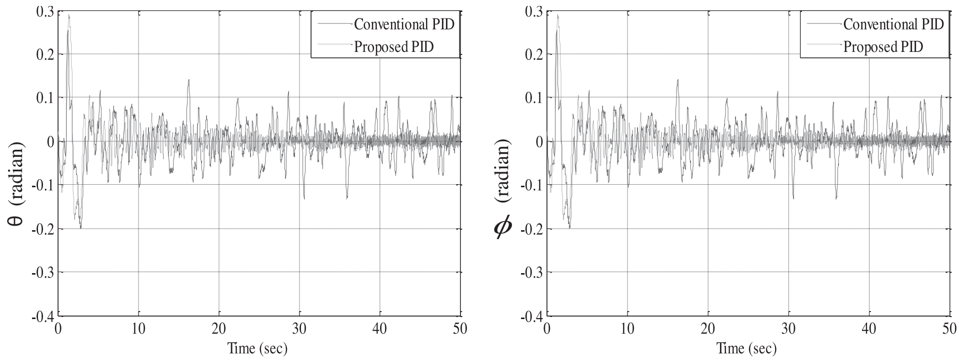
Table 8. Quantitative analysis for position control

Parameters	Gain Scheduling PID	Conventional PID
M_P	0.37	0.75
M_U	0.08	0.32
t_r(sec)	2.04	1.89
t_S(sec)	38.2	___
e_{SS}	0.015	___

(a) x Position

Parameters	Gain Scheduling PID	Conventional PID
M_P	0.37	0.77
M_U	0.08	0.32
t_r(sec)	2.04	1.87
t_S(sec)	38.2	___
e_{SS}	0.015	___

(b) y Position



(a) Angle θ (b) Angle ϕ

Fig. 9. Angle Vs Time graph with noise in controller

Table 9. Quantitative analysis for angle control

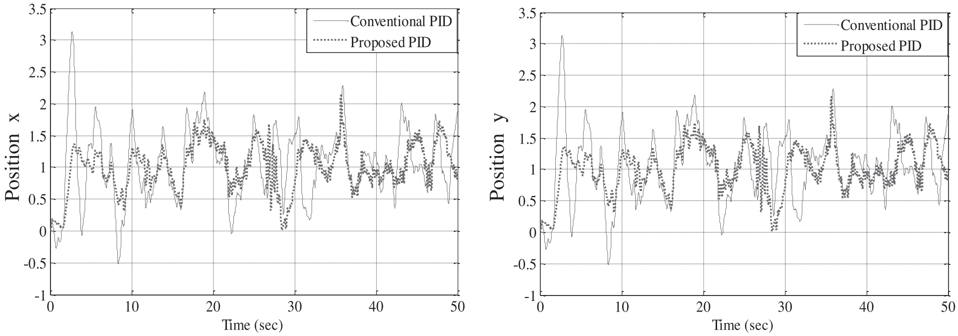
Parameters	Gain Scheduling PID	Conventional PID
M_P	0.29	0.25
M_U	-0.184	-0.2
t_S(sec)	46.3	___
e_{SS}	0.02	___

(a) Angle θ

Parameters	Gain Scheduling PID	Conventional PID
M_P	0.29	0.262
M_U	-0.18	-0.2
t_S(sec)	46.3	___
e_{SS}	0.02	___

(b) Angle ϕ

From Table 8, it can be revealed that using the proposed adaptive gain scheduling PID controllers the system response results in small maximum overshoot and undershoot and settle down to the steady-state value after some time. Whereas, the conventional PID controllers result in more chattering, and the system response does not settle down to the steady-state value. Similarly, Table 9 shows the quantitative analysis in case of angle control. A dash in the table indicates undefined values of settling time and steady-state error, which mean that the system will not reach to its steady-state value, while using conventional PID controllers which is not in case of the proposed adaptive gain scheduling controllers where the system settles down to the steady state value after some time.



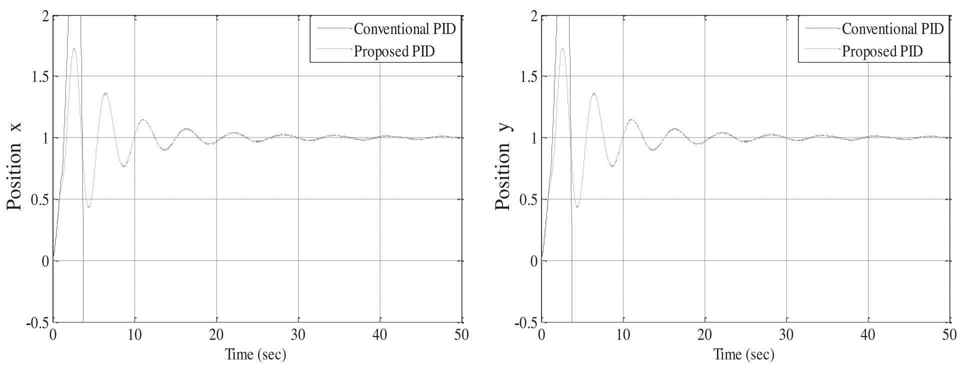
(a) *x* Position (b) *y* Position

Fig. 10. Position Vs Time graph with measurement noise in *x* and *y*

Figure 10 gives the simulation results considering measurement noise in some of the outputs (say *x* and *y*). The simulation results show that the proposed scheme results in less overshoot as compared to the conventional scheme.

Performance analysis of adaptive gain scheduling PID controller with friction

The frictional forces existing in an $X-Y$ inverted pendulum system are, the static and the coloumb friction (between the base and the track) and the viscous friction (between the pendulum and the base). The two type of friction models, i.e. the simple friction model and the exponential friction model are considered. The simulation results are shown from Figure 11 to Figure 14.



(a) *x* Position (b) *y* Position

Fig. 11. Position Vs Time graph considering simple friction model

The quantitative analysis of the simulation results is given in Table 10 and Table 13.

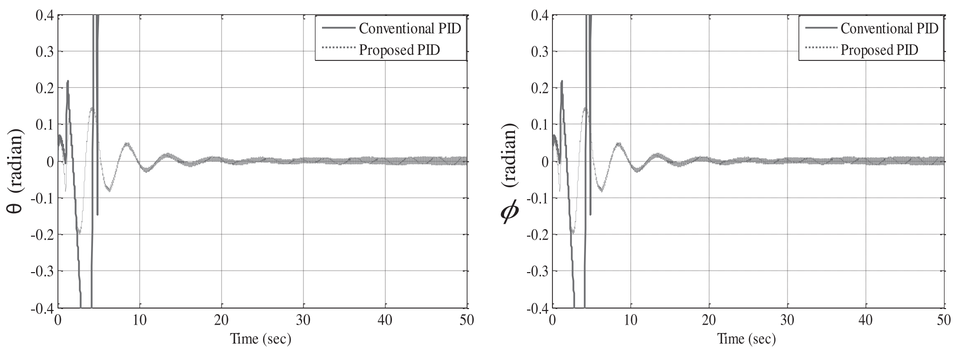
Table 10. Quantitative analysis for position control

Parameters	Gain Scheduling PID	Conventional PID
M_P	0.73	___
M_U	0.57	___
t_r (sec)	1.6	___
t_S (sec)	35.1	___
e_{SS}	0	___

(a) *x* Position

Parameters	Gain Scheduling PID	Conventional PID
M_P	0.73	___
M_U	0.57	___
t_r (sec)	1.6	___
t_S (sec)	35.1	___
e_{SS}	0	___

(b) *y* Position



(a) Angle θ (b) Angle ϕ

Fig. 12. Angle Vs Time graph considering simple friction model

Table 11. Quantitative analysis for angle control

Parameters	Gain Scheduling PID	Conventional PID
M_P	0.15	___
M_U	-0.2	___
t_S (sec)	11.62	___
e_{SS}	0	___

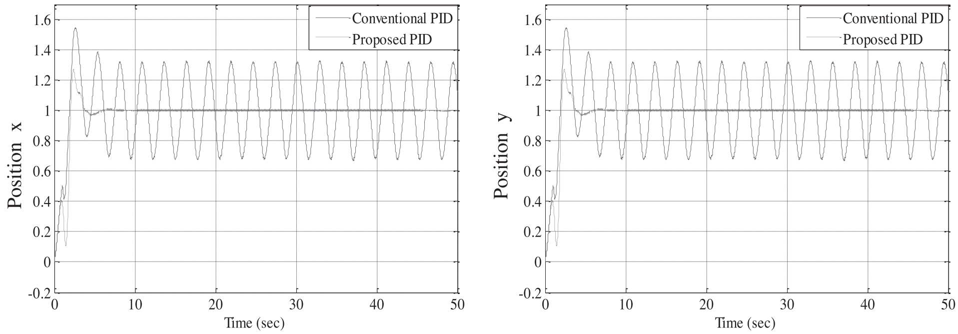
(a) Angle θ

Parameters	Gain Scheduling PID	Conventional PID
M_P	0.15	___
M_U	-0.2	___
t_S (sec)	11.62	___
e_{SS}	0	___

(b) Angle ϕ

Figure 11 and Figure 12 show that using the proposed adaptive gain scheduling PID controllers the system response is quite satisfactory as compared with the conventional PID controllers, whose response becomes unbounded in the presence of friction.

Figure 13 and Figure 14 give the simulation results when the exponential friction model is considered.



(a) x Position (b) y Position

Fig. 13. Position Vs time graph considering exponential friction model

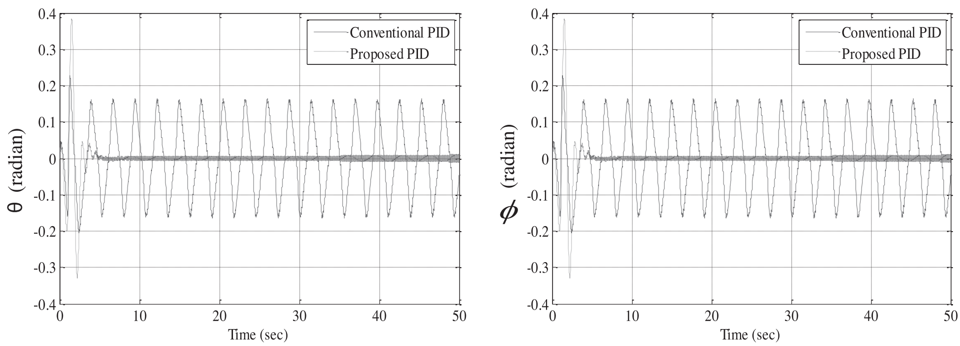
Table 12. Quantitative analysis for position control

Parameters	Gain Scheduling PID	Conventional PID
M_P	0.27	0.55
M_U	0.03	0.32
t_r(sec)	1.98	1.81
t_S(sec)	5.25	___
e_{SS}	0	___

(a) x Position

Parameters	Gain Scheduling PID	Conventional PID
M_P	0.27	0.55
M_U	0.03	0.32
t_r(sec)	1.98	1.81
t_S(sec)	5.25	___
e_{SS}	0	___

(b) y Position



(a) Angle θ (b) Angle ϕ

Fig. 14. Angle Vs time graph considering exponential friction model

Table 13. Quantitative analysis for angle control

Parameters	Gain Scheduling PID	Conventional PID
M_P	0.38	0.23
M_U	-0.33	-0.21
t_S (sec)	3.91	___
e_{SS}	0	___

(a) Angle θ

Parameters	Gain Scheduling PID	Conventional PID
M_P	0.38	0.23
M_U	-0.33	-0.21
t_S (sec)	3.91	___
e_{SS}	0	___

(b) Angle ϕ

The simulation results show the effectiveness of the proposed adaptive gain scheduling PID controllers as compared to the conventional controllers which gives an oscillatory response, when the exponential friction model is considered.

CONCLUSIONS

In this paper, adaptive gain scheduling PID controllers have been proposed for stabilizing an $X-Y$ inverted pendulum system. The performance of the proposed controllers has been compared to that of the conventional PID controllers given in literature, which has fixed gains for k_p, k_i and k_d. Simulation results show that the proposed controllers provide better performance than the conventional PID controllers in terms of different performance specifications under different conditions. In summary, the major contributions of this paper are as follows:

(1) The designing of adaptive gain scheduling PID controllers for stabilization of an $X-Y$ inverted pendulum.

(2) Checking the robustness by introducing a disturbance into the inverted pendulum and a band limited white noise.

(3) Performance analysis of the proposed scheme under the effect of friction.

APPENDIX A

(i) The simple friction model (Champbell, 2004)

The simple friction model considers three types of friction. These are, the static, the coulomb and the viscous friction. The static and the coulomb friction exist between the base and the track, while the viscous friction exists between the pendulum and the base. Mathematically the static model is represented as follows:

For x direction movement

$$F_{xfric} = (F_{xstatic} \text{ if } \dot{x} = 0)$$

$F_{xfric} = (F_{xcoloumb} + F_{xviscous} \text{ if } \dot{x} \neq 0)$

Here $F_{x\,static} = -F_{xapplied}$ if $\left|F_{x\,applied}\right| < \mu_s F_N$

$F_{xstatic} = -\mu_s F_N \text{ sgn}(F_{x\,applied})$ if $\left|F_{x\,applied}\right| \geq \mu_s F_N)$

The coulomb and the viscous friction are defined as:

$F_{x\,coloumb} = -\mu_C F_N \text{ sgn}(\dot{x})$

$F_{x\,viscous} = -\varepsilon \dot{x}$

For y direction movement

$F_{yfric} = (F_{ystatic} \text{ if } \dot{y} = 0)$

$F_{yfric} = (F_{ycoloumb} + F_{yviscous} \text{ if } \dot{y} \neq 0)$

Here $F_{y\,static} = -F_{yapplied}$ if $\left|F_{yapplied}\right| < \mu_s F_N$

$F_{ystatic} = -\mu_s F_N \text{ sgn}(F_{yapplied})$ if $\left|F_{yapplied}\right| \geq \mu_s F_N)$

The coulomb and the viscous friction are defined as:

$F_{ycoloumb} = -\mu_C F_N \text{ sgn}(\dot{y})$

$F_{yviscous} = -\varepsilon \dot{y}$

Here F_N is the magnitude of the normal force and is defined as, $F_N = (m + M)g$, μ_S =0.08328 is the coefficient of static friction, μ_C =0.04287 is the coefficient of Coulomb friction and ε =2.3156 the coefficient of viscous friction.

(ii) Exponential friction model (Champbell, 2004)

Another friction model known as the exponential friction model is defined as follows:

The exponential friction model is defined as follows:

$F_{xfric} = (F_{xstatic} \text{ if } \dot{x} = 0)$

$F_{xfric} = -[\mu_c + (\mu_s - \mu_c) e^{-(\frac{\dot{x}}{v_s})\gamma}] F_N \text{ sgn}(\dot{x}) - \varepsilon \dot{x} \text{ if } \dot{x} \neq 0$

Similarly $F_{yfric} = (F_{ystatic} \text{ if } \dot{y} = 0)$

$F_{yfric} = -[\mu_c + (\mu_s - \mu_c) e^{-(\frac{\dot{y}}{v_s})\gamma}] F_N \text{ sgn}(\dot{y}) - \varepsilon \dot{y} \text{ if } \dot{y} \neq 0$

Here γ = form factor=2 and v_s = Stribeck velocity=0.105

(iii) Disturbance

$d = 20 \sin(20\pi t)$

REFERENCES

Angeli, D. (2001). Almost global stabilization of the inverted pendulum via continuous state feedback, *Automatica, 37*, 1103–1108.

Astrom, K.J. & Furuta K. (1999). Swinging up a pendulum by energy control, *Automatica*, 3, 287-295.

Astrom, K. J. & Hagglund, T. (1995). PID Controllers: Theory, Design and Tuning, *Instrument Society of America, 2*, USA.

Bloch, A.M., Leonard, N.E. & Marsden, J.E. (2000). Controlled lagrangians and the stabilization of mechanical Systems I: the first matching theorem*, IEEE Transactions on Automatic Control, 45,* 2253–2270.

Champbell, S.A. (2004). Friction and the Inverted Pendulum Stabilization Problem, Department *of Applied Mathematics, University of Waterloo*, Canada, 1-21.

Chang, L.H. & Lee, A.C. (2007) Design of nonlinear controller for bi-axial inverted pendulum system, *IET Control Theory and Application, 1*, 979–986.

Ghosh, A. , Krishnan, T.R. & Subudhi, B. (2012). Robust PID compensation of an inverted cart-pendulum system: an experimental study, *IET Control Theory and Applications, 6*, 1145-1152

Ibanez, C.A. , Gutierrez, O. & Sossa-Azuela, H. (2006). Lyapunov Approach for the stabilization of the Inverted Spherical Pendulum, *45th IEEE Conference on Decision & Control* , 6133-6137

Kumar,V. & Jerome, J. (2013). Robust LQR Controller Design for Stabilizing and Trajectory Tracking of Inverted Pendulum, *Procedia Engineering, 64*, 169-178.

Liu, G.Y., Nesic, D. & Mareels, I. (2008). Non-linear stable inversion-based output tracking control for a spherical inverted pendulum, *International Journal of Control, 81,* 116–133.

Maravall, D. (2004). Control and stabilization of the inverted pendulum via vertical forces, Robotic welding, intelligence and automation. *Lecture notes in control and information sciences*, Berlin: Springer-Verlag, 299, 190–211.

Mason, P., Broucke, M. & Piccoli, B. (2008). Time optimal swing-up of the planar pendulum, *IEEE Transactions on Automatic Control, 53*, 1876–1886.

Muskinja, N. & Tovornik, B. (2006). Swinging up and stabilization of a real inverted pendulum, *IEEE Transaction on Industrial Electronics, 53*, 631-639.

Nagar, A., Bardini, M. & Rabaie, N.M. (2014). Intelligent control for nonlinear inverted pendulum based on interval type-2 fuzzy PD controller, *Alexandria Engineering Journal, 53*, 23-32.

Nasir, A. (2007). Modeling and controller design for an inverted pendulum system, *Master of Technology Degree, Faculty of Electrical Engineering*, Universiti Teknologi Malaysia.

Prasad, L.B. , Tyagi, B. & Gupta, H.O. (2011). Optimal Control of Nonlinear Inverted Pendulum Dynamical System with Disturbance Input using PID Controller & LQR, IEEE *International Conference on Control System, Computing and Engineering*, 540-545.

Tsai, C.C., Yu,C.C. & Chang, C.S. (2011). Aggregated hierarchical sliding-mode control for spherical inverted Pendulum, *Proceedings of 2011 8th Asian Control Conference (ASCC)*, 914-919.

Wai R.J. & Chang, L.J. (2006). Adaptive stabilizing and tracking control for a nonlinear inverted-pendulum system via sliding-mode technique, *IEEE Transactions on Industrial Electronics, 53*, 674-692.

Wang, J.J. (2011). Simulation studies of inverted pendulum based on PID controllers, *Simulation Modelling Practice and Theory, Elsevier 19* , 440-449.

Wang, J.J. (2012). Stabilization and tracking control of X–Z inverted pendulum with sliding-mode control, *ISA Transactions, 51*, 763-770.

Zhao, J. & Spong, M.W. (2001). Hybrid control for global stabilization of the cart-pendulum system, *Automatica, 37,* 1941-1951.

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