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جدولة مشروع بأنشطة وعلاقات بديلة

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الخلاصة

إن التعقيد الناشئ من إدارة مشاريع البناء ذات النطاق الواسع غالباً ما تتطلب عرض وتقييم لبرامج بديلة. غير أن أدوات الجدولة السائدة نادراً ما تراعي التمثيل الصريح للأنشطة والعلاقات البديلة من أجل تقييم أوسع. علاوة على ذلك، فإن التحري عن كيفية التعامل مع العلاقات المؤقتة المرتبطة بالأنشطة البديلة غير كافي. تم تطوير نموذج برمجة خطية للأعداد الصحيحة المركبة لعرض برامج البناء البديلة ومن ثم نشر مبدأ الحرجية للأنشطة لدمج العلاقات البديلة. إن مثال الحساب ودراسة المشاريع يبرهنان على أن مدة المشروع والتسلسل المثالي للأنشطة يمكن إستنباطهما بإستخدام الموديل المطور.

Project schedule with alternative activities and relationships

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ABSTRACT

The complexity arising from planning large-scale construction projects often requires representation and evaluation of alternative programs. However, prevailing schedule tools seldom consider and explicitly represent the alternative activities and relationships for further evaluation. Moreover, how to deal with the temporal relationships associated with alternative activities are also inadequately explored. A Mixed Integer Linear Programming model is developed to represent the alternative construction programs, and then the concept of criticality of activities is extended to incorporate the alternative relationships. The computation example and the case study demonstrate that project period and the optimal sequence of activities can be derived using the developed model.

INTRODUCTION

Project scheduling is crucial for timely delivery of construction works. The schedule of a project minimizes its span while guaranteeing the fulfillments of construction requirements. Meanwhile, a proper schedule can also increase productivity in-situ, reduce hazards and overtime, and finally lead to lower cost and better work environment. The critical path method (CPM) and precedence diagram method (PDM) were early approaches widely used in past decades.

Fulfillment of construction requirements should consider flexible utilization of multiple realization approaches or technical solutions. Scheduling tools such as reactive scheduling, stochastic project scheduling, and fuzzy project scheduling are available for scheduling under uncertainty (Herroelen & Leus, 2005). In addition, the resource constrained project scheduling problems implicitly depicted the parallel relationships among activities that compete the same construction resources (Lu & Li, 2003).

Meanwhile, alternative approaches can be applied for fulfilling construction requirements, including both functional and non-functional (Song & Chua, 2006). The functional requirements are necessary for fulfilling the construction intentions, while the non-functional ones refer to performance constraints such as capacity, productivity, and inventory. Specifically, alternative approaches can be semantically represented by alternative construction sequences/relationships, alternative activities, and alternative resource. This greatly exacerbates the complexity and difficulty for programming construction tasks.

Some studies focused on improving the representation of a schedule via enriching its attributes. Besides the minimum lag time of precedence relationships, Neumann and Schwindt

(1997) defined the maximum lag time to restrict the interval between two activities. Douglas and his fellow researchers (2006) introduced a negative lag to model parallel relationships. Plotnick (2006) utilized Reason/Why attributes in the relationship diagramming method (RDM) to depict how parallel relationships were derived from construction requirements. Fan & Tserng (2006) developed soft logic in the SOFTCPM prototype for inferring parallel sequences of construction activities.

On the other hand, the complex temporal constraints derived from construction requirements can be semantically described by a set of interval-based temporal relationships (Song & Chua, 2007). A number of complex temporal constraints have been developed, from which traditional precedence relationships can be derived, to improve the re-sequencing capability (Chua *et al*., 2003). Afterwards, PDM++ was developed to further describe more complex temporal constraints (Chua & Yeoh, 2011), and the inference engine ECLiPSe was applied for reasoning out optimal schedule (Chua *et al*., 2013).

Recently, a constraint integration reasoning framework has been developed to reason out conflicts or redundant constraints, and it presented a systematic method to classify the criticality of schedule constraints (Nguyen & Chua, 2013 & 2014). Lorterapong and Ussavadilokrit (2013) proposed a constraint satisfaction method to provide a framework for modeling temporal constraints and generating schedules.

Although several semantic representation schemata have been developed to describe alternative construction programming, they inadequately represent alternative construction methods in the same schedule. In particular, most of the previous works assume, by default, that all activities in a schedule should be executed, and the selective application of temporal relationships may not be adequately studied and explicitly represented.

In this regard, the incorporation of alternative construction methods into a schedule is described by exclusive existence and coexistence relationships for activities and relationships. Then, these relationships can be modeled by a mixed-integer linear programming (MILP). Furthermore, the temporal attributes and criticality of construction activities can be derived, and the optimal sequence can also be derived.

REPRESENTATION OF ALTERNATIVE CONSTRUCTION PROGRAM

As mentioned before, alternative scheduling programs require the description of mutual exclusion and coexistence relationships between activities and between temporal relationships.

Mutually exclusive existence of activities

Mutually exclusive existence between/among activities (denoted by XOR) means that only one of them will be executed. For example, the access to a construction zone can be realized by either a temporary cement road or macadam pavement. Thus, paving a cement road is a mutually exclusive activity with paving macadam road since any one of them can fulfill the requirement of temporary access.

The Boolean decision variable E_i^A is introduced to depict whether the ith activity is engaged in the project. The selection should be determined by scheduling algorithm to shorten the project span.

Mathematically, if E_i^A =1, the i^{th} activity is selected (See Equation 1):-

$$E_i^A = \begin{cases} 1 & \text{if selected} \\ 0 & \text{if not} \end{cases} \tag{1}$$

In general, the duration of an activity is positive. In this context, the zero duration means that the activity is not selected. In this way, the temporal relationships associated with all activities are not affected by the selection of activities. Mathematically, D_i^*, the duration of the i^{th} activity, is represented as follows:

$$D_i^* = E_i^A \times D_i \tag{2}$$

Additionally, the mutually exclusive existence among a set of *n* activities can be represented using the following equation:

$$\sum_{i=1}^{n} E_i^A = 1 \tag{3}$$

Equation (3) shows that one and only one decision variable E_i^A will be assigned to one, while the others should be zero, meaning that only one activity will be selected.

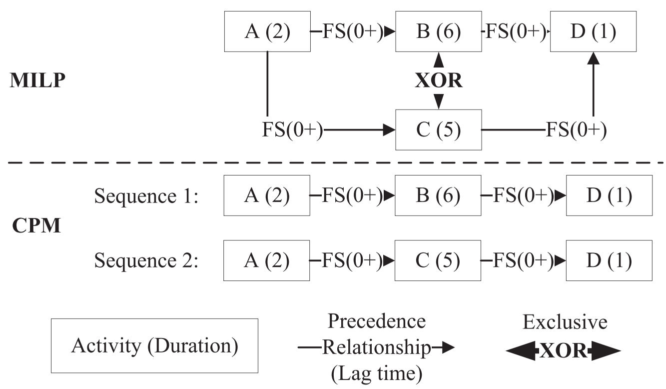


Fig. 1. Example of mutually exclusive activities

Fig. 1 illustrates an example that only one of two alternative tasks B and C can be selected for the project, while the activities A and D must be executed. Four Finish-to-Start (0+) (short for FS(0+)) generic relationships are utilized to represent the precedence relationships between four pairs of activities, that is, A and B, B and D, A and C, and C and D, which means that the latter cannot start until the former is completed.

The mutually exclusive existence between B and C can be represented as Equation (4):

$$E_B^A + E_C^A = 1 \tag{4}$$

Comparatively, if traditional scheduling tools like CPM are used for modeling the aforementioned scenario, two schedules, sequences 1 and 2, should be developed, whereas only one MILP model can describe two feasible solutions.

Coexistence of activities

Coexistence between/among activities (denoted by XNOR) is a relationship where two or more activities should be simultaneously selected into or excluded from the same schedule. For instance, if paving cement road is selected, curing the paved road should also be selected at the same time, and vice versa. Mathematically, the coexisting activities share the same Boolean decision variable E_i^A:-

$$E_1^A = E_2^A = ... = E_i^A = ... = E_n^A \tag{5}$$

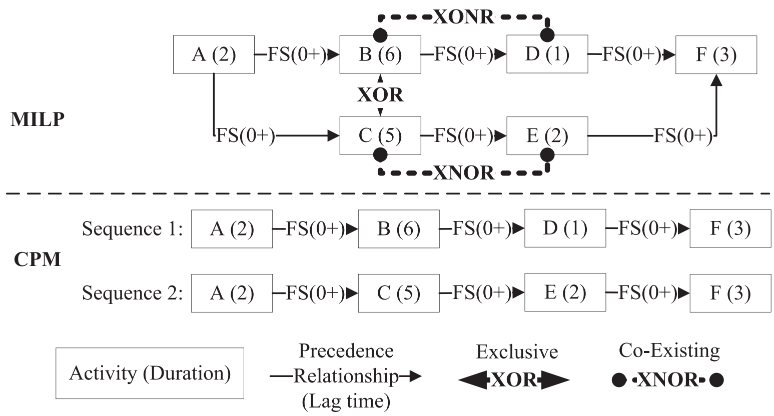


Fig. 2. Example of coexisting activities.

Fig. 2 shows an example where activity B coexists with D; meanwhile, C coexists with E. These scheduling constraints can be mathematically represented by the following equations:

$$E_B^A + E_C^A = 1 \tag{6}$$

$$E_B^A = E_D^A \tag{7}$$

$$E_C^A = E_E^A \tag{8}$$

In addition, there is a mutually exclusive relationship between activities B and C. Therefore, if B is selected, both C and E should be excluded, implying that two CPM schedules (sequences 1 and 2 in Fig. 2) are necessary to describe these relationships.

Mutually exclusive existence of relationships

Similar to the mutually exclusive relationships between/among activities, such relationships can also be used for describing alternative temporal relationships. XOR is used to represent the mutually exclusive relationships between two temporal relationships. For two road segment X and Y, the pavement can be carried out either from X to Y, or from Y to X. This can be represented by two mutually exclusive FS(0+) relationships, FS(0+) (A, B) and FS(0+) (B, A).

Mathematically, relationship *R(i, j)* between the i^{th} and the j^{th} activity is associated with a Boolean variable $E_{(i,j)}^R$, implying whether *R(i, j)* is selected. One is assigned to $E_{(i,j)}^R$ in Equation (9) indicating that the relationship *R(i, j)* is selected, while zero means exclusion of *R(i, j)* from the project planning:

$$E_{(i,j)}^R = \begin{cases} 1 & \text{if selected} \\ 0 & \text{if not} \end{cases} \tag{9}$$

Likewise, mutually exclusive relationships among *m* relationships can be mathematically represented as follows:

$$\sum_{k=1}^{m} E_k^R = 1 \tag{10}$$

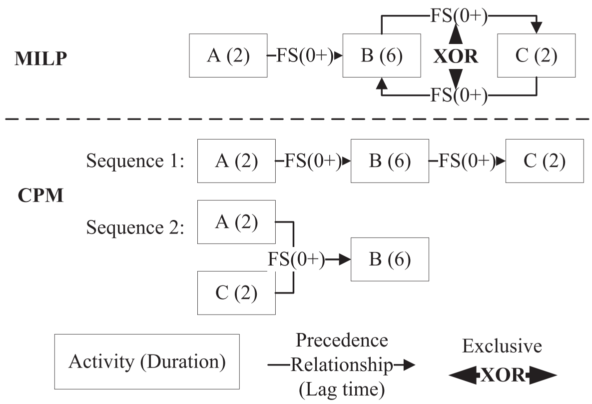


Fig. 3. Example of mutually exclusive existence relationships.

Fig. 3 illustrates that the disjoint relationship between activities B and C can be equally represented by two exclusive FS(0+) relationships, that is, FS(0+)(B, C) and FS(0+)(C, B). They can be mathematically represented by Equations (11) and (12), connected by logic operator XOR:

$$S_B + 6 \le S_C \tag{11}$$

$$\text{XOR } S_C + 2 \le S_B \tag{12}$$

Since the MILP algorithm requires all linear constraints be simultaneously satisfied, implying the AND relationship among them, in this regard, big-M approach can be applied to convert XOR logic into AND logic with the help of a Boolean decision variable $E_{(i,j)}^R$. In this way, Equations (11) and (12) can be rewritten as (13) to (16):

$$S_B + 6 - M(1 - E_{(B,C)}^R) \le S_C \tag{13}$$

$$\text{AND } S_C + 2 - M(1 - E_{(C,B)}^R) \le S_B \tag{14}$$

$$\text{AND } E_{(B,C)}^R + E_{(C.B)}^R = 1 \tag{15}$$

$$\text{AND } E_{(B,C)}^R, E_{(C.B)}^R \in \{0,1\} \tag{16}$$

Assigning one to $E_{(B,C)}^R$ means that Equation (13) is activated for scheduling, confining the early start of activity C and the late start of B. Otherwise, assigning zero to $E_{(B,C)}^R$ makes Equation (13) always satisfied, meaning that it applies no restriction for the start times of B and C. Meanwhile, Equation (14) can be similarly explained. Furthermore, Equation (15) defines the sum of $E_{(B,C)}^R$ and $E_{(C,B)}^R$ to be one, and this ensures that only one of two temporal relationships/constraints (Equations (11) and (12)) will be applied for scheduling. In this way, the mutually exclusive relationship between/among alternative temporal relationships can be modeled by the MILP.

Coexistence of relationships

Two or more temporal relationships may coexist. Similar to the representation of coexistence among activities, a set of coexisting relationships share the same value of Boolean decision variable $E^{R}_{(i,j)}$. For instance, two construction sequences are possible for paving three road segments X, Y, and Z, either from X to Y to Z, or from Z to Y to X. The former sequence can be described such that the precedence relationship FS(0+) (X, Y) coexists with FS(0+) (Y, Z), while the latter states that both FS(0+) (Z, Y) and FS(0+) (Y, X) should be simultaneously applied for scheduling.

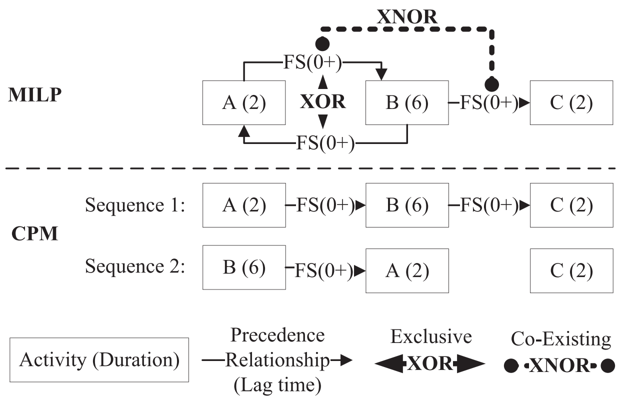


Fig. 4. Example of coexistent relationships.

Fig. 4 illustrates that activities A and B cannot overlap with each other. Only when A is finished before B, the commencement of C should wait until B is finished. On the other hand, there is no temporal constraints between B and C if A is executed after B is completed. These constraints are clearly depicted by sequences 1 and 2 in the figure. In particular, the dependence between FS(0+) (A, B) (represented by Equation (17)) and FS(0+) (B, C) (represented by Equation (18)) is a typical coexistence relationship, (represented by Equation (19)) :

$$S_A + 2 - M(1 - E^{R}_{(A,B)}) \le S_B \tag{17}$$

$$S_B + 6 - M(1 - E^{R}_{(B,C)}) \le S_C \tag{18}$$

$$E^{R}_{(A,B)} = E^{R}_{(B.C)} \tag{19}$$

PROJECT SCHEDULING WITH ALTERNATIVE PROGRAMS

Objective function of MILP

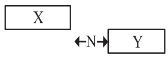
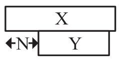
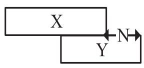
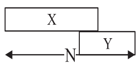
As introduced above, alternative activities and relationships can be represented as MILP constraints. The goal of the project schedule is to achieve the shortest period. Accordingly, the objective function of MILP model is to minimize the project finish (F_p) as follows:

$$MinZ = F_p \tag{20}$$

Constraint of MILP

Table 1 lists four basic precedence relationships and their linear inequalities. In this study, only Finish-Start (FS) relationship is used to represent temporal relationships since the other three can be converted into the FS relationship.

Table 1. Precedence relationships and MILP constraints.

Relationship	Diagram	MILP constraint	
		Minimum lag (lag≥0)	Maximum lag (lag<0)
Finish-Start (N)		$S_X + D_X + N \le S_Y$	$S_X + D_X \le S_Y$, $S_X + D_X + N \ge S_Y$.
Start-Start (N)		$S_X + N \le S_Y$	$S_X \le S_Y$, $S_X + N \ge S_Y$.
Finish-Finish (N)		$S_X + D_X + N \le S_Y + D_Y$	$S_X+D_X \le S_Y + D_Y$, $S_X + D_X + N \ge S_Y + D_Y$.
Start-Finish (N)		$S_X + N \le S_Y + D_Y$	$S_X \le S_Y + D_Y$, $S_X + N \ge S_Y + D_Y$.

Furthermore, the alternative activities and temporal relationships can be converted into MILP constraints. Every MILP constraint converted from temporal relationships can be rewritten as the following standard form:

$$a' \times S_i - a' \times S_j + D_i + L_{(i,j)} \le 0 \quad \forall R(i,j),\ 1 \le i \le n,\ 1 \le j \le n \tag{21}$$

$$a' \times S_i - a' \times S_j + E_i^A \times D_i + L_{(i,j)} \le 0 \quad \forall R(i,j),\ 1 \le i \le n,\ 1 \le j \le n \tag{22}$$

$$a' \times S_i - a' \times S_j + D_i + L_{(i,j)} - M(1 - E_{(i,j)}^R) \le 0 \quad \forall R(i,j),\ 1 \le i \le n,\ 1 \le j \le n \tag{23}$$

$$a' \times S_i - a' \times S_j + E_i^A \times D_i + L_{(i,j)} - M(1 - E_{(i,j)}^R) \le 0 \quad \forall R(i,j),\ 1 \le i \le n,\ 1 \le j \le n \tag{24}$$

$$E_i^A = E_j^A \text{ if the } i^{th} \text{ activity coexists with the } j^{th} \text{ activity } \forall i,j,\ 1 \le i \le n,\ 1 \le j \le n \tag{25}$$

$$E_i^A + E_j^A = 1 \text{ if the } i^{th} \text{ activity mutually excludes with the } j^{th} \text{ activity } \forall i,j,\ 1 \le i \le n,\ 1 \le j \le n \tag{26}$$

$$E_{(i,j)}^R = E_{(k,l)}^R \text{ if } R(i,j) \text{ coexists with } R(k,l) \quad \forall R(i,j), R(k,l)\ 1 \le i,j,k,l \le n \tag{27}$$

$$E_{(i,j)}^R + E_{(k,l)}^R = 1 \text{ if } R(i,j) \text{ mutually excludes with } R(k,l) \quad \forall R(i,j), R(k,l)\ 1 \le i,j,k,l \le n \tag{28}$$

$$E_i^A, E_{(i,j)}^A \in \{0,1\} \tag{29}$$

$$S_i \ge 0 \quad i = 1,2,......,n \tag{30}$$

$$S_i + D_i \le F_p \quad i = 1,2,......,n \tag{31}$$

a' : Coefficient that equals 1 or -1 .

S_i: Start time of the i^{th} activity.

D_i: Duration of the i^{th} activity.

$R_{(i,j)}$: Precedence relationship between the i^{th} and the j^{th} activity.

$L_{(i,j)}$: Lag time between the finish of the i^{th} activity and the start of the j^{th} activity.

E_i^A : Boolean variable for whether or not the i^{th} activity is selected.

$E_{(i,j)}^R$: Boolean variable for whether or not the relationship between the i^{th} and the j^{th} activities is selected.

M: A positive number big enough.

F_P : Project's finish time.

Equation (21) describes the constraint for activities and temporal relationships that must be performed. Equation (22) is a constraint for alternative activities, and Equation (23) is a constraint for alternative relationships. Equation (24) is for both alternative activities and alternative temporal relationships, with Boolean variables E_i^A and $E_{(i,j)}^R$ for selecting activities and relationships respectively, Equations (25) and (26) are descriptions of the mutually exclusive and coexisting relationships between the activities, respectively. Equations (27) and (28) describe the mutually exclusive and coexisting relationships of the precedence relationships. Equation (30) restricts that all activities' start times must be later than the project start time, that is, 0. Meanwhile, Equation (31) restricts all activities to be completed no later than the project's finish time, defined by F_P. In this way, the computed optimal value F_p^* is the shortest project span.

Criticality of activity

Once the minimum F_p^* is determined, further analysis of criticality can be conducted. In order to determine the early start (ES) for each activity, the goal function Z can be modified as follows:

$$MinZ = \sum_{i=1}^{n} S_i \tag{32}$$

The minimization of Z is to determine the early commencement of each activity. And the corresponding solution is called the early solution in this study.

The constraints used for finding early start times are kept nearly the same as those (Equations 21 to 31) used for deriving the project period, except for the F_P in Equation (31), which is replaced by the known project period F_p^* in Equation (33) as follows:

$$S_i + D_i \le F_p^* \tag{33}$$

Likewise, the maximization of the objective function Z in Equation (34) can produce the late start *(LS)* times for each activity, which is called the late solution in this study. The constraints are the same as those used for deriving the early starts:

$$MaxZ = \sum_{i=1}^{n} S_i \tag{34}$$

After the calculation of *ES* and *LS* of an activity, its criticality can then be determined by its float time. According to the traditional definition of the concept criticality, if the early and late start times of an activity are different, then this activity is non-critical. By contrast, in this study, an activity is non-critical only when it has an interval for floating. Particularly, even if the early start of activity is less than its late start, it may not have float time due to the incorporation of alternative relationships. This will be elaborated in the later section of the case study.

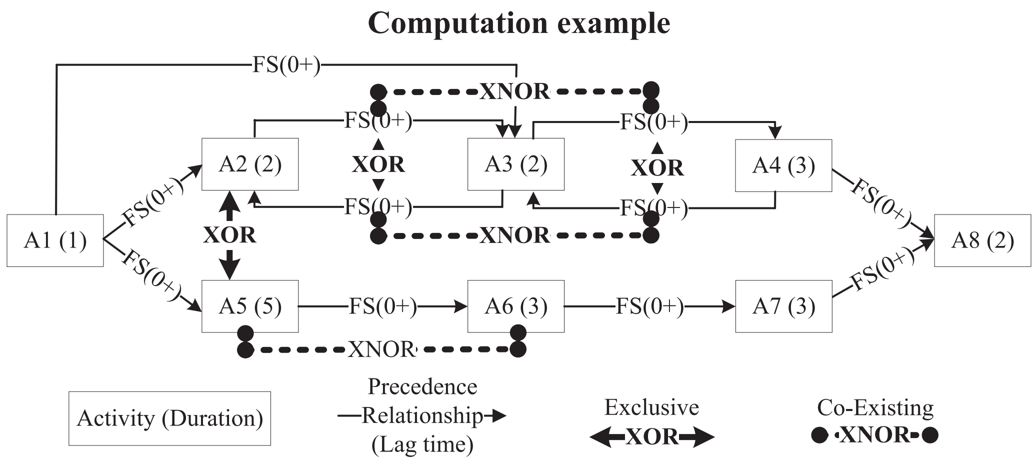


Fig. 5. Example with two construction methods.

Fig. 5 illustrates a computation example with eight activities (A1 to A8) and eleven precedence relationships listed in Table 2. Accordingly, the project schedule is modeled by Equations (35) to (55) :

$$Min\ Z = F_p \tag{35}$$

Subject to:

$$S_1 + 1 \le S_2 \tag{36}$$

$$S_1 + 1 \le S_3 \tag{37}$$

$$S_1 + 1 \le S_5 \tag{38}$$

$$S_2 + 2E_2^A - M(1 - E_{(2,3)}^R) \le S_3 \tag{39}$$

$$S_3 + 2 - M(1 - E_{(3,2)}^R) \le S_2 \tag{40}$$

$$S_3 + 2 - M(1 - E_{(3.4)}^R) \le S_4 \tag{41}$$

$$S_4 + 3 - M(1 - E_{(4,3)}^R) \le S_3 \tag{42}$$

$$S_4 + 3 \le S_8 \tag{43}$$

$$S_5 + 5E_5^A \le S_6 \tag{44}$$

$$S_6 + 3E_6^A \le S_7 \tag{45}$$

$$S_7 + 3 \le S_8 \tag{46}$$

$$E_2^A + E_5^A = 1 \tag{47}$$

$$E_5^A = E_6^A \tag{48}$$

$$E_{(2,3)}^R + E_{(3,2)}^R = 1 \tag{49}$$

$$E_{(3,4)}^R + E_{(4,3)}^R = 1 \tag{50}$$

$$E_{(2,3)}^R = E_{(3,4)}^R \tag{51}$$

$$E_{(4,3)}^R = E_{(3,2)}^R \tag{52}$$

$$E_2^A,\ E_5^A,\ E_6^A,\ E_{(2,3)}^R,\ E_{(3.4)}^R,\ E_{(4,3)}^R,\ E_{(3,2)}^R \in \{0,1\} \tag{53}$$

$$S_i \geq 0 \ \ i = 1,2,......,n \tag{54}$$

$$S_i + D_i \leq F_p \ \ i = 1,2,......,n \tag{55}$$

Equation (35) defines the goal function in detail to deliver the project as early as possible. Table 2 shows that Equations (36) to (46) are linear constraints converted from those eleven FS(0+) relationships illustrated in Fig. 5, using the conversion rules listed in Table 1.

Table 2. Precedence relationships and their linear constraints of the example

Relating activity	Related activity	Relationship	Linear constraint
A1	A2	FS (0+)	(36)
A1	A3	FS (0+)	(37)
A1	A5	FS (0+)	(38)
A2	A3	FS (0+)	(39)
A3	A2	FS (0+)	(40)
A3	A4	FS (0+)	(41)
A4	A3	FS (0+)	(42)
A4	A8	FS (0+)	(43)
A5	A6	FS (0+)	(44)
A6	A7	FS (0+)	(45)
A7	A8	FS (0+)	(46)

The Boolean variables E_2^A, E_5^A and E_6^A represent whether activities A2, A5, and A6 are selected for scheduling, respectively. Clearly, Equation (47) indicates that A2 and A5 are exclusive with each other. Equation (48) indicates that A5 and A6 coexist. Meanwhile, $E_{(2,3)}^R$, $E_{(3.4)}^R$, $E_{(4,3)}^R$, $E_{(3,2)}^R$ and denote whether the temporal relationships FS(0+)(A2, A3), FS(0+)(A3, A4), FS(0+)(A4, A3), and FS(0+)(A3, A2) are selected, respectively. Based on these four Boolean decision variables, Equation (49) describes the mutual exclusive relationship between FS(0+)(A2, A3) and FS(0+)(A3, A2), while Equation (50) represents either FS(0+)(A3, A4) or FS(0+)(A4, A3) can be selected for scheduling at the same time. Subsequently, Equations (51) and (52) abstract that FS(0+)(A2, A3) coexists with FS(0+)(A3, A4), and FS(0+)(A4, A3) with FS(0+)(A3, A2). In this way, all the mutual exclusive relationships and coexistence relationships in Fig. 5 can be modeled by linear constraints.

The aforementioned MILP problem can be solved by a linear programming solver. Finally, the resultant value of F_p^* is 7, the shortest project period.

Table 3. Results of Boolean variables of the example.

Boolean variable	Activity/relationship	Early solution	Late solution
E_2^A	A2	1	1
E_5^A	A5	0	0
E_6^A	A6	0	0
$E_{(2,3)}^R$	FS(0+)(A2, A3)	0	0
$E_{(3,2)}^R$	FS(0+)(A3, A2)	1	1
$E_{(3.4)}^R$	FS(0+)(A3, A4)	0	0
$E_{(4,3)}^R$	FS(0+)(A4, A3)	1	1

The true value of E_2^A means that activity A2 is selected in both early and late solutions while the false value of the resultant E_5^A and E_6^A indicates that activities A5 and A6 are excluded from scheduling. This is consistent with the fact A6 coexists with A5. Furthermore, the temporal relationship FS(0+)(A4, A3) is selected while FS(0+)(A3, A4) is excluded since these are two mutually exclusive relationships. Meanwhile, FS(0+)(A3, A2) is selected but FS(0+)(A2, A3) is excluded from scheduling.

Table 4. Criticality analysis of the example.

Activity	Early start	Early finish	Late start	Late finish	Float time	Criticality
A1	0	1	1	2	1	None-critical
A2	5	7	5	7	0	Critical
A3	3	5	3	5	0	Critical
A4	0	3	0	3	0	Critical
A7	1	4	2	5	1	None-critical
A8	4	6	5	7	1	None-critical

Besides, by replacing the objective Equation (35) with Equations (32) and (34) respectively, and substituting the constant project finish time 7 for the variable F_P in Equation (51), the *ES* and *LS* times of six selected activities (A1 to A4, and A7 to A8) can be computed and listed in Table 4, and consequently their resultant *ES* and *LS* times indicate that the three activities A2, A3, and A4 are on the critical path since they have equal ES and LS. On the other hand, the other three selected activities A1, A7, and A8 are non-critical.

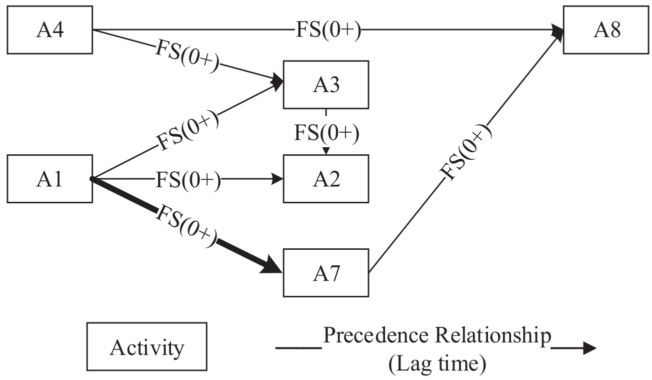


Fig. 6. Final solution.

Fig. 6 illustrated the derived schedule where six selected activities are connected by seven selected relationships. The resultant schedule network implies that even if an activity is not selected, the precedence relationships between its upper stream activities and downstream activities should remain in the schedule. As shown in Fig. 5, A1 should be executed before A5 and A6 that are excluded from the schedule, and A7 should be performed just after A6. Then, it can be intuitively inferred that A1 should be executed before A7. This inferred precedence relationship FS(0+)(A1, A7) remains in the resultant schedule, illustrated by the thick arrow in Fig. 6.

CASE STUDY

Problem description

The preparation works for excavating foundation pit to construct an underground traffic circle are presented to demonstrate the application of the developed scheduling model. Table 5 lists 12 activities with their durations to be scheduled.

Table 5. Activity list of the case

Index	Activity name	Duration (days)
1	Level ground	7
2	Install jet pipeline (for high-pressure jet)	1
3	Grout to default level (for high-pressure jet)	13
4	Remove jet pipeline (for high-pressure jet)	1
5	Install mixing equipment (for three-axis mixing)	1
6	Mix to default level (for three-axis mixing)	15
7	Remove mixing equipment (for three-axis mixing)	1
8	Construct fender piles	28
9	Transport of equipment for reinforcement	1
10	Reinforce Zone 1	12
11	Reinforce Zone 2	12
12	Cast grade beam	15

A foundation pit with the depth of about 12 to 14 meters should be excavated for constructing the traffic circle. In particular, the waterproof curtain should be constructed before earth excavation in order to prevent underground water from entering the construction zone, while the fender piles are constructed to maintain the stability of the soil slide. The waterproof curtain can be constructed by either high-pressure jet grouting or three-axis mixing, indicating the alternative construction programming.

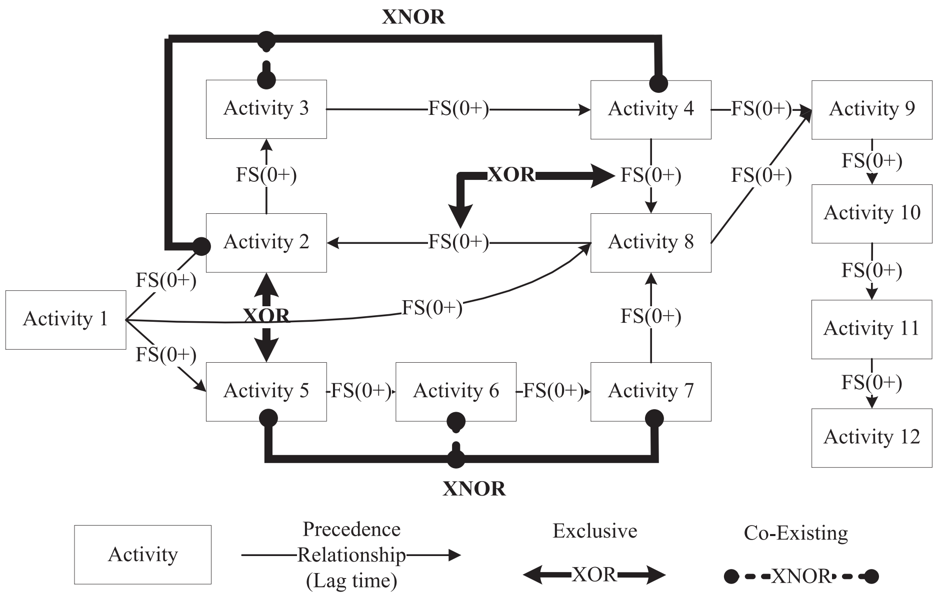


Fig. 7. Project scheduling network

Moreover, Fig. 7 illustrates the temporal relationships for planning the construction works. Numbering of the activities in the figure follows their indexes in Table 5. The leveling ground is the initial activity to prepare the workface of the construction site in detail, followed by constructing the waterproof curtain.

Two construction methods, either high-pressure jet grouting or three-axis mixing, are feasible for constructing the waterproof curtain. If the former is utilized, installing jet pipeline, grouting to default level and removing jet pipeline will be sequentially engaged in the project, which is represented by two FS(0+) relationships between activities 2 and 3 and between activities 3 and 4. Moreover, three XNOR relationships between those three activities indicate that they should simultaneously be selected or excluded in the project schedule. On the other hand, installing mixing equipment, mixing to default level, and removing mixing equipment can be executed in sequence to realize the latter method. Similarly, two FS(0+) relationships connect those three activities, which are linked by three XNOR relationships to define their coexistence (See Fig.7).

Furthermore, Fig. 7 also shows the XOR relationship between activities 2 and 5, meaning that only one of them can be selected into scheduling. In this way, each activity engaged in the high-pressure jet grouting approach cannot coexist with those in the three-axis mixing approach since the three activities in the same approach coexist, defined by the XNOR relationships. In addition, two FS(0+) relationships between activities 1 and 2 and between activities 1 and 5 indicate that the curtain wall should be constructed after the ground leveling is completed no matter which construction method is chosen.

Meanwhile, the fender piles should also be constructed after the ground is leveled, denoted by FS(0+) between activities 1 and 8. In particular, if the fender piles have been established, only high-pressure jet grouting can be applied since the blades for drilling of the three-axis mixing may damage the constructed fender piles.

The constructions of waterproof curtain and fender piles are followed by the reinforcement for the waterproof curtain. Due to the shortage of labor and equipment resources, the reinforcement work has to be divided into two segments, that is, Zone 1 and Zone 2. Considering the geological conditions, Zone 1 is reinforced before Zone 2. Subsequently, the grade beam will be casted. These precedence relationships can be represented by FS(0+). Finally, the foundation pit can be excavated.

The MILP model

The aforementioned temporal relationships and alternative sequencing conditions can be converted into linear constraints for the MILP scheduling model.

Table 6. Precedence relationships and linear constraints of the case

Relating activity	Related activity	Relationship	Linear constraint
1	2	FS (0+)	(57)
1	5	FS (0+)	(58)
1	8	FS (0+)	(59)
2	3	FS (0+)	(60)
3	4	FS (0+)	(61)
4	8	FS (0+)	(62)
4	9	FS (0+)	(63)
5	6	FS (0+)	(64)
6	7	FS (0+)	(65)
7	8	FS (0+)	(66)
8	2	FS (0+)	(67)
8	9	FS (0+)	(68)
9	10	FS (0+)	(69)
10	11	FS (0+)	(70)
11	12	FS (0+)	(71)

The precedence relationships between construction activities of the case are listed in the first 3 columns of Table 6, while the last column shows the indexes of the linear constraints used in the MILP model, described by Equations (56) to (78) :

$$Min\ Z = F_p \tag{56}$$

Subject to:

$$S_1 + 7 \le S_2 \tag{57}$$

$$S_1 + 7 \leq S_5 \tag{58}$$

$$S_1 + 7 \leq S_8 \tag{59}$$

$$S_2 + 1E_2^A \leq S_3 \tag{60}$$

$$S_3 + 13E_3^A \leq S_4 \tag{61}$$

$$S_4 + 1E_4^A - M(1 - E_{(4,8)}^R) \leq S_8 \tag{62}$$

$$S_4 + 1E_4^A \leq S_9 \tag{63}$$

$$S_5 + 1E_5^A \leq S_6 \tag{64}$$

$$S_6 + 15E_6^A \leq S_7 \tag{65}$$

$$S_7 + 1E_7^A \leq S_8 \tag{66}$$

$$S_8 + 28 - M(1 - E_{(8,2)}^A) \leq S_2 \tag{67}$$

$$S_8 + 28 \leq S_9 \tag{68}$$

$$S_9 + 1 \leq S_{10} \tag{69}$$

$$S_{10} + 12 \leq S_{11} \tag{70}$$

$$S_{11} + 12 \leq S_{12} \tag{71}$$

$$E_2^A + E_5^A = 1 \tag{72}$$

$$E_2^A = E_3^A = E_4^A \tag{73}$$

$$E_5^A = E_6^A = E_7^A \tag{74}$$

$$E_{(4,8)}^R + E_{(8,2)}^A = 1 \tag{75}$$

$$E_2^A,\ E_3^A,\ E_4^A,\ E_5^A,\ E_6^A,\ E_7^A,\ E_{(4,8)}^R,\ E_{(8,2)}^A \in \{0,1\} \tag{76}$$

$$S_i \geq 0 \;\; i = 1,2,\ldots\ldots,n \tag{77}$$

$$S_i + D_i \leq F_p \;\; i = 1,2,\ldots\ldots,n \tag{78}$$

Equation (56) defines the goal function to minimize the project finish time. Equations (57) to (71) are converted from FS(0+) relationships listed in Table 6. Subsequently, the mutually exclusive and coexistent relationships should be converted into constraints for the MILP model.

The Boolean variables E_2^A, E_3^A, and E_4^A as well as E_5^A, E_6^A, and E_7^A are used to represent the mutually exclusive existence of two construction methods. If the high-pressure jet grouting method is selected, a value of one is assigned to E_2^A, E_3^A, and E_4^A. If the three-axis mixing method is executed, a value of one is assigned to E_5^A, E_6^A, and E_7^A. To realize that goal, Equation (72) defines the sum of E_2^A and E_5^A as 1.

Meanwhile, the fender piles cannot be constructed concurrently with the works associated with the three-axis mixing method. In other words, if the fender piles are constructed later than the waterproof curtain, both construction methods can be utilized. In this case, two decision variables, $E^R_{(4,8)}$ and $E^A_{(8,2)}$, are used to represent this constraint via defining the mutually exclusive existence of two temporal relationships defined by Equations (62) and (67), respectively. $E^R_{(4,8)}$ being 1 means that the fender piles should be constructed after the waterproof curtain, while $E^A_{(8,2)}$ being 1 indicates that the waterproof curtain must be constructed after the fender piles. Accordingly, Equation (75) indicates that only one of these two relationships can be selected.

Once the earliest project finish F^*_P (i.e., project period) is determined, the variable F_P in Equation (78) is replaced with the resultant F^*_P. Then, the early start times for each activity can be computed by replacing the goal function defined by Equation (56) with Equation (32). The new goal function guarantees that each activity, if selected, will start as early as possible. Likewise, late start times for all activities can be calculated via replacing Equation (56) by Equation (33).

Results and discussion

The linear programming solver in the EXCEL software is adopted for solving the model. The earliest finish time of the project F_P is 90 days. And the resultant variables are computed and listed in Table 7.

Table 7. Boolean variables for selecting alternative programs of the case.

Variables	Early solution	Late solution
E^A_2	1	1
E^A_3	1	1
E^A_4	1	1
E^A_5	0	0
E^A_6	0	0
E^A_7	0	0
$E^R_{(4,8)}$	1	0
$E^A_{(8,2)}$	0	1

The resultant true values of E^A_2, E^A_3, and E^A_4 in Table 7 as well as the false values of E^A_5, E^A_6 and E^A_7 indicate that the construction method of high-pressure jet grouting is selected to ensure earlier delivery of the project no matter when the construction works will be scheduled, as early as possible or as late as possible. In this context, the fender piles should be constructed after the waterproof curtain for the early solution, whereas they should be constructed before the waterproof curtain for the late solution. This is implied by the different values of $E^R_{(4,8)}$ and $E^A_{(8,2)}$ in Table 7.

Most of the previous works assumed, by default, that all activities in a schedule should be executed using the approach developed in this study. The alternative activities and temporal relationships can be determined according to the construction requirement flexibly with the Boolean operators E^A_i and $E^R_{(i,j)}$.

Table 8. Criticality analysis of the case

Index	Activity	Early start	Early finish	Late start	Late finish	Criticality
1	Level ground	0	7	0	7	Critical
2	Install jet pipeline	7	8	35	36	Critical
3	Grout to default level	8	21	36	49	Critical
4	Remove jet pipeline	21	22	49	50	Critical
8	Construct fender piles	22	50	7	35	Critical
9	Transport of equipment for reinforcement	50	51	50	51	Critical
10	Reinforce Zone 1	51	63	51	63	Critical
11	Reinforce Zone 2	63	75	63	75	Critical
12	Cast grade beam	75	90	75	90	Critical

Table 8 lists the early and late start times of each selected activity. Three activities are not listed in Table 8 since they are associated with the construction method of three-axis mixing; they are not selected for scheduling in this case. Although the early start times of activities 2, 3, 4, and 8 are different from their late start times, they are still on the critical path since they have no float intervals to delay. In this regard, all selected nine activities are critical.

The difference between the early and late start times arises from the alternative temporal relationships that the fender piles can be constructed either before the installation of jet pipelines or after the removal of jet pipelines. In Table 7, $E^R_{(4,8)}/E^A_{(8,2)}$ is assigned by different values in the early and the late solutions. This implies that the float time of an activity should consider the selection of its associated alternative relationships. In this case, either in the early or the late solution, the fender pile work (activity 8) has no float time, which can be derived using the temporal data listed in Table 8. Similarly, the float times of activities 2 to 4 can be computed. So these four activities, with their early times being different from their late times, are critical.

This means that the criticality concept should be extended to consider the selection of alternative relationships besides the early and late start/finish times of an activity, which is seldom evaluated in the previous studies on complex temporal relationships.

CONCLUSIONS

This research has attempted to develop a MILP model to incorporate alternative activities and alternative relationships in one project schedule. In this way, alternative means for fulfilling construction requirements can be converted into MILP constraints. In particular, two types of Boolean operators, XOR and XNOR, are used to describe the mutual exclusive and coexisting relationship between activities/relationships, respectively. Then, the method to convert these two Boolean operators into MILP constraints is also developed. Moreover, the shortest period and the early and late start times of each activity can be derived by defining different objective functions. The results of the case study indicate that two or more construction methods can be incorporated into one schedule for exploring optimal sequences of construction works and concurrently determining better construction approaches. Meanwhile, this study suggests that the concept of the criticality of an activity should be extended to consider both its early and late times and its associated alternative relationships.

Currently, the MILP solver can only resolve a construction schedule of small-to-medium scales. Meta-heuristics algorithms are being explored for large-scale construction schedules with multiple alternatives of construction sequences. At the same time, the mathematical representation of temporal relationships is manually converted to linear constraints for MILP. In the future, a prototype software will be developed to realize the automatic conversation, and meanwhile the representation of more complex temporal relationships will be furthered to improve the capacity and efficiency for semantically representing more flexible scheduling ideas.

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REFERENCES

Chua, D.K.H., Nguyen, T.Q. & Yeoh, K.W. 2013. Automated construction sequencing and scheduling from functional requirements. Automation in Construction, **35**(11):79–88.

Chua, D.K.H., Shen, L.J. & Bok, S.H. 2003. Constraint-based planning with integrated production scheduler over internet. Journal of Construction Engineering & Management, **129**(3):293 -301.

Chua, D.K.H. & Yeoh, K.W. 2011. PDM++: planning framework from a construction requirements perspective. Journal of Construction Engineering & Management, **137**(4):266 -274.

Douglas III, E.E., Calvey, T.T., McDonald Jr, D.F. & Winter, R.M. 2006. The great negative lag debate. AACE International Transactions, PS02, www.calveyconsulting.com/files/PS02_Great_Negative_Lag_Debate.pdf.

Fan, S.L. & Tserng, H.P. 2006. Object-oriented scheduling for repetitive projects with soft logics. Journal of Construction Engineering & Management, **132**(1):35-48.

Herroelen, W. & Leus, R. 2005. Project scheduling under uncertainty: Survey and research potentials. European Journal of Operational Research, **165**(2):289-306.

Lorterapong, P. & Ussavadilokrit, M. 2013. Construction scheduling using the constraint satisfaction problem method. Journal of Construction Engineering & Management, **139**(4):414-422.

Lu, M. & Li, H. 2003. Resource-activity critical-path method for construction planning. Journal of Construction Engineering & Management, **129**(4):412-420.

Nguyen, T.Q. & Chua, D.K.H. 2013. Preemptive constraint analysis in construction schedules. Journal of Computing in Civil Engineering, **29**(5): 04014062.

Nguyen, Q.T. & Chua, D.K.H. 2014. Criticality of schedule constraints -- classification and identification for project management. Journal of Engineering Project & Production Management, **4**(1):17-25.

Plotnick, F.L. 2006. RDM-relationship diagramming method. AACE International Transactions, PS81.

Neumann, K. & Schwindt, C. 1997. Activity-on-node networks with minimal and maximal time lags and their application to make-to-order production. Operations Research-Spektrum, **19**(3):205-217.

Song, Y. & Chua, D.K.H. 2006. Modeling of functional construction requirements for constructability analysis. Journal of Construction Engineering & Management, **132**(12):1314-1326.

Song, Y. & Chua, D.K.H. 2007. Temporal logic representation schema for intermediate function. Journal of Construction Engineering & Management, **133**(4):277-286.

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