An improved firefly algorithm for identifying parameters of nonlinear empirical models

Xiaoxia Tian*,*, Chi Xiao* and Jingwen Yan****

** Computer and Information Engineering College, Hanshan Normal University, Chaozhou, China*
*** College of Engineering, Shantou University, Shantou, China*
***** Corresponding Author : txx@hstc.edu.cn*

Submitted : 28/03/2020
Revised : 23/11/2020
Accepted : 28/01/2021

ABSTRACT

The nonlinear model describes the vortex-induced resonance of long-span bridges under the action of natural wind. The identification accuracy of its parameters directly affects the understanding of vortex-induced vibration. Different algorithms have been used to solve this parameter identification problem, but their efficiency and accuracy are not satisfactory. In this work, a firefly algorithm based on local chaos search and brightness variant (FACLBV) was proposed. The characteristics of chaos made FACLBV search the widely local scope and improve the accuracy of the solution. FACLBV modified the fixed initial brightness, discarded the absorption coefficient of light intensity, linked the initial brightness of every firefly with the position of its solution space, and set the attraction of every firefly as a simple linear function, which reduced the complexity of the algorithm and improved its efficiency. In order to better verify the superiority of FACLBV, the simulation experiment included three parts: a comparison between FACLBV and other firefly algorithms, the verification of the parameters identified by FACLBV, and the nonparametric test between FACLBV and other intelligent algorithms. Simulation results showed that the performance of FACLBV is better than that of other algorithms.

Keywords: Firefly algorithm; Parameter identification; Nonlinear empirical models; Nonparametric test; Vortex-induced vibration

INTRODUCTION

For long-span bridges, vortex-induced vibration (VIV) is a kind of harmful vibration, which is a current research hotspot. Because of the complexity of fluid structure coupling, the VIV model is usually a nonlinear complex equation or system of equations (Marra et al., 2017; Xu et al., 2015; Zhu et al., 2017), and the parameter identification of the model is an optimization problem.

The grow-to-resonance method (GTR) (Ehsan & Scanlan, 1990) was initially used to identify parameters of Scanlan's nonlinear empirical model, but ubiquitous noise in wind tunnel tests had a very bad effect on the validity and accuracy of identified parameters. Marra et al. (2011) employed the fourth-order Runge–Kutta method to identify parameters of this model. With the given initial solution, parameters can be obtained by iterative calculations. This identification method has higher accuracy than GTR but has higher complexity. Wang et al. (2013) used Fourier transform to analyze the spectrum of wind tunnel test data, and obtained the amplitude and initial phase of each harmonic wave. With these amplitudes and initial phases as initial values, the parameters are identified by iterations. This method is also susceptible to noise and its accuracy is limited. Xiong et al. (2016) proposed an analytical identification method for parameters of the nonlinear self-excited aerodynamic system, which uses singular value decomposition (SVD) to determine the number of harmonics, and uses the non-linear fitting of the self-excited aerodynamic force to achieve the phase of the aerodynamic force. This method has strong anti-noise ability and high accuracy, but its calculation is complex. Zhu et al. (2013) identified parameters of their nonlinear model by the Levenberg–Marquardt algorithm (LMA). LMA belongs to a local search method, and the setting of its initial solution needs prior knowledge. Based on suitable initial solutions, LMA tries to minimize the error between the measured and reconstructed force data by iterations, and then the approximate solution can be obtained. LMA repeatedly calculates the inverse of the Hessian matrix in iterations, but it has high complexity.

Swarm intelligence algorithms have good optimization and rapid convergence, and especially they do not need to set initial solutions, so they are often used to solve many complex problems. In terms of identifying parameters of nonlinear models, many good intelligence algorithms have been proposed. These algorithms are divided into two categories: the hybrid algorithm and the improved algorithm. GALMA is a hybrid algorithm, which uses GA to exploit the solution space and LMA to explore the solution (Tian et al., 2017). NAPSO (Tian et al., 2018) and ISA (Tian et al., 2019) are the improved algorithms, which improve performance by adjusting an optimization strategy. NAPSO is an improved particle swarm optimization, which adaptively adjusts its search ability by the inertia weight function. ISA magnifies the energy function of simulated annealing 10^6-fold in order to ensure rapid convergence. These intelligence algorithms are better than the traditional methods because of their intelligence. Seeking a novel identification intelligent algorithm has become a research focus in recent years.

The firefly algorithm (FA), proposed by Yang (2010), is a new swarm intelligence optimization algorithm. The algorithm imitates the attractive characteristics of natural fireflies. It simulates the optimization process by attracting and moving between firefly individuals, but sometimes it will fall into the local minimum. In order to improve its performance, the standard FA has been improved.

In this work, an improved firefly algorithm is proposed to identify parameters of a nonlinear VIV model. The algorithm uses a chaotic operator to search locally, establishes adaptive brightness, and simplifies the calculation of attraction in order to improve the convergence and efficiency.

RELATED WORK

VIV Models of Long-span Bridges

The VIV of long-span bridges is a complex physical phenomenon. Many researchers tried to build a pure theoretical model to describe the VIV phenomenon, but such a model has not yet been proposed. Current research mainly focuses on modeling of wind tunnel tests or field measurements.

Scanlan (Scanlan, 1981) proposed a single-degree-of-freedom model to describe the effect of vortex shedding on the bridge deck. Due to fluid-structure coupling, the vortex-induced vertical force (VIVF) includes many nonlinear components. The model is simple linear so that it failed to describe the complex VIV phenomenon, especially lock-in, self-exciting and self-limited characteristics. In 1990, the linear model was improved by adding a nonlinear component describing the nonlinear damping force, the nonlinear empirical model is as follows (Ehsan & Scanlan, 1990).

$$f_{VI}=\frac{1}{2}\rho U^2(2D)\left[Y_1(1-\varepsilon(K)\frac{y^2}{D^2})\frac{\dot{y}}{U}+Y_2(K)\frac{y}{D}+\frac{1}{2}C_L(K)\sin(\omega_s t+\phi)\right] \tag{1}$$

where f_{VI} is the force per unit length, ρ is the air density, U is the velocity of the oncoming flow, D is the characteristic size of the structure section, ε is the nonlinear damping, ω_s is the circular oscillation frequency of vortex shedding, ϕ is the phase angle, y and $\dot{y}$ are the displacement and velocity of the vertical motion, respectively, K is the reduced frequency, Y_1 , Y_2 , and ε are the reduced frequency-depended coefficients, and t denotes time.

To fit the spectrum of test data, Zhu et al.(2013) added a new quadratic term to Scanlan's nonlinear model. In addition, they rewrote the nonlinear damping term. Zhu's nonlinear model is as follows.

$$f_{VI}=\frac{1}{2}\rho U^2(2D)\left[Y_1(1-\varepsilon(K)\frac{\dot{y}^2}{U^2})\frac{\dot{y}}{U}+Y_2(K)\frac{y}{D}+Y_3(K)\frac{y}{D}\frac{\dot{y}}{U}+\frac{1}{2}C_L(K)\sin(\omega_s t+\phi)\right] \tag{2}$$

where Y_3 is the new term, and $\frac{y^2}{D^2}$ is changed to $\frac{\dot{y}^2}{U^2}$.

The dimensionless form of Eq.(2) can be written as follows.

$$F_{VI}=m_r[Y_1(1-\varepsilon\eta'^2)\eta'+Y_2\eta+Y_3\eta\eta'+\frac{1}{2}C_L\sin(\omega_{vs}s+\psi)] \tag{3}$$

where $m_r=\rho D^2/m$, m is the mass per unit span, $\eta=y/D, s=tU/D$, $\omega_{VS}=\omega_s D/U$.

Let $x=(Y_1,\varepsilon,Y_2,Y_3,C_L,\omega_s,\psi)$, and based on the nonlinear fitting, the objective function of Eq.(3) can be

expressed

$$obj(x)=\left\|\tilde{F}_{VI}-\hat{F}_{VI}\right\|_2^2 \tag{4}$$

where $\left\|\bullet\right\|_2^2$ denotes the square of the L_2 norm, $\tilde{F}_{VI}$ indicates the measured VIVF, and $\hat{F}_{VI}$ indicates the VIVF reconstructed by Eq.(3).

For optimization problems, the equation has a minimum solution when the gradient is equal to zero. As far as Zhu's model is concerned, the main task is to find x to minimize Eq.(4). The parameters can be accurately identified by the following equation.

$$x^*=\arg\min_x obj(x) \tag{5}$$

Standard Firefly Algorithm and its Improvements

In FA(Yang, 2010) , fireflies search for and move to brighter partners in the solution space to realize the evolution. FA has three assumptions: 1) every firefly is unisex, 2) the attractiveness of every firefly is proportion to its brightness, and 3) the brightness of every firefly is determined by the value of the objective function for the specific problem. Therefore, firefly i will be attracted to the brighter firefly j , and the brightness is as follows:

$$\beta = \beta_0 e^{-\gamma d^2} \tag{6}$$

where β_0 is the initial brightness of the attracting firefly, usually $\beta_0 = 1$, γ is the light absorption coefficient, d is the distance between attracted firefly i and attracting firefly j . Therefore, the attracted firefly will move to the attracting firefly according to the following equation.

$$x_i = x_i + \beta(x_j - x_i) + \alpha(r - 0.5) \tag{7}$$

where x_i and x_j are the positions of firefly i and firefly j , respectively, r is a random number in [0,1], and α is the scale of the local search.

FA has the advantages of less adjustable parameters, simple structure and fast convergence. However, there are some shortcomings, such as a slow convergence rate, easy stagnation, and premature convergence.

Although FA has been applied in many fields, its optimization ability mainly depends on the inter-attraction between firefly individuals. Once the best firefly is captured by the local extremum, it will be difficult to get rid of. Especially in the early stage of evolution, the best firefly in the population will attract other fireflies to quickly approach it, which will greatly reduce the diversity of the population. At the later stage of FA, most fireflies gather near the optimal value, and the convergence rate of the population is too slow or even stagnates.

In order to enhance the performance of FA, researchers have improved FA, such as parameter change, strategy level change, and integration of other intelligent algorithms. The parameter α has a great effect on the local search ability of FA. If α is too large, it jumps out of the neighborhood space of the current solution, which makes FA unstable. If α is small, the search area is small, resulting in the inefficiency of FA. In order to make α in a reasonable range, researchers mainly focused on that α is viewed as a function related to the iteration, and this function maybe linear (Liu et al., 2015; Yan et al., 2012), non-linear (Baghlani et al., 2013; Shafaati & Mojallali, 2012; Yang, 2013; Wang et al., 2012), or chaotic operators (Coelho et al., 2011; Feng et al., 2013). The parameter β is the attraction step of firefly i moving towards firefly j . If $\beta = 0$, it does not move, if $0 < \beta < 1$, it moves to firefly j at β , if $\beta > 1$, it moves over firefly j in the direction of firefly i to firefly j . β is generally set as different functions related to the distance, and sometimes it is regarded as the chaotic function. The parameter γ is the absorption factor, which affects the value of β . γ is also set as various functions (Łukasik & Żak, 2009). Cheung et al.(2014) replaced the constant γ with a distance-based adaptive coefficient, designed a new adaptive coefficient based on gray relational analysis, and developed an adaptive firefly algorithm. In addition, Tilahun and Hong (2012) changed the moving direction of fireflies which is the best for the current optimal solution. Fu et al. (2015) used the strategy to generate a perturbation solution around the current optimal solution. Kazemzadeh-Parsi (2014) replaced the several worst solutions with the several random generated solutions. Wang and Chu (2019) first chose better fireflies as a new set, and then selected fireflies from the set to move with a certain probability strategy. Thus, every improved FA is the best method for a certain problem.

THE FIREFLY ALGORITHM BASED CHAOTIC LOCAL SEARCH AND BRIGHTNESS VARIANT (FACLBV)

Variant of the Brightness of Each Firefly

The assumption $\beta_0 = 1$ in FA does not reflect the actual situation. Since the location of fireflies directly affects their brightness and attractiveness, β_0 should be set to a function related to their location and the function is as follows.

$$\beta_0(x_i) = obj(x_i) \tag{8}$$

In other firefly algorithms, the attraction of the ith firefly to the jth firefly is $\beta = \beta_0 e^{-\gamma r_{ij}}$. Its role is to reduce the attraction of fireflies due to the increase of distance. This expression needs exponential operation, which makes the algorithm highly complex.

The initial brightness of every firefly is known, but in the process of optimization, the attractiveness of each firefly varies with its position in the solution space. Therefore, the attraction of each firefly will be improved.

$$\beta(x_i) = \frac{\beta_0(x_i) - \min\{obj(x)\}}{\max\{obj(x)\} - \min\{obj(x)\}} \tag{9}$$

The term $\min\{obj(x)\}$ is the minimum objective value in fireflies, $\max\{obj(x)\}$ is the maximum objective value. According to Eq.(9), the better the location of the firefly is, the brighter it is, and the shorter its step will be. The best firefly randomly moves in its neighborhood.

Local Search Based on Logistic Chaos

Chao optimization, a novel optimization technology, has been widely used in optimization problems in recent years. Chaos is the unique phenomenon of non-periodic motion in non-linear systems. It has an exquisite intrinsic regular chaotic motion. It has ergodicity, randomness and regularity. The chaotic variables can traverse all states without repetition, which effectively avoids premature falling into a local optimum and improves the ability of global optimization. These characteristics are used to improve the efficiency of the stochastic optimization algorithm, enhance the optimization ability of the algorithm, and ensure the stability of the algorithm. In this work, the logistic chaotic operator was used.

$$c_t = \theta c_{t-1}(1 - c_{t-1}) \tag{10}$$

where c_t is the chaotic operators of the t iteration, $\theta = 4$, the initial chaotic operator is between 0 and 1, and 0.25, 0.5, and 0.75 are expected.

In addition, the local search range decreases with the increase of iteration times. So, the scale factor is written as

$$\alpha = 0.99^t \alpha_0 \tag{11}$$

Therefore, the location of fireflies is updated by the following equation:

$$x_i^{t+1} = x_i^t + \beta(x_i - x_j) + ac_t \tag{12}$$

FACLBV Algorithm

The algorithmic pseudo-code of FACLBV is as follows:

Input: $\theta = 4$, c_0

Output: x_{best}

(1) Randomly generate an initial firefly population $\{x_i^0 \mid i = 1,\ldots,N\}$ and calculate $\{\beta_0(x_i^0) \mid i = 1,\ldots,N\}$.

(2) Calculate $\{I(x_i^0) \mid i = 1,\ldots,N\}$ and let $\beta(x_i^0) = I(x_i^0)$

(3) $\alpha_0 = 3$

(4) For t=1:Maxiter

{

Generate a new population $\{z_i \mid i = 1,\ldots,N\}$ and calculate $\{obj(z_i) \mid i = 1,\ldots,N\}$

Compute the current chaotic operator c_t

For i=1:N

For j=1:N

Calculate $\beta(x_i^t)$

If $obj(x_i^{t-1}) > obj(x_j^{t-1})$

Calculate x_i^t

Generate a temporary firefly $tmpfly$ location x_i^t and compute $obj(tmpfly)$

If $obj(tmpfly) < obj(z_i)$ then $z_i \leftarrow tmpfly$

If $obj(z_i) < obj(x_{best})$ then $x_{best} \leftarrow z_i$

Select the N best fireflies from the new population $\{z_i \mid i = 1,\ldots,N\}$ and the current population $\{x_i^t \mid i = 1,\ldots,N\}$ to form the next generation population

Compute α according to Eq.(11)

}

(5) Output x_{best}

SIMULATION RESULTS

This selection includes three parts: one is the comparison of FACLBV with other FAs, the second is the verification of FACLBV validity, and the is the nonparametric test for FACLBV and different optimization algorithms.

The Comparison of FACLBV With Other FAs

In this work, based on wind tunnel test data and Zhu's nonlinear model, FACLBV was compared with other six algorithms in performance. In Table 1, those algorithms are described in detail. For FAN, in each evolution, in addition to the contemporary population, a new population was randomly generated. After evolution, the best N fireflies were selected from the contemporary and new populations formed the next generation population. In FACP, the chaotic operator was used to generate the initial population. FACL means the chaotic operator was used in the local search. For FABV, the initial brightness of each firefly relates to its own position. In FAGV, the parameter γ adaptively varies with the firefly's position.

Table 1. Description of different improved algorithms.

Algorithm	Description
FAN	Multipopulation strategy, α_0=3 and $\alpha = 0.99^t \alpha_0$, $\beta_0 = 1, \gamma = 1$
FACP	The initial population based the chaotic strategy, α_0=3 and $\alpha = 0.99^t \alpha_0$, $\beta_0 = 1$, $\gamma = 1$
FACL	c_0=0.6 and $c_t = 4c_{t-1}(1-c_{t-1})$, α_0=3 and $\alpha = 0.99^t \alpha_0, \beta_0 = 1$, $\gamma = 1$
FABV	α_0=3 and $\alpha = 0.99^t \alpha_0$, $\gamma = 1$, $\beta = (obj(x_i) - \min\{obj(x)\})/(\max\{obj(x)\} - \min\{obj(x)\})$,
FAGV	α_0=3 and $\alpha = 0.99^t \alpha_0$, $\beta_0 = 1$, $\beta_0(x_i) = obj(x_i)$, $\gamma = (\beta_0(x_i) - \min\{obj(x)\})/(\max\{obj(x)\} - \min\{obj(x)\})$
FACLBV	Multipopulation strategy, α_0=3 and $\alpha = 0.99^t \alpha_0$, $\beta_0(x_i) = obj(x_i)$, $\gamma = 1$, c_0=0.6 and $c_t = 4c_{t-1}(1-c_{t-1})$, $\beta = (\beta_0(x_i) - \min\{obj(x)\})/(\max\{obj(x)\} - \min\{obj(x)\})$

For the above algorithms, a population had ten fireflies, the maximum number of iterations was 30, and the number of runs was 20. Table 2 shows the running results of different algorithms, μ denotes the mean, and SD denotes the standard deviation.

Table 2. Simulation results ($Time$: the CPU time).

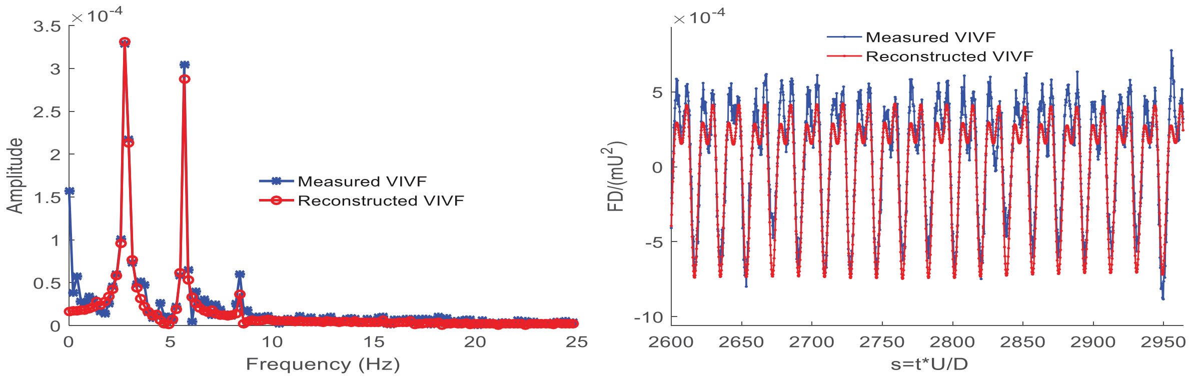
Algorithm		μ	SD	**Max**	**Min**
FA	obj	1.1921E-04	1.9393E-06	1.2346E-04	1.1633E-04
	$Time$	29.3373	0.9466	31.7766	27.8206
FAN	obj	1.1791E-04	1.3335E-06	1.2018E-04	1.1526E-04
	$Time$	29.8554	1.6798	32.8126	26.8312
FACP	obj	1.1923E-04	1.9201E-06	1.2535E-04	1.1639E-04
	$Time$	26.1583	1.8900	31.7789	22.8484
FACL	obj	1.1530E-04	6.8478E-07	1.1655E-04	1.1440E-04
	$Time$	28.6282	0.8469	30.0058	26.73
FAGV	obj	1.1841E-04	1.5327E-06	1.2110E-04	1.1596E-04
	$Time$	26.0908	1.4629	28.758	23.0305
FABV	obj	1.1829E-04	2.1026E-06	1.2224E-04	1.1523E-04
	$Time$	27.9779	2.6700	32.8406	24.9617
FACLBV	obj	1.1470E-04	5.8561E-07	1.1647E-04	1.1392E-04
	$Time$	24.4298	2.7951	27.9535	18.0469

Table 2 shows the simulation results of twenty runs for different algorithms. It can be seen that the logistic operator employed in local searching makes FACL and FACLBV better than those of the other algorithms in the objective value, while the logistic operator employed in the initial population makes the objective value of FACP the worst. No matter the objective value or the running time, the adaptive strategy makes FACLBV, FABV, and FAGV better than FA. The multi-population strategy improves the objective value of FAN, but it increases its running time. Hence, FACLBV is the best.

The Verification of Validity of Identified Parameters

Parameters of Zhu's model were identified by FACLBV, the validity and accuracy of parameters were used to assess FACLBV's performance.

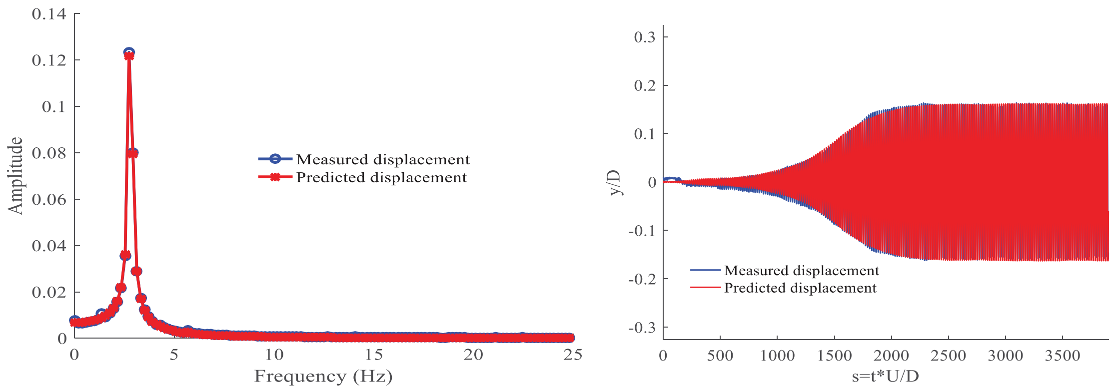
First, the time history of $\hat{F}_{VI}$ was reconstructed by Eq.(3) with identified parameters, $\hat{F}_{VI}$ was compared with $\tilde{F}_{VI}$. Figure 1 shows the comparison of frequency and time domains. According to Figure 1(a), $\hat{F}_{VI}$ has the same frequency components as $\tilde{F}_{VI}$. In Figure 1(b) the curve of $\hat{F}_{VI}$ is similar to that of $\tilde{F}_{VI}$.



(a) **(b)**

Figure 1. Comparison of VIVFs. (a) Frequency domain. (b) Time domain.

With $\hat{F}_{VI}$, the Newmark-β was used to predict the time history of vertical displacement. Fig 2 shows the measured displacement compared with the predicted displacement. Figure 2(a) shows that both of the measured and predicted displacements had the same spectrum. In Figure 2(b), the envelope of the predicted displacement well fitted that of the measured displacement, particularly at the maximum displacement. The validity and accuracy of identified parameters were verified from VIVFs and displacements. Therefore, FACLBV is valid.



(a) **(b)**

Figure 2. Comparison of displacements. (a) Frequency domain. (b) Time domain.

The Nonparametric Test of Different Algorithms (García et al., 2009)

The nonparametric test was used to evaluate the performance of the above FAs. The fit coefficient R^2 was used to evaluate the algorithm performance, according to $\hat{F}_{VI}$ and $\tilde{F}_{VI}$.

Using FACLBV as the key, the Wilcoxon test was used to evaluate the performance of different algorithms. $\mathbf{W^+}$ is given by the sum of all of the positive ranks, $\mathbf{W^-}$ is given by the sum of all of the negative ranks. **R** denotes that the null hypothesis is rejected, **A** is that the null hypothesis cannot be rejected.

Tables 3 and 4 are results of the nonparametric test for different FAs. In Table 3, FA, FACP, and FAN have the same W^-, and it is zero. FABV is one, and FAGV is two. Therefore, their asymptotic significances (Asymp.Sig) are zero, which means they have significant differences compared with FACLBV at all levels. While the W^- of FACL is 75, and W^+ is 135, FACL and FACLBV have no significant differences at different levels. Table 4 shows the mean rank of different FAs, the Chi-Square is 70.877 and the Asymp.Sig is zero. FACLBV has the highest value, and FACP has the lowest value. Therefore, it can be seen from Tables 3 and 4 that FACLBV is the best.

Table 3. Wilcoxon signed ranks test for FAs.

Algorithm	W^+	W^-	a=0.01	a=0.02	a=0.05	a=0.1
FA	210	0	**R**	**R**	**R**	**R**
FACP	210	0	**R**	**R**	**R**	**R**
FABV	209	1	**R**	**R**	**R**	**R**
FAGV	208	2	**R**	**R**	**R**	**R**
FAN	210	0	**R**	**R**	**R**	**R**
FACL	135	75	**A**	**A**	**A**	**A**

Table 4. Friedman test for FAs.

Algorithm	Mean Rank
FACLBV	6.60
FACL	6.15
FAN	3.60
FAGV	3.23
FABV	3.05
FA	2.73
FACP	2.65

GALMA, NAPSO, ISA, and FACLBV also were compared by the nonparametric test. With the same data and simulation environment, they were used to identify parameters of Zhu's nonlinear model, R^2 is employed to evaluate their performance.

In Table 5, FACLBV is superior to GALMA at different levels. For GALMA, the Asymp.Sig is zero, FACLBV and GALMA have significant difference at all levels. The Asymp.Sig of NAPSO is 0.037, FACLBV and NAPSO have no significant differences at different levels except for a=0.01 and a=0.02. While FACLBV and ISA have no significant difference at all levels.

Table 5. Wilcoxon signed ranks test.

Algorithm	W^+	W^-	a=0.01	a=0.02	a=0.05	a=0.1
GALMA	55	0	**R**	**R**	**R**	**R**
NAPSO	48	7	**A**	**A**	**R**	**R**
ISA	35	20	**A**	**A**	**A**	**A**

Table 6 shows the results of Friedman test for the four algorithms. The Chi-square is 16.2 and the Asymp.Sig is 0.001. FACLBV has the maximum mean rank, while GALMA has the minimum mean rank. Therefore, FACLBV is the best, followed by ISA.

Table 6. Friedman test for the four algorithms.

Algorithm	Mean Rank
FACLBV	3.4
ISA	3.1
NAPSO	2.2
GALMA	1.3

CONCLUSION

In the work, FACLBV was proposed to identify parameters of Zhu's nonlinear model. FACLBV increased the convergence rate to obtain the global optimal. FACLBV used the chaotic operator to escape from the local optimal for its particularly inherent randomness and ergodicity. FACLBV linked the initial brightness of each firefly to its position, rather than setting it to constant 1. In addition, to enhance the efficiency of FACLBV, the parameter β was simplified by overlooking the light absorption parameter. The simulation results demonstrated that FACLBV can quickly and effectively obtain the global or near-global optima compared with the other mentioned algorithms.

In the field of civil engineering, optimization is an attractive topic, and the efficient optimization should be developed depending on the analysis of the specific engineering problem. Further work is to apply FACLBV and develop a metahybrid algorithm to solve the optimization problems of the anti-wind study for long-span bridges.

ACKNOWLEDGEMENT

The project was supported by Natural Science Foundation of Guangdong Province(Grant Nos. 2020A151501672), Guangdong Education Department's Innovative and Strong School Project (Grant Nos. 2017KTSCX121), Doctor start up fund (Grant No.: QD20190916), and School level project of Hanshan Normal University (Grant No.: 2018GY20).Thanks for the data provided by Dr. Xiaoliang Meng. Thanks to Dr. Edward C. Mignot, Shandong University, for linguistic advice.

CONFLICTS OF INTEREST

The authors declare no conflict of interest.

REFERENCES

Baghlani, A., Makiabadi, M.H., &Rahnema, H. 2013. A new accelerated firefly algorithm for size optimization of truss structures. Scientia Iranica. 20(6):1612-1625.

Cheung, N. J. , Ding, X. M. , & Shen, H. B. 2014. Adaptive firefly algorithm: parameter analysis and its application. Plos One 9(11):e112634.

Coelho, L.D.S., Bernert, D.L.D.A., &Mariani, V.C. 2011. A chaotic firefly algorithm applied to reliability-redundancy optimization. Proceedings of the IEEE Congress on Evolutionary Computation. New Orleans, LA, USA.

Ehsan, F., &Scanlan, R.H. 1990. Vortex-induced vibrations of flexible bridges. Journal of Engineering Mechanics. 116(6):1392-1411.

Feng Yanhong, Liu Jianqin, & HE Yichao. 2013. Chaos-based dynamic population firefly algorithm. Journal of Computer Applications. 33(3): 796-799.

Fu, Q., Liu, Z., Tong, N., Wang, M., &Zhao, Y. 2015. A novel firefly algorithm based on improved learning mechanism, International Conference on Logistics Engineering, Management and Computer Science. Shenyang, Liaoning, China.(pp. 1343-1351).

García, S., Molina, D., Lozano, M., &Herrera, F. 2009. A study on the use of non-parametric tests for analyzing the evolutionary algorithms' behaviour: a case study on the CEC'2005 Special Session on Real Parameter Optimization. Journal of Heuristics. 15(5):617-644.

Kazemzadeh-Parsi, M.J. 2014. A modified firefly algorithm for engineering design optimization problems. Iranian Journal of Science & Technology Transactions of Mechanical Engineering. 38(M2):403-421.

Liu, C., Zhao, Y., Gao, F., &Liu, L. 2015. Three-dimensional path planning method for autonomous underwater vehicle based on modified firefly algorithm. Mathematical Problems in Engineering. 2015:1-10.

Łukasik, S., &Żak, S. 2009. Firefly algorithm for continuous constrained cptimization tasks. Lecture Notes in Computer Science 5796:97-106.

Marra, A.M., Mannini, C., &Bartoli, G. 2011. Van der Pol-type equation for modeling vortex-induced oscillations of bridge decks. Journal of Wind Engineering & Industrial Aerodynamics. 99(6-7):776-785.

Marra, A.M., Mannini, C., &Bartoli, G. 2017. Wind tunnel modeling for the vortex-induced vibrations of a yawed bridge tower. Journal of Bridge Engineering. 22(5):04017006.

Scanlan, R.H., 1981. State-of-the-art methods for calculating flutter, vortex-induced, and buffeting response of bridge structures. Logits, Federal Highway Administration, Washington, DC.

Shafaati, M., &Mojallali, H. 2012. Modified firefly optimization for IIR system identification. control engineering & applied informatics.14(4):59-69.

Tian, X., Yan, J., &Xiao, C. 2018. Parameter identification of the vortex-induced vertical force model using a new adaptive pso. Mathematical Problems in Engineering. 2018(PT.14):1-9.

Tian, X., Yan, J., Yang, Y., Xiao, C., &Zhou, Q. 2019. Parameter identification of a nonlinear model using an improved version of simulated annealing. International Journal of Distributed Sensor Networks. 15(2): 1550147719832788.

Tian, X., Yan, J., &Zhou, Q. 2017. A novel method of parameter identification based on nonlinear empirical model for vortex- induced vibration. Journal of Engineering Research. 5 (4): 44-58.

Tilahun, S.L., &Hong, C.O. 2012. Modified firefly algorithm. Journal of Applied Mathematics. 2012(2012):2428-2439.

Wang, C., & Chu, X. 2019. An Improved Firefly Algorithm With Specific Probability and Its Engineering Application. IEEE ACCESS.

Wang, G., Guo, L., Hong, D., Luo, L., &Wang, H. 2012. A modified firefly algorithm for UCAV path planning. International Journal of Hybrid Information Technology 5.

Wang ,Q.， Liao, H.， Li, M.， &Ma, C. 2013. Empirical mathematical model for nonlinear motion-induced aerodynamic force of bridge girder. Journal of Southwest Jiaotong University. 48(2), 271-277.

Xiong, L., Liao, H.,Wang, Q., &Ma, C. 2016. Analytic identification of bridge nonlinear motion-induced aerodynamic parameter. Journal of Southwest Jiaotong University. 51: 824-831.

Xu, K., Ge, Y., &Zhang, D. 2015. Wake oscillator model for assessment of vortex-induced vibration of flexible structures under wind action. Journal of Wind Engineering & Industrial Aerodynamics. 136:192-200.

Yan, X., Zhu, Y., Wu, J., &Chen, H. 2012. An improved firefly algorithm with adaptive strategies. Advanced Science Letters. 16(1), 249-254.

Yang, X.S., 2010. Firefly algorithms for multimodal optimization. Mathematics. 5792, 169-178.

Yang, X.S., 2013. Multiobjective firefly algorithm for continuous optimization. Engineering with Computers. 29(2), 175-184.

Zhu, L.D., Meng, X.L., Du, L.Q., &Ding, M.C. 2017. A simplified nonlinear model of vertical vortex-induced force on box decks for predicting stable amplitudes of vortex-induced vibrations. Engineering. 3(6):854-862. DOI: 10.1016/j.eng.2017.06.001.

Zhu, L.D., Meng, X.L., Guo, &Z.S. 2013. Nonlinear mathematical model of vortex-induced vertical force on a flat closed-box bridge deck. Journal of Wind Engineering & Industrial Aerodynamics. 122:69-82. DOI: 10.1016/j.jweia.2013.07.008.