Effect of statistical scatter in the elastic properties on the predictability of first ply failure of a polymer composite pressure vessel

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ABSTRACT

Statistical variations in the elastic properties of composites are unavoidable. This work is concerned with the assessment of the probabilistic first ply failure of pressure vessel in response to the variations of elastic properties of T700 carbon/epoxy composite among the published literature. Initially, deterministic failure pressure is estimated using Finite Element Analysis software ANSYS. The uncertainties with respect to elastic properties are quantified by using Statistical software MINITAB. Then, the probability design system platform in ANSYS software has been utilized to perform probabilistic failure analysis by Monte Carlo simulations. The statistical results in terms of the mean and standard deviation of first ply failure pressure are computed. The probabilistic first ply failure pressure range determined is within 3σ limits with 99.997% confidence level. It is helpful to the designers in assessing the factor of safety in the case of anticipated variations in properties.

Keywords: Elastic properties; First-ply failure; Finite element analysis; Pressure vessel; Statistical analysis.

INTRODUCTION

Polymer composites are the advanced composites that find valuable applications in high pressure storage and aircraft applications due to their superior stiffness/weight ratio, strength, corrosion resistance, etc. (Dahl *et al*., 2019; Rafiee *et al.,* 2020). Evaluation of the first ply failure pressure of composite cylinders is essential for designers as experimentation is not always feasible. The accuracy of prediction of first ply failure pressure greatly depends on the elastic and strength properties of composites. But it is obvious to have variation in properties of composite materials in response to the variations or errors in volume fractions and fiber angles. Soden *et al.* (2004) warn about the unavoidable variability in the design arising from loading, material property, and manufacturing variations that include orientation of fiber and thickness of the laminate. Uncertainty concepts have been prescribed by Lin *et al.* (2000) with regard to structural design. Vanaerschot *et al.* (2015) describe the manufacturing uncertainties and their effects on composite stiffness. Rafiee and Torabi (2018) considered uncertainties in the manufacturing of composites and their effects on properties. They have predicted the mean value of first ply failure pressure very close to the deterministic value. In an earlier study, Rafiee and Amini (2015) have conducted a stochastic study by varying fiber winding angles and volume fractions of composites in the failure prediction of composite cylinders under internal pressure. It is reported that about 10 % error in volume fraction leads to a 22 % variation in failure pressure. Similarly, a 10% error in winding angle also brought about the same variation in the failure pressure. Probabilistic analysis is found to be of interest in other similar design fields also. Krishnaveni *et al.* (2014) have followed a stochastic analysis procedure in assessing the failure pressure of high strength metallic rocket motor cases. They have considered scattering in yield strength, ultimate strength, and thickness of metallic cylindrical pressure vessels and evaluated reliability-based safety factors. They identified the most accurate failure prediction formula for the failure of metallic pressure vessels of different materials under specified reliability of 95%, 99%, and 99.99%.

The material considered in the present work is polymer composite T700 carbon/epoxy. T700 carbon fiber is known to replace T300 due to better densification and possessing higher tensile strength. Cylinders wound by T700 fibers are suitable for long life combined with high reliability of 99% (Xiaobing and Yongbo, 2015). Variability in the properties of composite T700 carbon/epoxy, among the ones fabricated by different researchers, is considered in the present analysis. Material properties are considered to be orthotropic and linear. Hence, unit pressure is applied in the analysis, and stress calculations are done by linear extrapolation. The analysis would result in different first ply failure pressure values while using different properties. Hence, it is the designer's concern to have a statistical approach that would take the variability into account and build confidence in the design. The procedure followed in the present work towards this task uses two software tools, namely, MINITAB and ANSYS Probabilistic Design System (PDS) module. MINITAB is used to find the best distribution, mean, and coefficient of variation (COV) of the property variations by conducting the Anderson Darling test. PDS module of ANSYS is utilized to compute the first ply failure pressure of composite cylinder using the above information through Monte Carlo simulation with a specified number of iterations. The results are further postprocessed using MINITAB software for getting the statistical result of first ply failure pressure.

DETERMINISTIC ANALYSIS

The pressure vessel considered for the demonstration of the procedure is a metal lined 11-layered composite pressure vessel. For the deterministic evaluation of first ply failure pressure of cylindrical composite pressure vessel (p_{fpf}), the input parameters include radius, the thickness of laminae, orientations of laminae, Young's moduli E_1, E_2, E_3, shear moduli G_{12}, G_{23}, G_{13}, and Poisson's ratios $\nu_{12}, \nu_{23}, \nu_{13}$. It shall be noted that $E_3 = E_2$, $G_{13} = G_{12}$ and $\nu_{13} = \nu_{12}$. These elastic properties are taken from Wu *et al.* (2015) and included in Table 1. The strength properties considered in this analysis are presented in Table 2. Metallic liner is specified as isotropic material with bilinear stress-strain behavior. The elastic and strength properties (Young's modulus E, Poisson's ratio ν, tangent modulus E_t, yield strength σ_{ys} and ultimate tensile strength σ_{ult}) of the aluminum liner are shown in Table 3. The pressure vessel is modeled as a shell of revolution through 90^o. The element type used is SHELL281 of ANSYS software. The geometric details of the pressure vessel including radius, layer angle, and thickness are taken from Liu *et al.* (2014) and presented in Table 4. Mesh convergence study has been made, and accordingly, the model has been meshed with an element size of 1mm. One end of the cylinder is constrained in the axial direction. The other end is applied with meridional stress per unit thickness that corresponds to an internal pressure of 1 MPa. The internal pressure of 1 MPa is applied on the inner surface. Both the lateral edges are applied with symmetric boundary conditions. Figure 1 shows the geometric model created in the ANSYS software.

Table 1. Elastic properties of T700 carbon/epoxy composite according to various authors.

Reference	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	G_{23} (GPa)	υ_{12}	υ_{23}	p_{fpf}
Farhood *et al.* (2017)	135.0	8.50	4.40	3.05	0.34	0.31	45.62
Ramanjaneyulu (2018)	130.0	6.90	5.60	4.50	0.30	0.32	49.86
Reddy *et al.* (2015)	110.0	9.00	3.58	4.50	0.32	0.25	44.77
Wang *et al.* (2016)	145.2	6.99	2.67	2.30	0.26	0.44	49.74
Wu *et al.* (2015)	154.1	10.30	7.09	3.79	0.28	0.49	43.71
Xu *et al.* (2009)	181.0	10.30	7.00	4.00	0.28	0.49	41.19
Yingjun *et al.* (2010)	128.0	10.50	5.00	5.00	0.28	0.40	41.09

Table 2. Strength properties of T700 carbon/epoxy composite (Wu *et al.,* 2015).

X_T, MPa	Y_T , MPa	X_C, MPa	Y_C, MPa	S, MPa
2500	60	1250	186	85

Table 3. Properties of 6061-T6 aluminium liner (Wu *et al.,* 2015).

E, MPa	ν	E_t, MPa	σ_{ys}, MPa	σ_{ult}, MPa
70000	0.33	600	246	324

Table 4. Geometric details of metal lined composite pressure vessel (R_i=91.5 mm) (Liu et al., 2014).

Layer #	**Layer angle, (°)**	**Thickness, mm**	**Layer #**	**Layer angle, (°)**	**Thickness, mm**
1	liner	5	7	22	0.87
2	90	2.1	8	27	0.87
3	12.3	0.87	9	32	0.87
4	15.4	0.87	10	38	0.6
5	18.6	0.87	11	90	0.54
6	90	2.1			

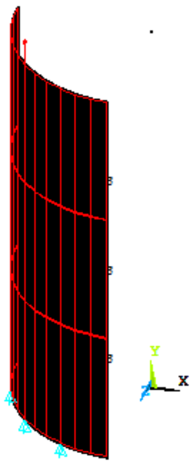

Figure 1. Geometric model of the pressure vessel.

The analysis is run, and hoop stress (σ_X), meridional stress (σ_Y), and the shear stress (τ_{XY}) on a node at mid-length of a lateral edge are noted for all 11 layers. These stress values are transformed into the local coordinate system using the formulae given in matrix form.

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \tau_{12}\end{bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2-n^2\end{bmatrix}\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix} \tag{1}$$

where $m = \cos\theta$ and $n = \sin\theta$; θ is the ply orientation with respect to the axis of the cylinder. σ_1 and σ_2 are normal stresses along with fiber and transverse to fiber, respectively. τ_{12} is the shear stress. Using the values of $p_1 = X_T/\sigma_1$, $p_2 = Y_T/\sigma_2$ and $p_3 = S/\tau_{12}$ of all the eleven layers, the minimum pressure of either p_1 or p_2 or p_3 is the first ply failure pressure, where X_T, Y_T and S are the strength values in fiber, transverse, and shear directions. The source literature reports that the first ply failure pressure tested is approximately 40 MPa (Liu *et al.,* 2014). The last column of Table 1 shows the deterministic failure pressure evaluated using different elastic properties. It may be noted that the deterministic first ply failure computed with the properties presented by Liu *et al.* (2014) (the same as Wu *et al.,* 2015) is 43.71 MPa.

PROBABILISTIC ANALYSIS

Among all the inputs mentioned in the deterministic analysis, statistical variation is considered for $E_1, E_2, G_{12}, G_{23}, \nu_{12}$ and ν_{23}. A few researchers dealing with T700 carbon/epoxy composite pressure vessels have been identified, and the properties observed are enlisted in Table 1. Apart from the elastic properties mentioned above, the strength properties are also required to find out the failure pressure. It is now required to find the most suitable distribution such as Normal, Log-Normal, Weibull, and Gamma for the input variables. Anderson-Darling statistic (AD) test gives two parameters known as AD static value and P-value. The P-value signifies the incorrectly rejecting probability. It needs to be higher than the prescribed significance usually 0.05. Figure 2 shows the significance of the P-value. If it falls below 0.05, it is to be concluded that the particular distribution is not valid. AD value depends on the sample size and the cumulative distribution function of the variable concerned. While selecting the best distribution, it has to be seen that P is more than 0.05, as well as the highest and AD, is the most minimum. In order to carry out the probabilistic failure analysis in ANSYS, all the parameters are needed to be stored as scalar parameters. Following the procedure outlined above, property input, shell definition, modeling, meshing, application of constraints, and loading are done specifying the parameters. The analysis is run, and the typical node number is identified to pick hoop, meridional, and shear stresses. Using the "get scalar data" command of ANSYS, stress values of all the layers are fetched.

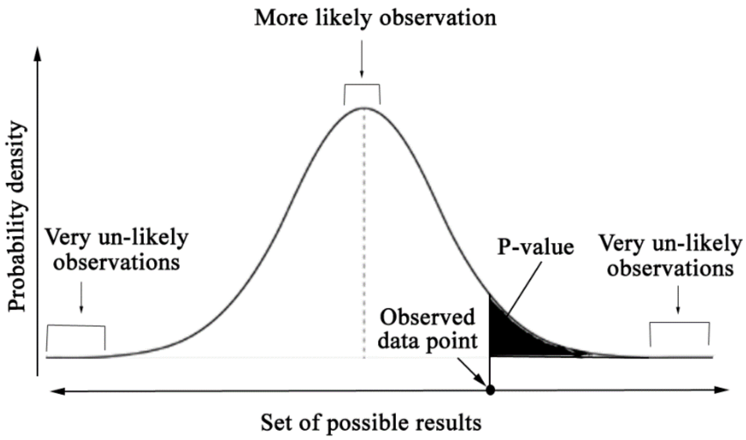

Figure 2. Significance of P-value.

Now, the ANSYS program stored as a text (.lgw) is utilized to make a macro file (.mac). Followed by this, the macro file is assigned to the ANSYS PDS section, and the mean and standard deviation (SD) of input parameters are specified as random inputs. Random output parameters, that is, hoop stress (σ_X), meridional stress (σ_Y) and the shear stress (τ_{XY}), are chosen. Now, the program is made to run the Monte Carlo Simulation routine with a specified number of simulations. Monte Carlo Simulation performs the statistical analysis by taking random values within the range of parameters that are considered to be uncertain. Multiples of simulations would be necessary depending on the uncertainties and number of variables. It paves way for a realistic understanding of the risk due to the variabilities. The analysis takes approximately 2 min/simulation. After the solution is done, numerical results are stored in a text file (.pdrs), and from the ANSYS PDS section, the reports are generated. The output values of layer-wise, iteration wise σ_X, σ_Y and τ_{XY} are transformed into the local coordinate system of fiber (σ_1, σ_2 and τ_{12}) using the matrix in equation (1). By comparing the stress values with the strength values (as demonstrated in the previous section), first ply failure pressure is recorded layer-wise. If the analysis is for 200 simulations, 200 values of failure pressure are available. These values are subjected to the Anderson Darling goodness of fit test in order to find the best distribution, mean, and SD of the first ply failure pressure.

STATISTICAL RESULTS AND INTERPRETATION

The random variables as found in Table 1 are analyzed to find the fittest distribution. Figure 3 shows the probability plots of the AD test for the input variable E_1. Out of 14 types of distributions inbuilt in the MINITAB software, based on previous experience, only four are considered. It may be noted that the highest P-value and the lowest AD value are associated with the Log-Normal distribution. Similar analyses are carried out for all the elastic properties. Table 5 summarizes AD statistics and P-values for all the 6 variables considered. Table 6 shows the names of the distributions, mean, COV, and SD of all the six variables. It may be noted that SD = (Mean x COV)/100. Though reasonably good solution convergence was achieved with a single loop of 100 simulations, another trial was carried out with a loop of 200 simulations. The results are presented only for the loop of 200 simulations. It is to be noted that Rafiee and Torabi (2018) carried out 100 simulations. Figure 4 shows the convergence phenomenon with respect to the number of simulations. It is comparable to the one presented by Rafiee and Torabi (2018).

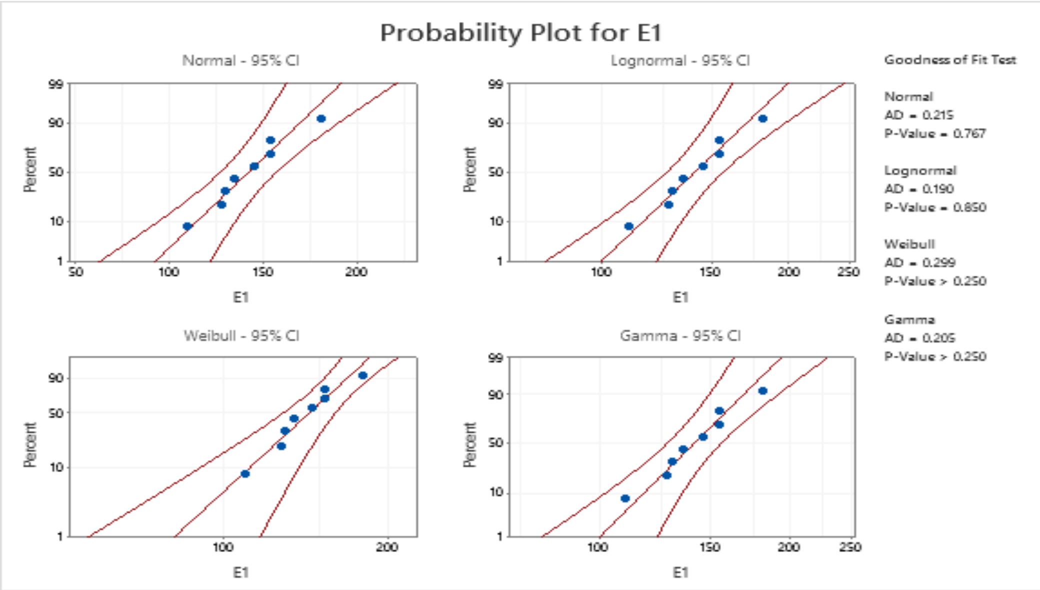

Figure 3. Probability plot for Young's modulus E_1.

Table 5. AD Statistic and P-value for T700 carbon/epoxy composite (closest fit shown in bold).

Properties	Statistic	Normal	Log Normal	Weibull	Gamma
E_1 (GPa)	AD	0.251	0.190	0.299	0.206
	P-Value	0.767	0.850	> 0.250	> 0.250
E_2 (GPa)	AD	0.640	0.678	0.760	0.741
	P-Value	0.069	0.047	0.038	0.055
G_{12} (GPa)	AD	0.196	0.209	0.239	0.224
	P-Value	0.823	0.774	> 0.250	> 0.250
G_{23} (GPa)	AD	0.258	0.374	0.263	0.362
	P-Value	0.590	0.306	> 0.250	> 0.250
υ_{12}	AD	0.544	0.512	0.640	0.562
	P-Value	0.110	0.133	0.060	0.164
υ_{23}	AD	0.454	0.477	0.554	0.526
	P-Value	0.194	0.167	0.140	0.196

Table 6. Statistical input parameters of T700 carbon/epoxy composite.

Attributes	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	G_{23} (GPa)	υ_{12}	υ_{23}
MEAN	142.18	9.099	5.049	3.877	0.2921	0.3987
COV	15.12	16.590	32.860	24.080	9.1100	23.7700
SD	21.50	1.509	1.659	0.934	0.0266	0.0948
Distribution	Log-Normal	Normal	Normal	normal	Log-Normal	Normal

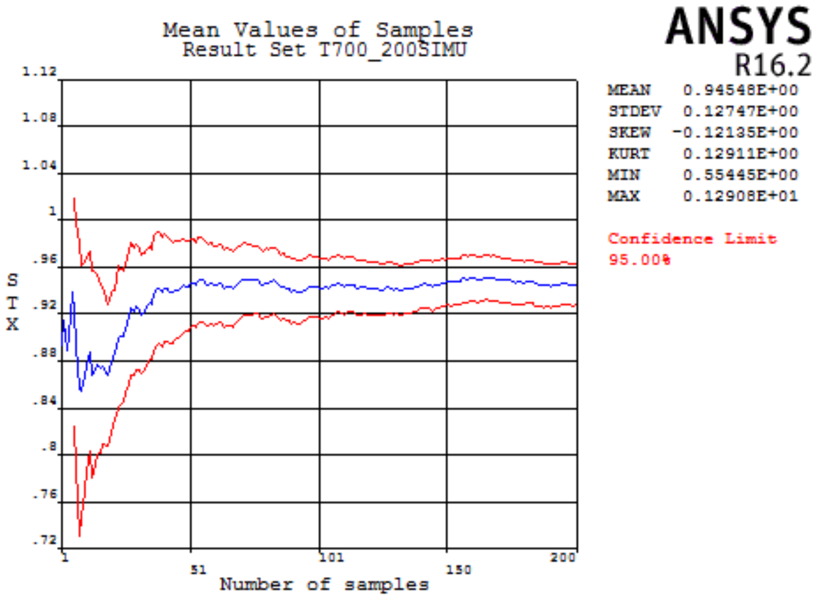

Figure 4. Solution convergence for hoop stress (σ_X shown as STX).

The statistical analysis gives interesting information on the significance of input parameters on the output parameters. Figure 5 displays the results of sensitivity analysis for the first failed ply. This ply is having fiber oriented at 18.6° with the cylinder axis, and hence, it is closer to the meridional direction. Hence, hoop stress is not sensitive to E_1, whereas it is highly sensitive to E_2. Figure 6 shows the histogram of the hoop stress and displays mean, SD, etc. All the 200 values of each of hoop stress, meridional stress, and shear stress belonging to all the eleven layers are transformed into the local coordinate system, and in comparison of these values with the respective strength values X_T, Y_T and S, 200, the values of first ply pressure are picked. These values are subjected to AD goodness of fit analysis using MINITAB software, and the statistical output is presented in Figures 7 and 8 and Table 7. From Figure 7, the best fit is identified as Log-Normal with AD=0.298 and P=0.586. The probabilistic first ply failure pressure obtained is of mean 45.47 MPa and SD 3.636 corresponding to the deterministic value of 43.71 MPa. In other words, the probabilistic first ply failure pressure is 45.47 $\pm$ 1.818 MPa. It may be compared with the work of Rafiee and Torabi (2018) as a prediction of mean 89.9 MPa with SD 2.65 MPa corresponding to the deterministic value of 87 MPa.

Table 7. Probability results of first ply failure pressure in MPa (AD=0.298 and P=0.586).

Count	Mean	COV	SD	Minimum	Maximum	Distribution
200	45.469	8.00	3.636	37.226	59.732	Log-Normal

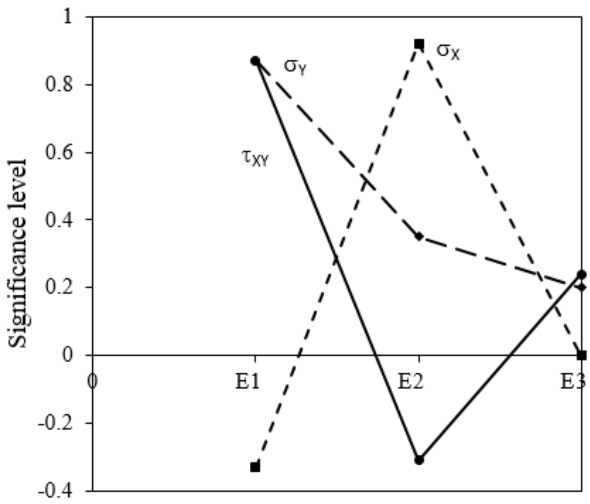

Figure 5. Significance level of Elastic properties on the computed stresses.

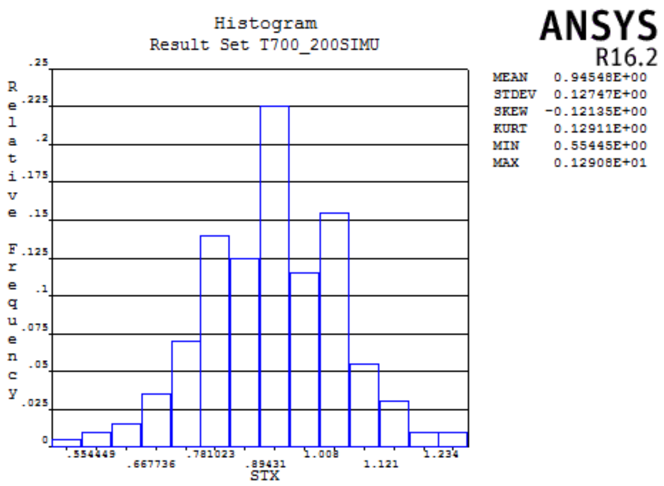

Figure 6. Typical interim result. Histogram for hoop stress (shown as STX).

Figure 8 clearly shows that the distribution of predicted first ply failure pressure through the proposed probabilistic analysis falls well above the distribution of deterministic failure pressure. This agreement is encouraging the use of elastic properties within the specified range with confidence. The predicted failure pressure range at 3σ level is shown in Figure 9 for the composite pressure vessels along with deterministic failure pressure for the input range of E_l values. It is clear from Figure 9 that 200 simulations of failure data generated fall within the predicted probabilistic first ply failure pressure range at 3σ limits. Further, using the MINITAB software, the correlation between global stresses and the first ply failure pressure has been examined.

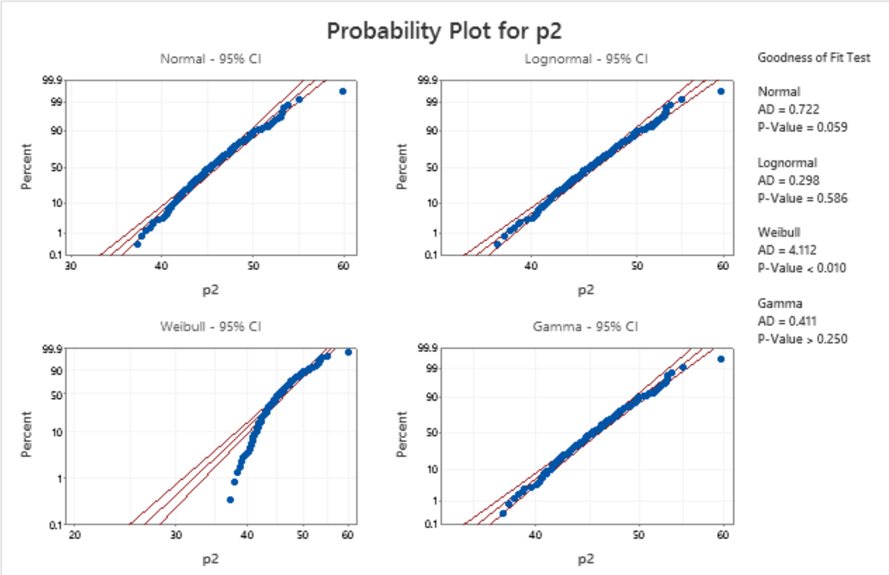

Figure 7. Probability plot for first ply failure pressure (shown as p2).

It is found that the hoop stress has a Pearson coefficient of -0.938 and a P-value of 0.000 with the first ply failure pressure. A P-value <0.05 is an indication that the correlation is significant, and a negative correlation means a decrease of first ply failure pressure with the increase of hoop stress. These observations are true because the hoop stress is the major cause of failure with the ply orientation almost along the meridional direction.

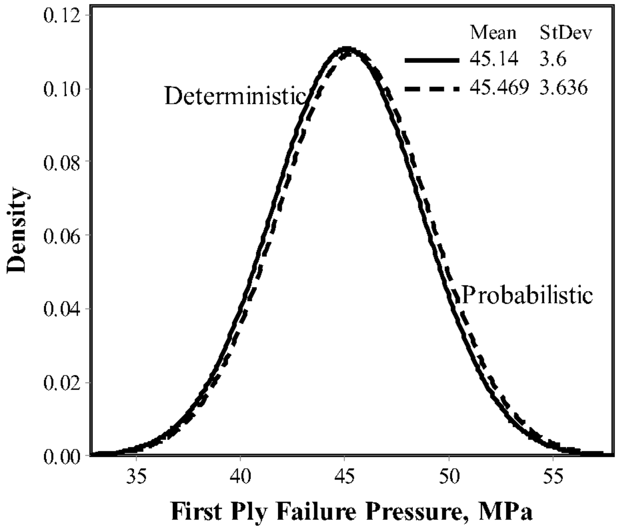

Figure 8. Probabilistic and deterministic distribution plot for failure pressure.

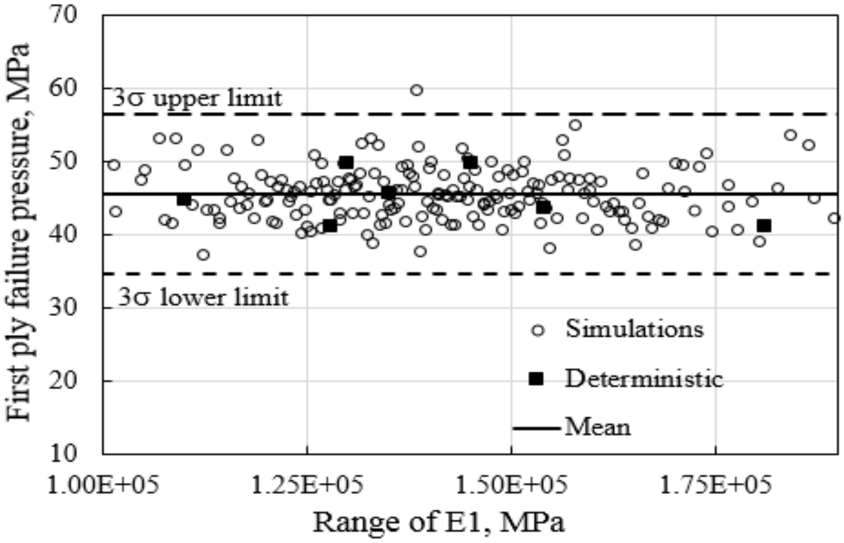

Figure 9. Comparison of deterministic and statistical results.

CONCLUSION

An attempt is made to handle the statistical variations in the input properties and their effect on the predictability of the first ply failure of the T700 carbon/epoxy composite pressure vessel. The intermediate results and their correlations have been shown to conform to the general understanding of the relationships between elastic properties, global stresses, and local stresses. The probabilistic first ply failure pressure is predicted as mean 45.47 MPa with an SD of 3.636 MPa that is close to the deterministic value of 43.71 MPa. Probabilistic predictions by Monte Carlo simulations have been found to be within 3σ limits with a confidence level of 99.997%. The agreement between the statistical results and deterministic results is very much encouraging as it indicates the usability of the properties within the range considered in this work. This analysis helps assess the factor of safety in the case of anticipated variations in the elastic properties.

REFERENCES

Dahl, E., Becker, J.S., Mittelstedt, C., & Schürmann, H. 2019. A new concept for a modular composite pressure vessel design. Composites Part A: Applied Science and Manufacturing 124: 105475.

Farhood, N.H., Karuppanan, S., Ya, H.H., & Ariff Baharom, M. 2017. Burst Pressure Investigation of Filament Wound Type IV Composite Pressure Vessel. AIP Conference Proceedings: 030017-1–030017-9.

Krishnaveni, A., Christopher, T., Jeyakumar, K., & Jebakani, D. 2014. Probabilistic Failure Prediction of High Strength Steel Rocket Motor Cases. Journal of Failure Analysis and Prevention 14: 478–490.

Lin, K.Y., Du, J., & Rusk, D. 2000. Structural Design Methodology Based on Concepts of Uncertainty. NASA, Washington.

Liu, P.F., Xing, L.J., & Zheng, J.Y. 2014. Failure analysis of carbon fiber / epoxy composite cylindrical laminates using explicit finite element method. Composites Part B 56: 54-61.

Rafiee, R. & Amini, A. 2015. Modeling and experimental evaluation of functional failure pressures in glass fiber reinforced polyester pipes. Computational Materials Science 96: 579-588.

Rafiee, R. & Torabi, M.A. 2018. Stochastic Prediction of Burst Pressure in Composite Pressure Vessels. Composite Structures 185: 573-583.

Rafiee, R., Rashedi, H., & Rezaee, S. 2020. Theoretical study of failure in composite pressure vessels subjected to low-velocity impact and internal pressure. Frontiers of Structural and Civil Engineering 14: 1349–1358.

Ramanjaneyulu, V., Murthy, V.B., Mohan, R.C., & Raju, C.N. 2018. Analysis of composite rocket motor case using finite element method. Materials Today: Proceedings 5(2): 4920-4929.

Reddy, S, S., Yuvraj, C., & Prahlada Rao, K. 2015. Design, Analysis, Fabrication and Testing of CFRP with CNF Composite Cylinder for Space Applications. International Journal of Composite Materials 5(5): 102-128.

Soden, P.D., Kaddour, A.S., & Hinton, M.J. 2004. Recommendations for designers and researchers resulting from the world-wide failure exercise. Composites Science and Technology 64: 589–604.

Vanaerschot, A., Lomov, S., Moens, D., & Vandepitte, D. 2015. Variability in Composite Materials Properties. Applied Mechanics and Materials 807: 23-33.

Wang, L., Wang, B., Wei, S., Hong, Y., & Zheng, C. 2016. Pred iction of long-term fatigue life of CFRP composite hydrogen storage vessel based on micromechanics of failure. Composites Part B 97: 274-281.

Wu, Q.G., Chen, X.D., Fan, Z.C., & Nie, D.F. 2015. Stress and Damage Analyses of Composite Overwrapped Pressure Vessel. Procedia Engineering. 130: 32-40.

Xiaobing, M. & Yongbo, Z. 2015. Life Prediction on a T700 Carbon Fiber Reinforced Cylinder with Limited Accelerated Life Testing Data. Mathematical Problems in Engineering 1: 1-9.

Xu, P., Zheng, J.Y., & Liu, P.F. 2009. Finite element analysis of burst pressure of composite hydrogen storage vessels. Materials and Design 30(7): 2295-2301.

Yingjun, W., Minqing, S., Zixiong, Z., & Sirong, Z. 2010. Finite Element Modeling of Carbon Fiber Reinforced Polymer Pressure Vessel. International Conference on Educational and Network Technology 259-262.