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# تحليل الموثوقية وتقييم نظام الفرامل على أساس المخاطر المنافسة

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## الخلاصة

موثوقية نظام الفرامل يؤثر مباشرة على سلامة السيارة. خطأ تحليل شجرة هو الطريقة المستخدمة لتقليل احتمال الحوادث المرورية الناجمة عن فشل نظام الفرامل. فشل المعدلات توزيع وايبل من مكونات نظام الفرامل للسيارات السباق ولذلك من خلال الحصول على احتمال أقصى تقدير. يتم تأسيس خطأ شجرة من فشل الرئيسي للنظام الفرامل. وأخيرا، يتم تحليل موثوقية نظام الفرامل وتقييمها من خلال تحليل خطأ شجرة إلى توفير مرجعية لتصميم أمثل لنظام الفرامل.

# Reliability analysis and evaluation of a brake system based on competing risks

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## ABSTRACT

The reliability of a brake system directly affects the safety of a vehicle. Fault tree analysis is a method used to reduce the probability of traffic accidents caused by brake system failure. For the brake system components of a racing car, the failure parameters of the Weibull distributions are obtained through maximum likelihood estimation. A fault tree is then established according to the failure principle of the brake system. Finally, the reliability of the brake system is analysed and evaluated by fault tree analysis to provide a reference for the design and development of brake system.

## INTRODUTION

The brake system is an important device that ensures safety of vehicle. Vehicle manoeuvrability is influenced by the performance of the brake system, and many accidents are caused by brake system failures. Disc brakes are applied in many brake systems; hence, several optimisation plans for analysing and studying the brake system have been proposed in recent years (Jegadeeshwaran & Sugumaran, 2015).

The fault tree is a special tree logical causality diagram that uses event, logic gate and other symbols to describe the causal relationship among events in a system (Matthew & Heffernan, 2006). The fault tree of a non-service air braking system is set up using fault tree analysis techniques and the assessment method for the reliability of such air braking systems (Sun *et al*., 2007; Tu *et al*., 2010). The assessment shows that the techniques can clearly deduct the failure modes of non-service air braking systems, identify the weaknesses during operation and reflect the influence degree of each fault mode. The rule set for a fuzzy classifier is obtained using a fault tree (Jegadeeshwaran & Sugumaran, 2013; Sugumaran & Ramachandran, 2007). Fault trees are related to the disruption of energy delivery from generators to specific load points. The quantitative evaluation of fault trees, which represents a standpoint for assessing power delivery reliability, enables the identification of the most important elements in a power system (Yan *et al*., 2009). The construction of fault trees begins with the top event and then traces backward to the primary and undeveloped events of the components based on the structural configuration of the system and the extended decision tables stored in a database (Wang & Liu, 1993; Xu *et al*., 2010). This method is currently applied to analyse the reliability of automotive brake systems (Sugumaran *et al*., 2006).

Reliability is an important index for measuring product safety, quality and fatigue life; research on this variable is involved in an extensive field. The new method is technically based on the strengthening and damaging features of vehicle components under a loading spectrum whilst combining dynamic strength equations with the residual strength of vehicle components

(Nicholas, 2002; Zheng & Lu, 2012). The system reliability level matches the requirement for differential systems, which considerably reduces cost, as demonstrated by using the stress–strength interference and low-load strengthening models (Liu & Huang, 2006; Xie W *et al*., 2014). The rationality of reliability reallocation is verified according to the subsystem importance coefficient (Huang & Zhang, 2014; Liu & Chen, 2005). Based on the stress–strength interference model, automobile welding structure reliability has been studied and discussed with theories of probability and reliability (Song *et al*., 2014; Allella *et al*., 2004). A generalised reliability analysis model, which considers multiple competing causes, is highly necessary (Liu *et al*., 2014; Wu & Yan, 2014). Existing methods are difficult to use in resolving the reliability apportionment problem because of data insufficiency and uncertainty in terms of the relationships among the components of a mechanical system (Kuo & Wan, 2007). Reliability analysis is helpful in optimising the design of a brake system, which leads to an improved design of such systems (Chen *et al*., 2016; Gavin & Zaicenco, 2007).

Reliability analysis is widely applied to many fields, such as system engineering, electronic computer technology and testing technology. But the literature about reliability analysis and evaluation through competing risks is less. This paper takes the brake system for example and carries out reliability analysis and evaluation using the theory of competing risks.

The parameters of the key components of a brake system are described in Section 2. The values of failure parameters are obtained via maximum likelihood estimation in Section 3. The fault tree of the brake system of a racing car is obtained by analysing its failure principle in Section 4. The reliability of a brake system is analysed and evaluated in Section 5 according to Sections 3 and 4. The discussion and conclusion are drawn based on the results of the reliability analysis and evaluation in Section 6.

## PARAMETERS OF BRAKE SYSTEM

The brake system of a racing car consists of a pedal, a balance lever, a main cylinder and a brake on each wheel, among others. The main components of a brake are a friction block, a brake disc, a brake clamp body and so on. A simple hydraulic brake system is adopted in the aforementioned racing car, whilst the front and rear wheels adopt the floating disc type calliper disc brake.

The key parameters of the racing car are provided in Table 1, and the parameters of the key components of the brake system are listed in Table 2.

**Table 1.** Key parameters of the racing car

| Key parameters | Values |
|---|---|
| Mass (full) | 250 kg |
| High centre of mass | 0.330 m |
| Wheel base | 1.575 m |
| Centre of mass to the front axle | 0.819 m |
| Centre of mass to the rear axle | 0.756 m |
| Front axle load | 1176 N |
| Rear axle load | 1274 N |

**Table 2.** Parameters of the key components of the brake system

| Key parameters | Values |
|---|---|
| Master cylinder piston diameter | 15.9 mm |
| Front calliper piston diameter | 25 mm |
| Front calliper piston number | 8 |
| Rear calliper piston diameter | 25 mm |
| Rear calliper piston number | 4 |
| Brake disc working radius | 86.25 mm |
| Tire radius | 254 N |

## PARAMETER ESTIMATION

Weibull distribution can reasonably reflect the failure of a large number of products and materials, and the shape parameter of it can make data fitting of failure has good adjustment. Its analytical expression of reliability function is very convenient on mathematical processing, especially the linearization of reliability function by logarithmic transformation.

If the brake system breaks or fails, then certain failure parameters follow the Weibull distribution, such as oil in the friction plate, the failure of the calliper piston to home, lack of braking action and the failure of the oil valve. The failure cause of brake system follows Weibull distribution.

The probability density function of the Weibull distribution $f(t)$ can be expressed as follows:

$$f(t)=\frac{k}{b}\left(\frac{t-a}{b}\right)^{k-1}e^{-\left(\frac{t-a}{b}\right)^{k}},(a\leq t;k,b>0) \tag{1}$$

where *k* is a shape parameter, *b* is a scale parameter, *a* is a location parameter and  is a random variable.

When *t* follows the three parameter-Weibull distribution, the cumulative distribution function *F(t)* can be expressed as follows:

$$F(t)=1-\exp\left[-\left(\frac{t-a}{b}\right)^{k}\right],t>0 \tag{2}$$

When In*(T)* follows the smallest extreme value distribution, the corresponding probability density function $f(t;\mu,\sigma)$ and the cumulative distribution function $F(t;\mu,\sigma)$ can be expressed as follows:

$$f(t;\mu,\sigma)=\frac{1}{\partial t}j_{sev}\left[\frac{\ln(t-a)-\mu}{\sigma}\right],F(t;\mu,\sigma)=F_{sev}\left[\frac{\ln(t-a)-\mu}{\sigma}\right],t>0 \tag{3}$$

where $\mu=\ln(b),-\infty<\mu<+\infty,\sigma=1/k>0$ is a location parameter.

A random sample of $n$ identical systems is placed on a life test. The observed data are presented $(x_1,\delta_1),(x_2,\delta_2),...,(x_n,\delta_n)$ . $(x_i,\delta_i)$ as . is the realization of the observed data $(X_i,\Delta_j)$ $X_i$, is minimum time for system failure and

$$\Delta_j = \begin{cases} j & observed\ \ failure\ due\ to\ cause\ j \\ 0 & observed\ censoring \end{cases}$$

. For example, the brake system failure is detected at $x_1$ time on account of failure of the oil valve, the observed data is presented as $(x_1, \delta_4)$, where $\delta_4$ is the failure of the oil valve.

The unknown parameter, which is denoted by $\theta^{(j)} = (\mu^{(j)}, \sigma^{(j)})$, is the maximum likelihood estimator for system $i$ that has failed from cause $j$. The parameters are estimated by maximum likelihood estimation as follows:

$$L_i(\theta^{(j)}) = \left\{ f^{(j)}(x_i) \prod_{\substack{j=1 \\ j \neq j}}^{m} R^{(j)}(x_i) \right\}^{I_i(\delta_i = j)} \left\{ R^{(j)}(x_i) \right\}^{I_i(\delta_i = 0)} = \left\{ f^{(j)}(x_i) \right\}^{I_i(\delta_i = j)} \prod_{\substack{1=0 \\ 1 \neq j}}^{m} \left\{ R^{(j)}(x_j) \right\}^{I_i(\delta_i = 1)}, I_i(\Delta_i = j) = \begin{cases} 1 & \Delta_i = j \\ 0 & otherwise \end{cases} \tag{4}$$

$$\lambda_j(t) = -\frac{\partial \log R(t_1, ..., t_m)}{\partial t_j} \bigg|_{t_1 = \cdots = t_m = t} \tag{5}$$

$$R_j(t) = \Pr(T > t) = \exp\left( -\sum_{j=1}^{k} \Lambda_j(t) \right) \tag{6}$$

where $\Lambda_j(t) = \int_0^t \lambda_j(s) ds$ is the cumulative cause-specific hazard.

$$F_j(t) = \Pr(T \leq t, D = j) = \int_0^t \lambda_j(u) R(u) du, j = 1, 2, ..., k \tag{7}$$

$$f_j(t) = \lambda_j(t) F_j(t) \tag{8}$$

Thus, the likelihood function $L(\Theta)$ for all failures is calculated by multiplying all single failure likelihood function factors as follows:

$$L(\Theta) = \prod_{j=1}^{m} L(\theta^{(j)}) = \prod_{j=1}^{m} \prod_{i=1}^{n} L_i(\theta^{(j)}) \tag{9}$$

where $\Theta = (\theta(1), \theta(2), ..., \theta(m))$ is an unknown parameter of failures.

The likelihood function Weibull distribution of failures $L\left(\theta^{(j)}\right)$ can be obtained using Eq. (10) as follows:

$$L\left(\theta^{(j)}\right) = \prod_{i=1}^{n} \left\{ \left\{ \frac{1}{\sigma^{(j)} t_i} \varphi_{sev} \left[ \frac{\log(t_i) - \mu^{(j)}}{\sigma^{(j)}} \right] \right\}^{I_i(\delta_i = j)} \times \prod_{\substack{1=0 \\ 1 \neq j}}^{m} \left\{ 1 - \Phi_{nor} \left[ \frac{\log(t_i) - \mu^{(j)}}{\sigma^{(j)}} \right] \right\}^{I_i(\delta_i = 1)} \right\} \tag{10}$$

Data on oil in the friction plate, the failure of the calliper piston to home, the lack of braking action and the failure of the oil valve can be collected through the racing car. The failure parameters are estimated using the maximum likelihood estimation method as shown in Table 3.

**Table 3.** Failure distributions and parameter estimate values

| Failures | Distribution functions | Estimate values of parameters |
|---|---|---|
| Oil in the friction plate | Weibull distribution | $a = 3405.756; b = 182386; k = 0.799$ |
| Failure of the calliper piston to home | Weibull distribution | $a = 30.865; b = 125865; k = 1.112$ |
| Lack of braking action | Weibull distribution | $a = 110.259; b = 139162; k = 0.730$ |
| Failure of the oil valve | Weibull distribution | $a = 1391.891; b = 67988; k = 0.816$ |

## FAULT TREE OF BRAKE SYSTEM

The logic relationship between the system and the element function can be analysed by studying the function and failure modes of the system. A system is called a serial system only when all the subsystems are normal in a system composed of $N$ subsystems. A system is called a parallel system if it is effective on any one of the subsystems; the system can work in a system that consists of $N$ subsystems.

Engine oil is used as the medium in the hydraulic brake system of the racing car. The brake system produces a brake torque by transferring the force to the brake pedal. The main failures can be analysed through the working process and principle of a brake system, and the specific causes are presented in Table 4.

**Table 4.** Failures and the corresponding fault causes of the brake system

| Failures | Fault causes |
|---|---|
| Oil in the friction plate | The friction plate is provided by the manufacturer, and its appearance is determined by the factory. Oil pollution is caused by improper operation and installation. |
| Failure of the calliper piston to home | The oil finds it difficult to return because brake fluid viscosity is extremely thick. The oil cannot return to the brake pipe plug wheel cylinder because braking clearance is too narrow. |
| Lack of braking action | Thus, brake movement disorders may be attributed to several reasons: limited pedal, pedal fracture, extremely large push rod and master cylinder piston clearance, disengaged and insufficient brake fluid, absence of brake fluid and brake pipe serious leakage. |
| Failure of the oil valve | The brake pipe is jammed. The load proportion valve is faulty. The brake pressure distribution valve is invalid or the pressure regulator is not adjusted. The main piston of the cylinder or the seal of the oil return valve is bad. |

Failures include oil in the friction plate, the failure of the calliper piston to home, the lack of braking action and the failure of the oil valve. The fault tree for the brake system of the racing car is established based on the work principle of a brake system. The results are shown in Figure 1.

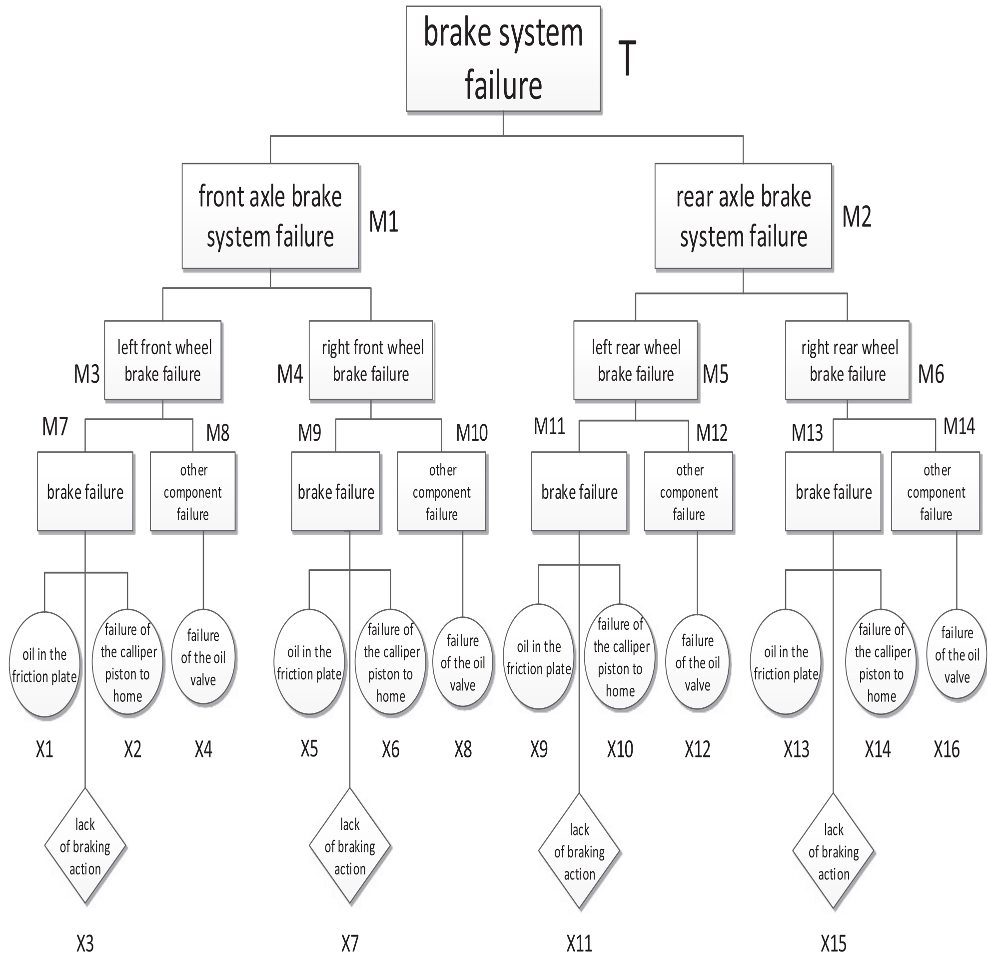


**Fig. 1.** Fault tree for the brake system of the racing car

## RELIABILITY ANALYSIS AND EVALUATION

### Reliability analysis of the brake system

The reliability function of variable $x$ is calculated as follows:

$$R(x) = e^{-\left(\frac{x-a}{b}\right)^{k}}, (a \le x; k, b > 0). \tag{11}$$

The cumulative distribution function of failure reliability $F(x)$ is calculated as follows:

$$\mathrm{F}(x) = 1 - e^{-\left(\frac{x-a}{b}\right)^{k}}, (a \le x; k, b > 0). \tag{12}$$

The values of the random variables with two points $x_i(t)$ can be expressed as follows:

$$x_i(t) = \begin{cases} 1; & \text{event } i \text{ occurs at } t \text{ moment} \\ 0; & \text{event } i \text{ does not occur at } t \text{ moment} \end{cases} .$$

The expected value of random event $x$ is calculated as follows:

$$Q_i(t) = E[x_i(t)] = 0 \cdot p(x_i = 0) + 1 \cdot p(x_i = 1) = p[x_i(t) = 1].$$

In the same manner, the top event is described with a two-point distribution as follows:

$$F[X(t)] = \begin{cases} 1; & \textit{top event occurs at t moment} \\ 0; & \textit{top event does not happen at t moment} \end{cases}$$

where $X(t) = \{x_1(t), x_2(t), ...., x(t)\}$ is the vector in the bottom event.

The expected value of random variable $\Phi(X)$ for the top event is calculated as follows:

$$P(top) = E\Phi[X(t)] = p\{\Phi[X(t)] = 1\} = p[\Phi(x_1, x_2, ....., x_n) = 1]$$
$$= g(Q_1(t), Q_2(t), ...., Q_n(t)) = g(Q(t))$$

$$Q(t) = \{Q_1(t), Q_2(t), ...., Q_n(t)\} \quad (13)$$

Structure function can be found for any fault tree. One of the most effective means is to use the minimal cut set or the maximum set of a fault tree to describe structure function.

The AND gate of a fault tree is $\Phi(X) = x_1 x_2 .... x_n = \prod_{i=1}^{n} x_i$ . (14)

The OR gate of a fault tree is $\Phi(X) = 1 - (1 - x_1)(1 - x_2)...(1 - x_n) = 1 - \prod_{i=1}^{n} (1 - x_i)$. (15)

The minimal cut set of the top event $C_j$ is calculated as follows:

$$C_j = \bigcap_{i \in C_j} x_i , \quad (16)$$

Whereis the basic event number of the minimum cut set $C_j$ .

The structure function of fault tree $\Phi(X)$ can be expressed as follows:

$$\Phi(X) = \bigcup_{j=1}^{n} C_j = \bigcup_{j=1}^{n} \bigcap_{i \in C_j} x_i . \quad (17)$$

The fault tree is completely qualitatively as described by structure function $\Phi(X)$. The top event necessarily happens when any basic event of the minimum cut concentration occurs. Approximately 3500 km is required in testing, training, commissioning and competing for a system according to experience. Thus, reliability analysis is completed for 3500 km.

According to the estimated values of the parameters, the probability of the basic events can be calculated using Equation (12) as follows:

$$P(X1)=1-R(x)=1-e^{-\left(\frac{x-a}{b}\right)k}=1-e^{-\left(\frac{3500-3405.756}{182386}\right)^{0.798645}}=0.0024.$$

Similar results can be obtained to according to the estimated values of the parameters as follows:

$$P(X2)=0.0182,$$

$$P(X3)=0.0642,$$

$$P(X4)=0.0572,$$

$$P(X1)=P(X5)=P(X9)=P(X13)=0.0024,$$

$$P(X2)=P(X6)=P(X10)=P(X14)=0.0182,$$

$$P(X3)=P(X7)=P(X11)=P(X15)=0.0642,$$

$$P(X4)=P(X8)=P(X12)=P(X16)=0.0572.$$

The probability of the intermediate event is calculated using Equation (15) according to the OR gate of the fault tree as follows:

$$P(M7)=1-(1-P(X1))(1-P(X2))(1-P(X3))=0.0834,$$

$$P(M7)=P(M9)=P(M11)=P(M13)=0.0834,$$

$$P(M8)=P(M10)=P(M12)=P(M14)=0.0572.$$

Similarly, the probability of other events is calculated as follows:

$$P(M3)=1-(1-P(M7))(1-P(M8))=0.1358,$$

$$P(M3)=P(M4)=P(M5)=P(M6)=0.1358,$$

$$P(M1)=P(M2)=0.2532.$$

Finally, the reliability of the entire system is obtained as follows:

$$P(T)=P(M1)P(M2)=0.064.$$

The unreliable degree of the entire system is 0.064. Therefore, the reliability of the entire brake system is 0.936.

## Failure mileages of different reliabilities

According to the competition risks theory in test of product life, any one failure factor can cause failure of product with m failure factor. When the brake system of car fails, the probability of failure factor of brake system failure can be analysed according to the competition risks theory.

Different reliability conditions and failure mileages are obtained using Equation (3) according to competition risks theory and the parameters in presented Table 3.

$$R_1(t) = 1 - \Phi_{sev}\left[\frac{\ln(t - 3405.756) - 12.113}{1.252}\right] \tag{18}$$

$$R_2(t) = 1 - \Phi_{sev}\left[\frac{\ln(t - 30.865) - 11.743}{0.899}\right] \tag{19}$$

$$R_3(t) = 1 - \Phi_{sev}\left[\frac{\ln(t - 110.259) - 11.843}{1.370}\right] \tag{20}$$

$$R_4(t) = 1 - \Phi_{sev}\left[\frac{\ln(t - 1391.891) - 11.127}{1.226}\right] \tag{21}$$

The mean mileage to failure $MMTF$ can be obtained as follows:

$$MMTF = \exp(\mu + 0.5\sigma^2) \tag{22}$$

The failure mileages of different reliabilities include $R(0.5)$, and $MMTF$ are listed in Table 5.

**Table 5.** Failure mileages of different reliabilities

| Numbers | Failures | R(0.5) /km | R(0.9)/km | MMTF/km |
|---|---|---|---|---|
| *X1* | Oil in the friction plate | 185631.245 | 39648.704 | 399113.122 |
| *X2* | Failure of the calliper piston to home | 125900.223 | 39489.248 | 188527.719 |
| *X3* | Lack of braking action | 139217.438 | 23875.923 | 355400.335 |
| *X4* | Failure of the oil valve | 69374.025 | 15366.549 | 144206.327 |

Table 5 shows that when reliability is 90%, the order of failure mileage is $X4 > X3 > X2 > X1$. When reliability is 50%, the order of failure mileage is $X4 > X2 > X3 > X1$. According to the average mileage, the order of failure mileage is $X4 > X2 > X3 > X1$. The factor that most likely leads to failure can be drawn by analysing different reliabilities; the failure of the corresponding cause can then be obtained.

## CONCLUSIONS

The fault tree of brake system is established according to the structure and failure reason of brake system of the racing car. Weibull distribution parameters of failure reasons of brake system are estimated using the maximum likelihood estimation method. Reliability analysis of brake system is carried out based on parameter estimation and fault tree of brake system. The reliability of brake system is 0.936 when the mileage of car reaches 3500 km. The failure causes probability of brake system is different under different reliability according to the principle of competition failure.

The reliability of brake system of racing car is analysed using fault tree, and this provides effective reference for the design and development of brake system. Fault tree can also be applied to reliability analysis and evaluation of car other system, such as the transmission system, lubrication system and cooling system. At the same time, the fault tree can also be applied in lots of fields combined with other related theory.

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## REFERENCES

**Jegadeeshwaran, R. & Sugumaran, V. 2015.** Fault diagnosis of hydraulic brake system using statistical features and support vector machines. Mechanical Systems and Signal Processing **52**(1): 436-446.

**Matthew, E. & Heffernan. 2006.** Analyzing and simulating brake rotor temperatures: A technique applied to a formula SAE vehicle. FSAE International.

**Tu, J.L., Duan, R.X. & Dong, D.C. 2010.** An approach for construction of fault trees using rough set and fuzzy clustering algorithm. Applied Mechanics and Materials **11**(35):186-191.

**Sun, W., Chen, J. & Li, J. 2007.** Decision tree and PCA-based fault of rotating machinery. Mechanical Systems and Signal Processing **21**(3):1300-1317.

**Sugumaran, V. & Ramachandran, K.I. 2007.** Automatic rule learning using decision tree for fuzzy classifier in fault diagnosis of roller bearing. Mechanical Systems and Signal Processing **21**(5): 2237-2247.

**Jegadeeshwaran, R. & Sugumaran, V. 2013.** Comparative study of decision tree classifier and best first tree classifier for fault diagnosis of hydraulic brake system using statistical features. Measurement **46**(9): 3247- 3260.

**Yan, S.T., Guo, B. & Cui, Q.C. 2009.** Reliability analysis of roll bending machine based on fuzzy fault tree. Machinery & Electronics **11**(11): 23-26.

**Xu, R., Che, J.G., Yang, Z.B. & Zuo, X.Y. 2010.** The fault tree analysis and its application in the system reliability analysis. Command Control & Simulation **2**(32): 112-115.

**Wang, J.D. & Liu, T.S. 1993.** A component behavioural model for automatic fault tree construction. Reliability Engineering and System Safety **42**(1): 87-100.

**Sugumaran, V., Muralidharan, V. & Ramachandran, K.I. 2006.** Feature selection using decision tree and classification through proximal support vector machine for fault diagnostics of roller bearing. Mechanical Systems and Signal Processing **21**(2): 930-942.

**Zheng, S.L. & Lu, X. 2012.** Lightweight design of vehicle components based on strengthening effects of low-amplitude loads below fatigue limit. Fatigue and Fracture of Engineering Materials Structures **35**(3): 269-277.

**Nicholas, T. 2002.** Step loading for very cycle fatigue. Fatigue and Fracture of Engineering Materials Structures **25**(8): 861-869.

**Xie, W., Liao, H.T. & Jin, T.D. 2014.** Maximizing system availability through joint decision on component redundancy and spares inventory. European Journal of Operational Research **11**(237): 164-176.

**Liu, C.H. & Huang, H. 2006.** An interval analysis for the mesco ductile fracture. Key Engineering Materials **2**(324): 971-974.

**Liu, C.H. & Chen, Q. 2005.** Interval analysis of fuzzy-random heat conduction in the composite tubes. Applied Mathematics and Mechanics **10**(26): 1312-1318.

**Huang, X. & Zhang, Y. 2014.** A probability estimation method for reliability analysis using mapped Gegenbauer polynomials. Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability **228**(1): 72-82.

**Allella, F., Chiodo, E. & Lauria, D. 2004.** Optimal reliability allocation under uncertain conditions with application to hybrid electric vehicle design. International Journal of Quality and Reliability Management **22**(6): 626-641.

**Song, S., Coit, D.W., Feng, Q. & Peng, H. 2014.** Reliability analysis for multi-component systems subject to multiple dependent competing failure processes. Reliability, IEEE Transactions **63**(1): 331-345.

**Wu, J.N. & Yan, S.Z. 2014.** An approach to system reliability prediction for mechanical equipment using fuzzy reasoning Petri net. Journal of Risk and Reliability **1**(228): 39-51.

**Liu, X.T., Zheng, S.L., Feng, J., Huang, H., Chu, J. & Zhao, L.H. 2014.** Reliability analysis and evaluation of automobile welding structure. Quality and Reliability Engineering International **30**(8): 1293-1300.

**Kuo, W. & Wan, R. 2007.** Recent advances in optimal reliability allocation. IEEE transactions on systems, Man, and Cybernetics-Part A: Systems and Humans **37**(2): 143-156.

**Gavin, H.P. & Zaicenco, A. 2007.** Performance and reliability of semi-active equipment isolation. Journal of Sound and Vibration **306**(1): 74-90.

**Chen, T., Zheng, S.L., Luo, H.W., Liu, X.T. & Feng, J.Z. 2016.** Reliability analysis of multiple causes of failure in presence of independent competing risks. Quality and Reliability Engineering International **32**(2): 363-372.

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