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الأمثلة نموذج متعدد المنتجات EPQ مع التسليم المتعددة المحسن

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الخلاصـة

يقدم هذا البحث على الحل الأمثل لنموذج عملية الانتاج الاقتصادي للمنتج المتعدد (EPQ) مع مواد الخردة وأسلوب التسليم المتعددة المحسن. والبحث هو امتداد لعمل Chiu et al. في عام (2013a) من خلال دمج خطة المحسنة n+1 الى خطة تسليم المتعدد في نموذجهم بهدف خفض تكاليف إجراء جرد من قبل البائع. وفي ظل مثل هذا أسلوب المحددة لشحن لكل منتج يتم تسليمة بإضافية الى السلع مكتملة الصنع من خلال جهوزية الإنتاج لتلبية الطلب على المنتجات النفطية للفترة المطلوبة. بعد ذلك، عندما تأتي عملية إعادة صياغة الى النهاية (أي بقية الكثير الإنتاج مضمونة الجودة) يتم تسليم عدد n أقساط ثابتة من السلع تامة الصنع للمشترين في فترة زمنية محددة. وأهداف هذه الدراسة هي تحديد دوران الإنتاج الأمثل للوقت والتي تقلل متوسط تكلفة لوحدة الزمن وتكشف عن تأثير معدل العشوائية وتحسين أسلوب التسليم على مدى دورة الإنتاج الأمثل. وبمساعدة نموذج رياضي مستمد بشكل مغلق، لقد تم تحديد زمن دورة مشتركة الأمثل. ومن خلال المثال العددي، وأظهرت الاستخدامات العملية للنتائج التي تم الحصول عليها ووفرة كبيرة في تكاليف العائدة الى حملة الأسهم.

Optimization of a multi-product EPQ model with scrap and an improved multi-delivery policy

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ABSTRACT

This paper considers optimization of a multi-product economic production quantity (EPQ) model with scrap items and an improved multi-delivery policy. We extend the work of Chiu *et al.* (2013a) by incorporating an improved $n + 1$ multi-delivery plan into their model with an aim at reducing vendor's inventory holding cost. Under such a specific shipment policy, for each product an extra delivery of finished goods is made during production uptime to satisfy the product demand for the period of vendor's uptime. Then, when rework process comes to an end (i.e., the rest of the production lot is quality assured) n fixed quantity installments of finished goods are delivered to buyers at a fixed time interval. The objectives of this study are to determine the optimal rotation production cycle time that minimizes the long-run average system cost per unit time and reveals the effects of random scrap rate and the improved delivery policy on the optimal production cycle time. With the help of a mathematical model a closed-form optimal common cycle time is derived. Through a numerical example, practical usages of obtained results and significant savings in vendor's stock holding cost are demonstrated.

Keywords: Multi-product EPQ model; multi-delivery plan; optimization; rotation cycle time; scrap.

INTRODUCTION

A multi-product economic production quantity (EPQ) model with scrap items and an improved multi-delivery plan is studied in this paper. The conventional EPQ model (Taft, 1918) considers a perfect manufacturing process for a single product and a continuous issuing policy of finished items. EPQ model employed a mathematical technique and optimization process, balanced production setup and inventory holding costs, and derived a most economic production lot that minimizes total production cost for proposed manufacturing system. Despite its simplicity in assumption, the concept of traditional EPQ model has since been adopted and applied industry-wide (Wagner & Whitin, 1958; Hadley & Whitin, 1963; Osteryoung *et al*., 1986; Tersine, 1994).

In real-life manufacturing environments, due to different unpredictable factors, production of defective items seems to be inevitable. Mak (1985) employed mathematical modeling to examine an inventory system, where the number of units of acceptable quality in a replenishment lot is uncertain and the demand is partially captive. He assumed that the fraction of the demand during the stock-out period which can be backordered is a random variable whose probability distribution is known. The optimal replenishment policy was synthesized for such a system. A numerical example was used to illustrate the theory. The results indicated that the optimal replenishment policy is sensitive to the nature of the demand during the stock-out period. Schwaller (1988) studied an EOQ model by including both fixed and variable inspection costs, for finding and removing a known proportion of defective items in incoming lots. Hariga and Ben-Daya (1998) considered the economic production quantity problem in the presence of imperfect processes. The time to shift from the in-control state to the out-of-control state was assumed to be flexible, and they provided distribution-based and distribution-free bounds on the optimal cost. For the exponential case, they compared the optimal solutions to approximate solutions proposed in the literature. Inderfurth *et al*. (2006) investigated a deterministic problem of planning the production of new and recovering defective items of the same product manufactured on the same facility. The processing of a batch includes two stages: the regular production and the rework process. While waiting for rework, defective items deteriorate and there is a given deterioration time limit. Deterioration results in an increase in time and cost for performing rework processes. The objective of their study was to find batch sizes and positions of items to be reworked such that overall production-inventory costs are minimized. A polynomial dynamic programming algorithm was presented to solve this problem. Other studies that addressed different aspects of imperfect production systems can also be found in Chiu *et al*. (2013b); Kamsu-Foguem *et al*. (2013); Sarkar & Sarkar (2013); Lin *et al*. (2013).

The continuous finished items distribution policy in conventional EPQ model is unrealistic when finished goods need to be distributed to outside customers in supply chains environment. Multiple or periodic delivery policy is commonly adopted in such a vendor-buyer system. Goyal (1977) investigated an integrated inventory model for a single supplier - single customer problem. A method was proposed for solving those inventory problems, wherein a product made by a single supplier is procured by a single customer. A numerical example was provided to verify this solution process. Schwarz *et al.* (1985) determined the fill-rate of a one-warehouse N-identical retailer distribution system. An approximation model was adopted from a prior study to maximize system fill-rate subject to a constraint on system safety stock. As a result, properties of fill-rate, policy were suggested to provide management, when looking into system optimization. Aderohunmu *et al*. (1995) demonstrated that a co-operative batching policy, based on cost information exchange between the vendor and the buyer, can significantly reduce total cost in the just-in-time (JIT) environment. The impact of such co-operation on total costs (including ordering, set-up, transportation and inventory holding costs) was examined for a long-term supply relationship. They indicated that joint optimization of both the vendor and the buyer's operations do not necessarily result in a common lot size. Sensitivity of the resulting cost savings due to the exchange of cost information to changes in the relevant operating parameters was also analyzed. Swenseth & Godfrey (2002) demonstrated that the freight rate functions presented in the literature can be incorporated into inventory replenishment decisions without compromising the accuracy of the decision. They concluded that these functions can be incorporated without adding undue complexity to the decision process. Abdul-Jalbar *et al*. (2008) considered a multi-echelon inventory system in which one vendor supplies an item to multiple buyers. They assumed that the vendor produces the item at a finite rate and customer demand occurs at each buyer at a constant rate. The objective is to determine the order quantities at the buyers and the production and shipment schedule at the vendor in order to minimize the average total cost per unit time. The problem was formulated in terms of integer-ratio policies and a heuristic procedure was developed. Both solution procedures were illustrated with a numerical example. Performance of the heuristic for computing integer-ratio policies was demonstrated. Additional studies that addressed various aspects of periodic or multi-delivery issues in the vendor-buyer integrated systems can also be found in Chiu *et al*. (2013c); Cedillo-Campos & Sánchez-Ramírez (2013).

In manufacturing sector, for the purpose of maximizing machine utilization, vendors often manufacture multiple products in turn on a single machine. Gordon & Surkis (1975) presented an approach to determine control policies for a multi-item inventory environment, where the items are ordered from a single supplier and the demand for items are subject to severe fluctuations. The time between orders can either be fixed or based on accumulating a fixed order quantity for all products. Their model balanced carrying and stock-out costs. An operational system structure was developed and a simulation procedure was used to determine the appropriate value of their inventory factor in the model. Zahorik *et al*. (1984) studied a multi-item, multi-level production scheduling problem with linear costs and production and inventory constraints at one key facility. Two multi-item problems were considered, one in which the constraint was on shipping capability and the other in which there was a final stage bottleneck machine. A multi-item facilities-in-series problem was formulated as a linear program. A 3-period result was used as the basis for a rolling heuristic for T-period problems. Conditions under which this heuristic fails to find optimal solutions were discussed, and computational comparisons to standard linear programming were provided. Byrne (1990) presented an approach to the multi-item production lot sizing problem based on the use of simulation, to model the interactions occurring in the system. A search algorithm was used to adjust the lot sizes on the basis of the results of previous simulation runs, with the objective of achieving a minimum total cost solution. Zipkin (1995) explored the performance of a multi-item production-inventory system. He compared two alternative policies, representing different modes of collecting and utilizing information, then derived a closed-form measure of performance for one of them, the familiar first-come-first-served (FCFS) policy, and proposed a comparable approximation for the other, the longest-queue (LQ) policy. These results were illustrated, tested through simulations, and used to address several basic managerial issues. Khoury *et al*. (2001) considered a production facility that is used to produce more than a single item in order to maximize its utilization. They assumed the existence of capacity constraint and analyzed the lot scheduling problem under such an insufficient capacity. A two-product problem using the common cycle approach was examined. Extension to any number of products is discussed. Clausen & Ju (2006) studied an economic lot and delivery scheduling problem (ELDSP), where a supplier produces and delivers components of different types to a consumer in batches. The objective is to determine the cycle time, which minimizes the total cost per time unit. The purpose of their study includes the determination of the production sequence of the component types within each cycle. Computational behavior of two published algorithms; a heuristic and an optimal algorithm were investigated. With large number of component types, the optimal algorithm has long running times, so they developed a hybrid algorithm to assist them to find optimal solution in a more efficient manner. Guchhait *et al*. (2010) considered a multi-item inventory model of breakable items where demands of the items are stock dependent, and breakability rates increase linearly with stock and nonlinearly with time. Due to non-linearity and complexity of the problem, their model was solved numerically and final decisions were made using genetic algorithm (GA). They considered both fuzzy and stochastic uncertain inventory costs, and a chance constrained approach was used to deal with simultaneous presence of stochastic and fuzzy parameters. Different numerical examples were used to illustrate different cases of the problem. Björk (2012) developed a fuzzy multi-item economic production quantity (EPQ) model, where a company has to decide the size of some production batches under uncertain cycle times. The uncertainty was handled with triangular fuzzy numbers and an analytical solution was found to the optimization problem. Chiu *et al.* (2013a) derived the optimal common production cycle time for a multi-item finite production rate (FPR) model with random scrap rate and *n* fixed quantity multiple delivery policy. Mathematical modeling along with optimization technique is used to solve the problem. A closed-form optimal common production cycle time that minimizes the expected system costs is obtained. Effect of scrap rate on the optimal common cycle time was investigated and discussed through a numerical example. Studies related to different aspects of the multi-item production planning and optimization issues can also be found in Jodlbauer & Reitner (2012); Chiu *et al*. (2013d).

Aiming at lowering vendor's inventory holding cost, this paper extends Chiu *et al.*'s work (2013a) by incorporating an improved n+1 shipment policy into their model. Under the proposed policy, one extra delivery of finished items is made in vendor's production uptime to satisfy buyer's product demands during the uptime. Then, upon completion of production, n fixed quantity installments of finished items are delivered to buyers at a fixed time interval. The objectives are to determine the optimal common production cycle time that minimizes the long-run average system cost per unit time, and study the effects of random scrap rate and improved delivery policy on the optimal cycle time and on the system costs.

MODELING AND FORMULATIONS

In this paper a multi-product economic production quantity model with random scrap rate and an improved multi-delivery plan under the rotation cycle policy is studied. Description of the proposed model is as follows: consider L products are manufactured in turn on a single machine and all items are screened and inspection cost is included in unit production cost C_i (where i = 1, 2, … , L). During production uptime of product i there is a x_i portion of random scrap items produced at a rate d_i. Under the regular operating policy, shortages are not allowed, so the constant production rate for product i, P_i must satisfy $(P_i - d_i - \lambda_i) > 0$, where λ_i is the annual demand rate for product i, and d_i can be expressed as $d_i = x_i P_i$. For the purpose of lowering producer's inventory holding cost, this study extends the work of Chiu *et al.* (2013a) by adopting a specific n+1 multi-shipment policy, and under the proposed n+1 delivery policy an initial shipment of finished goods is to meet customer's product demands during producer's uptime; then upon completion of regular production n fixed quantity installments of the finished products are transported to customers, at a fixed time interval t_n (see Figure 1).

Figure 2 depicts producer's on-hand inventory level of perfect quality items of product i in the proposed n+1 delivery model (in blue lines) and the expected reduction in producer's stock holding costs (green shaded area) in comparison with that in Chiu *et al.* (2013a) (in black lines).

Additional cost variables used in this study include: production setup cost K_i (where i = 1, 2, … , L), unit holding cost h_i, disposal cost C_{Si} per scrapped item, the fixed delivery cost K_{1i} for product i per delivery, and unit shipping cost C_{Ti} for each product i. Other parameters used in this paper are listed as follows:

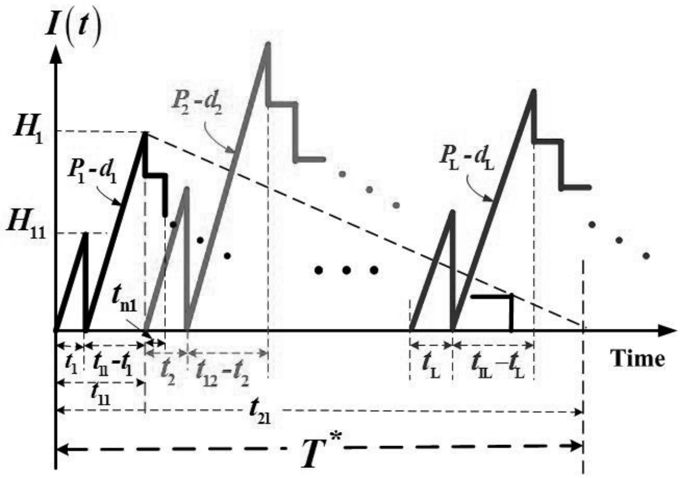

Fig. 1. On-hand inventory of perfect quality items in the proposed multi-product EPQ model under rotation cycle time policy

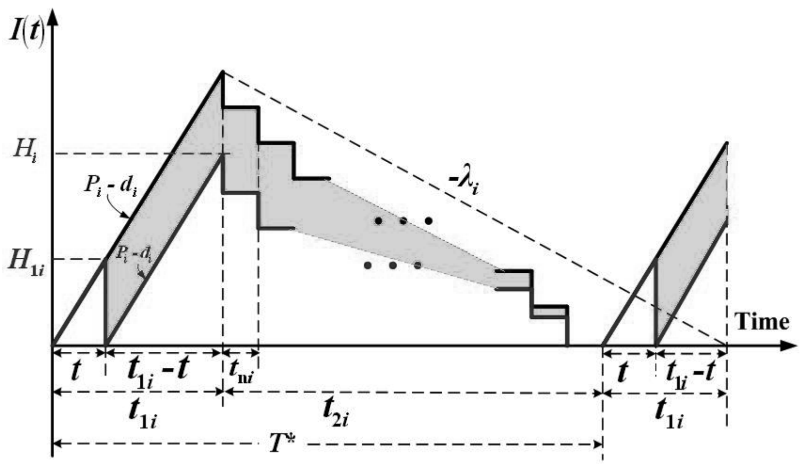

Fig. 2. Expected reduction in producer's stock holding costs (in green) for each product i in the proposed model in comparison with Chiu et al.'s model (2013a)

T = rotation cycle time - the decision variable,

t_i =time needed to produce enough items of product i to meet customer's demand during uptime t_{1i},

t_{1i} = production uptime for product i,

t_{2i} = the delivery time for product i,

H_{1i} = level of on-hand inventory in units of product i for meeting customer's demand during uptime t_{1i},

H_i = maximum level of on-hand inventory of product *i* when the regular production ends,

n = number of fixed quantity installments of the finished lot to be delivered to customers in each cycle,

Q_i = production lot size per cycle for product *i*,

t_{ni} = fixed interval of time between each installment of finished product *i* being delivered during t_{2i},

$I(t)$ = on-hand inventory of perfect quality items at time *t*,

$I_S(t)_i$ = on-hand inventory of scrap items for product *i* at time *t*,

$TC(Q_i)$ = total production-inventory-delivery cost per cycle for product *i*,

$E[TCU(Q_L)]$ = total expected production-inventory-delivery cost per unit time for *L* products in the proposed model under the rotation cycle time policy,

$E[TCU(T)]$ = total expected production-inventory-delivery cost per unit time for *L* products in the proposed system using the rotation cycle time *T* as decision variable.

By observing from Figure 1, the following formulas can be directly obtained for any $i = 1, 2, \ldots, L$:

$$T = t_{1i} + t_{2i} = \frac{Q_i}{\lambda_i} \tag{1}$$

$$t_i = \frac{H_{1i}}{P_i - d_i} = \frac{\lambda_i t_{1i}}{P_i - d_i} \tag{2}$$

$$t_{1i} = \frac{Q_i}{P_i} = \frac{H_{1i} + H_i}{P_i - d_i} \tag{3}$$

$$t_{2i} = T - t_{1i} = n t_{ni} \tag{4}$$

$$H_{1i} = \lambda_i t_{1i} \tag{5}$$

$$H_i = (P_i - d_i)(t_{1i} - t_i) \tag{6}$$

$$\lambda = \sum_{i=1}^{L} \lambda_i \tag{7}$$

Figure 3 illustrates on-hand inventory of scrap items in the proposed multi-product EPQ model under rotation cycle time policy. From Figure 2, the total scrap items of product i can be obtained as follows:

$$d_i t_{1i} = x_i Q_i \tag{8}$$

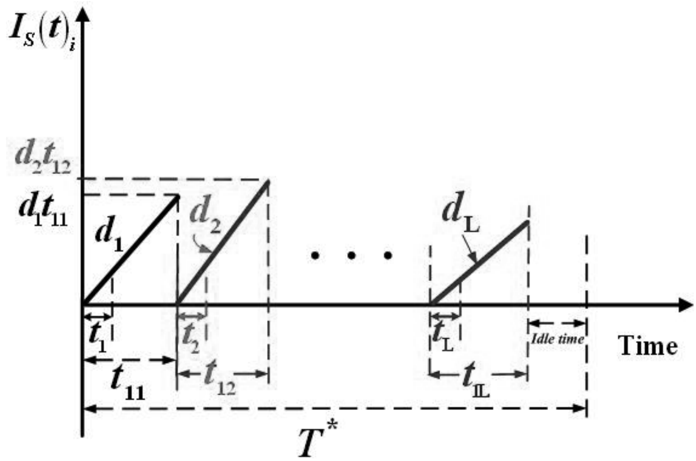

Fig. 3. On-hand inventory of scrap items in the proposed multi-product EPQ model under rotation cycle time policy

Total delivery cost for product i (i.e., n+1 shipments) in a given production cycle is

$$(n+1)K_{1i} + C_{\mathrm{T}i}Q_i \tag{9}$$

The variable holding costs for finished items of product i during delivery time t_{2i} are as follows (Chiu *et al*., 2009):

$$h_i\left(\frac{n-1}{2n}\right)H_i t_{2i} \tag{10}$$

Total production-inventory-delivery cost per cycle for L products, consists of production setup cost, variable production cost, variable disposal cost, fixed and variable delivery cost, holding cost during production uptime t_{1i}, and holding cost for finished goods kept during the delivery time t_{2i}. Hence, total $TC(Q_i)$ for L products are

$$\sum_{i=1}^{L} TC(Q_i) = \sum_{i=1}^{L} \left\{ \begin{array}{l} K_i + C_i Q_i + C_{Si}(x_i Q_i) + (n+1)K_{1i} + C_{Ti} Q_i \\ + h_i \left[\frac{H_{1i}}{2}(t_i) + \frac{H_i}{2}(t_{1i} - t_i) + \frac{d_i t_{1i}}{2}(t_{1i}) + \left(\frac{n-1}{2n} \right) H_i t_{2i} \right] \end{array} \right\} \tag{11}$$

The defective rate *x* is assumed to be a random variable with a known probability density function; hence, to take the randomness of *x* into account the expected value of *x* is used in this study. Substituting all variables from equations (1) to (10) in Eq. (11) and with further derivations the expected E[$TCU(Q_L)$] can be obtained as

$$E\left[TCU(T)\right] = \sum_{i=1}^{L} \left\{ \begin{array}{l} C_i \lambda_i + \frac{K_i}{T} + C_{Si} \lambda_i E[x_i] + C_{Ti} \lambda_i + \frac{(n+1)K_{1i}}{T} \\ + \frac{h_i T \lambda_i^2}{2} \left\{ \lambda_i \left(\frac{1}{P_i} \right)^2 \left[\frac{2\lambda_i}{P_i(1 - E[x_i])} \right] + \frac{1}{P_i} + \left(1 - \frac{1}{n} \right) \left[\left[\frac{1}{\lambda_i} - \frac{2}{P_i} \right] \right] \right\} \end{array} \right\} \tag{12}$$

Derivation of the optimal rotation cycle time

As stated earlier, the objective of this study is to determine the optimal rotation production cycle time to minimize the long-run average system cost per unit time (i.e., Eq. (12)). In order to obtain the optimal rotation cycle time *T** one must first prove the existence of the minimum of the expected cost function E[*TCU*(*T*)]. Because the system cost has one decision variable, *T*, differentiating E[*TCU*(*T*)] with respect to *T* gives the first and the second derivative of E[*TCU*(*T*)] as

$$\frac{dE[TCU(T)]}{dT} = \sum_{i=1}^{L} \left\{ \begin{array}{l} -\frac{K_i}{T^2} - \frac{(n+1)K_{1i}}{T^2} \\ + \frac{h_i \lambda_i^2}{2} \left\{ \lambda_i \left(\frac{1}{P_i} \right)^2 \left[\frac{2\lambda_i}{P_i(1 - E[x_i])} \right] + \frac{1}{P_i} + \left(1 - \frac{1}{n} \right) \left[\left[\frac{1}{\lambda_i} - \frac{2}{P_i} \right] \right] \right\} \end{array} \right\} \tag{13}$$

$$\frac{d^2 E[TCU(T)]}{dT^2} = \sum_{i=1}^{L} \left\{ \frac{2\left[K_i + (n+1)K_{1i}\right]}{T^3} \right\} \tag{14}$$

The second-order derivative is positive because K_i, *n*, K_{1i}, and *T* are all positive. Thus, *E[TCU(T)]* is a convex function for all *T* different from zero. It follows that the optimal rotation production cycle time *T** can be obtained by setting the first derivative of *E[TCU(T)]* as equal to zero:

$$\frac{dE[TCU(T)]}{dT}=\sum_{i=1}^{L}\left\{\begin{array}{l}-\frac{K_i}{T^2}-\frac{(n+1)K_{1i}}{T^2}\\+\frac{h_i\lambda_i^2}{2}\left\{\lambda_i\left(\frac{1}{P_i}\right)^2\left[\frac{2\lambda_i}{P_i(1-E[x_i])}\right]+\frac{1}{P_i}+\left(1-\frac{1}{n}\right)\left[\left[\frac{1}{\lambda_i}-\frac{2}{P_i}\right]\right]\right\}\end{array}\right\}=0 \tag{15}$$

or

$$\frac{1}{T^2}\sum_{i=1}^{L}\left[K_i+(n+1)K_{1i}\right]=\sum_{i=1}^{L}\frac{h_i\lambda_i^2}{2}\left\{\lambda_i\left(\frac{1}{P_i}\right)^2\left[\frac{2\lambda_i}{P_i(1-E[x_i])}\right]+\frac{1}{P_{1i}}+\left(1-\frac{1}{n}\right)\left[\left[\frac{1}{\lambda_i}-\frac{2}{P_i}\right]\right]\right\} \tag{16}$$

With further derivations one obtains

$$T^*=\sqrt{\frac{2\sum_{i=1}^{L}\left[K_i+(n+1)K_{1i}\right]}{\sum_{i=1}^{L}\frac{h_i\lambda_i^2}{2}\cdot\left\{\lambda_i\left(\frac{1}{P_i}\right)^2\left[\frac{2\lambda_i}{P_i(1-E[x_i])}\right]+\frac{1}{P_i}+\left(1-\frac{1}{n}\right)\left[\left[\frac{1}{\lambda_i}-\frac{2}{P_i}\right]\right]\right\}}} \tag{17}$$

Effect of production setup time on the rotation cycle time

The production setup time is relatively short in general, when compared to the production uptime. However, if setup time becomes a factor, one has to check whether the production cycle length is long enough to account for the setup and production times of L products (Chiu *et al*., 2009). Let S_i denote production setup time for product i; Eq. (18) must hold in order to ensure that each cycle has sufficient time for the setup and production of L products.

$$\sum_{i=1}^{L}\left[S_i+(Q_i/P_i)\right]<T \tag{18}$$

Substituting Eq. (1) in Eq. (18) gives

$$T>\frac{\sum_{i=1}^{L}S_i}{1-\sum_{i=1}^{L}(\lambda_i/P_i)}=T_{\min.} \tag{19}$$

Hence, if the setup time becomes a significant factor in production planning, one should choose the optimal rotation cycle time from max[T*, $T_{\min}$] (Chiu *et al*., 2009).

NUMERICAL EXAMPLES

In order to lessen comparison efforts for readers the same numerical example is used in this section as in Chiu et al. (2013a). Consider a production plan for making five different products in turn on a single machine under a rotation cycle time policy. Assume that for product i = 1, 2, … , 5, respectively, the annual production rate P_i is 58000, 59000, 60000, 61000, and 62000; the demand rate λ_i is 3000, 3200, 3400, 3600, and 3800 per year; the random defective rate x_i during the production processes follows the uniform distribution over the intervals of [0, 0.05], [0, 010], [0, 0.15], [0, 0.20], and [0, 0.25]; all defective items are not repairable; and the disposal cost C_{Si} is \$20, \$25, \$30, \$35, and \$40. Other system variables used for product i = 1, 2, … , 5 include the following:

K_i = the production setup costs are \$3800, \$3900, \$4000, \$4100, and \$4200, respectively.

h_i = unit holding costs are \$10, \$15, \$20, \$25, and \$30, respectively.

C_i = unit production costs are \$80, \$90, \$100, \$110, and \$120, respectively.

K_{1i} = fixed costs per delivery are \$1800, \$1900, \$2000, \$2100, and \$2200, respectively.

C_{Ti} = unit transportation costs are \$0.10, \$0.20, \$0.30, \$0.40, and \$0.5, respectively.

n = number of shipments per cycle; in this study, it is assumed to be a constant 3 (i.e., n+1 = 4).

Applying Eq. (17) the optimal rotation cycle time T^* = 0.7279 (years) can be obtained, and from computation of Eq. (12), one finds that the expected system cost for making L products E[$TCU(T^*)$] = \$2,097,903. The variation of the average defective rate of x_i effects on the expected system cost E[$TCU(T)$] and on various components of E[$TCU(T^*)$] is illustrated in Figure 4. It is noted that, as average defective rate increases, the disposal cost (for the scrap items) and the expected system cost E[$TCU(T^*)$] increases significantly.

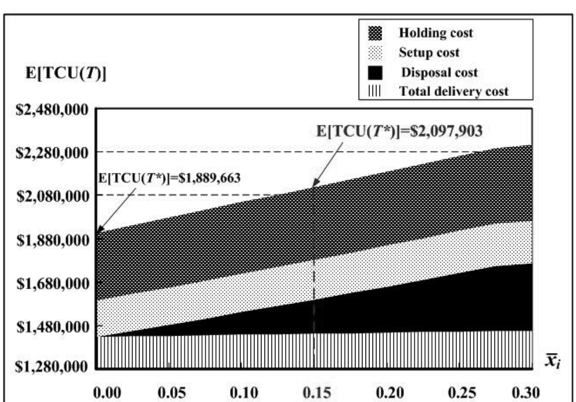

Fig. 4. Variation of average defective rate effects on the expected system cost E[$TCU(T)$] and on various components of E[$TCU(T)$]

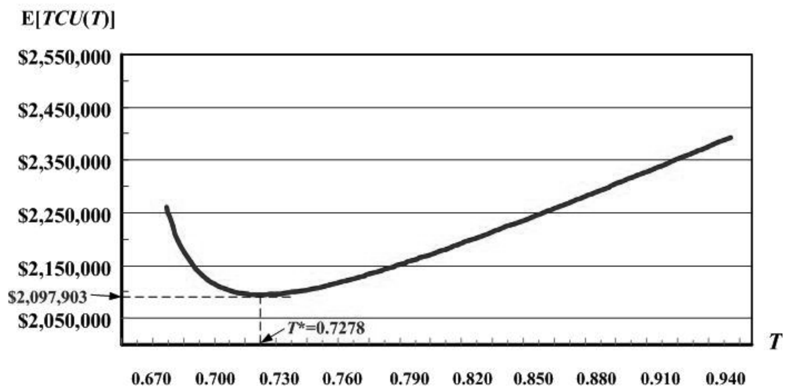

Fig. 5. Variation of rotation production cycle time T effects on the expected system cost E[TCU(T)]

Variation of rotation production cycle time *T* effects on the expected system cost E[*TCU*(*T*)] is depicted in Figure 5. The proposed model is aimed at reducing the producer's inventory holding cost for each product *i* during the production cycle (see Figure 2). In this example, the total system cost savings are $15,291 or 3.89% of total other related costs (i.e., total system cost E[*TCU*(*T*)] excludes variable production cost $\lambda_i C_i$). A further analysis shows that the percentage of reduction of total holding cost is 16.7% (i.e., reduced from $98,906 [Chiu *et al*., 2013a] to $82,424; see Fig. 6). Further analysis indicates that the relative reductions of inventory holding cost for each product are $429, $1,351, $2,776, $4,712, and $7,251; or 5.72%, 10.68%, 14.78%, 18.19%, and 21.17%, respectively.

In summary, through the aforementioned numerical example, significant savings in the vendor's stock holding cost are demonstrated in our research result.

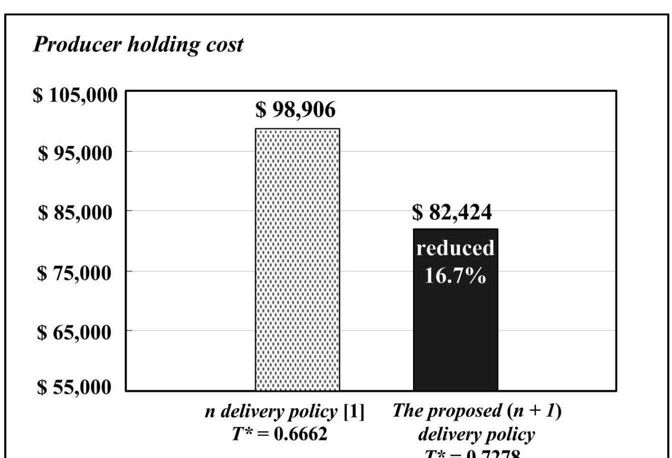

Fig. 6. Comparison of total inventory holding costs of the proposed n+1 delivery policy with that in Chiu et al.'s model (2013a)

An additional example to demonstrate the applicability of the proposed model is as follows. Suppose management of a manufacturing firm has different manufacturing process options for a multi-product production system, and associated with these options are different mean scrap rates over the interval of [0.03, 0.30]. Applying the research results obtained from the present study, the effects of various scrap rates on the optimal rotation cycle time T and on the expected system cost E[TCU(T)] can be determined and illustrated as in Figure 7. It is noted that as mean scrap rate increases, the optimal rotation cycle time T^* decreases slightly, but the expected system cost E[TCU(T)] increases significantly.

In summary, through the aforementioned numerical example, effects of mean scrap rates on the optimal rotation cycle time and on the expected system cost can all be obtained by the use of results of the proposed study.

CONCLUDING REMARKS

With a primary intention of reducing the vendor's inventory holding cost, this study extends the work of Chiu *et al*. (2013a) by incorporating an improved n + 1 multi-delivery plan into their model. Mathematical modeling and optimization techniques are used to solve the proposed problem. As a result, the optimal rotation production cycle time that minimizes the long-run average system cost per unit time is derived. Through a numerical example, the practical usage of the obtained result is demonstrated (see Figures 4 and 5) and the significant savings in the vendor's stock holding cost as compared to prior work (Chiu *et al*., 2013a) is confirmed (see Figures 2 and 6). In conclusion, the objective of lowering the vendor's holding cost has been successfully accomplished. For future study, one interesting direction will be to consider the effect of stochastic demand rates on the optimal common cycle time.

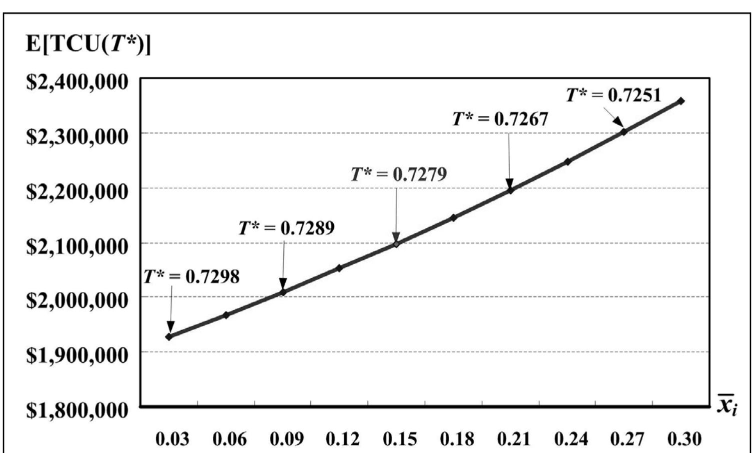

Fig. 7. Effects of various scrap rates on the optimal rotation cycle time T* and on the expected system cost E[TCU(T)]

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REFERENCES

Abdul-Jalbar, B., Gutiérrez, J.M. & Sicilia, J. 2008. Policies for a single-vendor multi-buyer system with finite production rate. Decision Support Systems **46**(1): 84-100.

Aderohunmu, R., Mobolurin, A. & Bryson, N. 1995. Joint vendor-buyer policy in JIT manufacturing. Journal of the Operational Research Society **46**(3): 375-385.

Björk, K.M. 2012. A multi-item fuzzy economic production quantity problem with a finite production rate. International Journal of Production Economics **135**(2): 702-707.

Byrne, M.D. 1990. Multi-item production lot sizing using a search simulation approach. Engineering Costs and Production Economics **19**: 307-311.

Chiu, Y.S.P., Chiu, S.W., Li, C.Y. & Ting, C.K. 2009. Incorporating multi-delivery policy and quality assurance into economic production lot size problem. Journal of Scientific & Industrial Research **68**(6): 505-512.

Chiu, S.W., Kuo, Y.Y., Huang, C.C. & Chiu, Y.S.P. 2013a. Random scrap rate effect on multi-item finite production rate model with multi-shipment policy. Research Journal of Applied Sciences, Engineering and Technology **5**(22): 5164-5169.

Chiu, S.W., Chou, C.L. & Wu, W.K. 2013b. Optimizing replenishment policy in an EPQ-based inventory model with nonconforming items and breakdown. Economic Modelling **35**: 330-337.

Cedillo-Campos, M. & Sánchez-Ramírez, C. 2013. Dynamic self-assessment of supply chains performance: An emerging market approach. Journal of Applied Research and Technology **11**(3): 338-347.

Chiu, Y.S.P., Chen, K.K., Cheng, F.T. & Ting, C.K. 2013c. Reexamination of "combining an alternative multi-delivery policy into economic production lot size problem with partial rework" using an alternative approach. Journal of Applied Research and Technology **11**(3): 317-323.

Chiu, Y.S.P., Lin, H.D., Cheng, F.T. & Hwang, M.H. 2013d. Optimal common cycle time for a multi-item production system with discontinuous delivery policy and failure in rework. Journal of Scientific & Industrial Research **72**(7): 435-440.

Clausen, J. & Ju, S. 2006. A hybrid algorithm for solving the economic lot and delivery scheduling problem in the common cycle case. European Journal of Operational Research **175**: 1141-1150.

Gordon, G.R. & Surkis, J. 1975. A control policy for multi-item inventories with fluctuating demand using simulation. Computers and Operations Research **2**(2): 91-100.

Goyal. S.K. 1977. Integrated Inventory Model for a Single Supplier - Single Customer Problem. International Journal of Production Research **15**(1): 107-111.

Guchhait, P., Maiti, M.K. & Maiti, M. 2010. Multi-item inventory model of breakable items with stock-dependent demand under stock and time dependent breakability rate. Computers and Industrial Engineering **59**(4): 911-920.

Hadley, G. & Whitin, T.M. 1963. Analysis of Inventory Systems. Prentice-Hall, Englewood Cliffs, N.J., USA.

Hariga, M. & Ben-Daya, M. 1998. Note: the economic manufacturing lot-sizing problem with imperfect production processes: bounds and optimal solutions. Naval Research Logistics **45**: 423-433.

Inderfurth, K., Janiak, A., Kovalyov, M.Y. & Werner, F. 2006. Batching work and rework processes with limited deterioration of reworkables. Computers and Operations Research **33**(6): 1595-1605.

Jodlbauer, H. & Reitner, S. 2012. Optimizing service-level and relevant cost for a stochastic multi-item cyclic production system. International Journal of Production Economics **136**(2): 306-317.

Kamsu-Foguem, B., Rigal, F. & Mauget, F. 2013. Mining association rules for the quality improvement of the production process. Expert Systems with Applications **40**(4): 1034-1045.

Khoury, B.N., Abboud, N.E. & Tannous, M.M. 2001. The common cycle approach to the ELSP problem with insufficient capacity. International Journal of Production Economics **73**: 189-199.

Lin, H-D., Pai, F.Y. & Chiu, S.W. 2013. A note on "intra-supply chain system with multiple sales locations and quality assurance." Expert Systems with Applications **40**(11): 4730-4732.

Mak, K.L. 1985. Inventory control of defective products when the demand is partially captive. International Journal of Production Research **23**(3): 533-542.

Osteryoung, S., Nosari, E., McCarty, D. & Reinhart, W.J. 1986. Use of the EOQ model for inventory analysis. Production and Inventory Management **27**: 39-45.

Sarkar, B. & Sarkar, S. 2013. An improved inventory model with partial backlogging, time varying deterioration and stock-dependent demand. Economic Modelling **30**(1): 924-932.

Schwaller, R. 1988. EOQ under inspection costs, Production and Inventory Management **29**: 22-24.

Schwarz, L.B., Deuermeyer, B.L. & Badinelli, R.D. 1985. Fill-rate optimization in a one-warehouse N-identical retailer distribution system. Management Science **31**(4): 488-498.

Swenseth, R.S. & Godfrey, R.M. 2002. Incorporating transportation costs into inventory replenishment decisions. International Journal of Production Economics **77**(2): 113-130.

Zahorik, A., Thomas, L.J. & Trigeiro, W.W. 1984. Network programming models for production scheduling in multi-stage, multi-item capacitated systems. Management Science **30**(3): 308-325.

Taft, E.W. 1918. The most economical production lot, Iron Age **101**: 1410-1412.

Tersine, R.J. 1994. Principles of Inventory and Materials Management. PTR Prentice-Hall: New Jersey, USA: 120-131.

Wagner, H. & Whitin, T.M. 1958. Dynamic version of the economic lot size model. Management Science **5**: 89-96.

Zipkin, P.H. 1995. Performance analysis of a multi-item production-inventory system under alternative policies. Management Science **41**(4): 690-703.

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